where the summation is taken over all possible cyclic permutations of the arguments $y_1, \ldots, y_n$, and ℓ is the number of interchanges necessary to go from $y_j \ldots y_n y_1 \ldots y_{j-1}$ to $y_1 \ldots y_n$.

We seek the solution of Eq. (1) in the form

$$G_n(x_1, \ldots, x_n;\ y_1, \ldots, y_n \mid A) = -\sum_P (-1)^l [G_1(x_1, y_j \mid A)\, G_{n-1}(x_2, \ldots, x_n;\ y_{j+1}, \ldots, y_n, y_1, \ldots, y_{j-1} \mid A)$$

$$+ G_{1n}(x_1, \ldots, x_n;\ y_j, \ldots, y_n, y_1, \ldots, y_{j-1} \mid A)]. \tag{2}$$

The equation for G_{1n} in the momentum representation can be solved exactly by a method used previously.[3] We get as the result

$$G_{1n}(p_1, \ldots, p_{2n-1} \mid A) = -\sqrt{4\pi}\frac{eu^{\mu_1}}{(2\pi)^2}\int_0^\infty d\nu e^{-\varepsilon\nu} \exp\left\{-i\left[m - \sum_{i=1}^{2n-1}(up_i)\right]\nu + f(\nu)\right\} \exp\left\{-\sqrt{4\pi}\frac{eu^\mu}{(2\pi)^2}\int \frac{e^{-i(up)\nu} - 1}{(up)} A_\mu(p)\, dp\right\}$$

$$\times \int D_{\mu_1\nu_1}(k) \left\{G_1(p_2 \mid A)\left[\frac{\delta G_{n-1}(p_3, \ldots, p_{2n-1} \mid A)}{\delta A_{\nu_1}(k)}\right]_{A \to T_x A}\right\}_{A \to A^{T_\nu}} \exp\left\{i\left(k + p_2 - \sum_{i=1}^{2n-1} p_i\right)x\right\} dx\, dk. \tag{3}$$

It follows that the probability for the emission of n low-energy photons is given by the Poisson formula

$$w_n = e^{-W}\frac{W^n}{n!}, \quad W = \frac{2e^2}{\pi}\ln\frac{E_2}{E_1}, \tag{4}$$

where the energy of the photons lies between E_1 and E_2.

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ON THE SPEED OF PROPAGATION OF ELECTROMAGNETIC WAVES AT AUDIO FREQUENCIES

Ia. L. AL'PERT and S. V. BORODINA

Institute of Terrestrial Magnetism, the Ionosphere, and Radio-Wave Propagation

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THE speed of electromagnetic waves has been investigated experimentally in various researches, over a frequency range approximately from 10^{15} cps (optical waves) to 10^6 or 10^5 cps (so-called medium radio waves). Recently we have determined the speed of electromagnetic waves under natural conditions at lower, audio frequencies, namely from about 3×10^4 to 10^3 cps; in addition, the investigations are about to be extended down to frequencies of some tens of cycles per second.

The method used in our work was that of complete harmonic analysis of photo-oscillograms of individual atmospheric events $E(t, r)$, taken at various distances from their sources (thunderstorm discharges).[1,2] The distances r from the sources were determined by means of three direction-finders. The method consists in the following.

The spectral density of the signal $E(t, r)$ can be written in the form

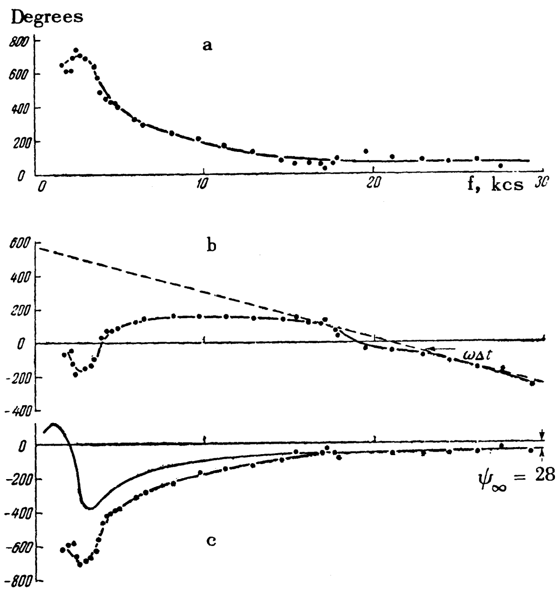

FIG. 1

$$Q(\omega, r) = A(\omega, r)\, e^{i\varphi(\omega, r)} = \int_0^\infty E(t, r)\, e^{-i\omega t}\, dt, \tag{1}$$

where $A(\omega, r)$ and $\varphi(\omega, r)$ are determined by harmonic analysis of $E(t, r)$. If the radiated signal $E(t, r)$ is assigned the "standardized" spectral density $A(\omega, 0) \times e^{i\varphi(\omega, 0)}$, then from (1) one immediately obtains the propagation function

$$E(\omega, r)\, e^{-i\Phi(\omega, r)} = \frac{A(\omega, r)}{A(\omega, 0)}\, e^{i[\varphi(\omega, r) - \varphi(\omega, 0)]}, \tag{2}$$

which describes the variation of the amplitude and phase of the received spectrum of the wave. The complete phase is

$$\Phi(\omega, r) = \omega \int_0^r \frac{dr}{v(\omega, r)} = \frac{\omega r}{\bar{v}(\omega, r)} = \frac{\omega r}{c} + \psi(\omega, r), \tag{3}$$

where v and $\bar{v}$ are respectively the differential and the mean phase velocity of the wave, and where $\psi(\omega, r)$ is the so-called supplementary phase, describing the deviation of the phase of the wave and of its speed from their values in free space, where $v = c$. Since

$$E(t, r) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\omega, r)\, e^{i[\omega t + \varphi(\omega, r)]} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(\omega, r)\, A(\omega, 0)\, e^{i[\omega(t - r/c) - \psi(\omega, r) + \varphi(\omega, 0)]} d\omega \tag{4}$$

does not depend on the linear term $\omega r/c$ in the phase (it leads only to a uniform displacement of $E(t, r)$ along the time axis), the Fourier analysis determines the value of the phase only to within a term $\omega r/c$, i.e.,

$$\varphi(\omega, r) = -\psi(\omega, r) + \varphi(\omega, 0). \tag{5}$$

Consequently, the mean phase velocity is equal to

$$\bar{v}(\omega, r) = c \Big/ \left[1 - \frac{\varphi(\omega, r) - \varphi(\omega, 0)}{\omega} \frac{c}{r}\right]. \tag{6}$$

In this connection it is important to take the following into consideration. Usually the recorded signal $E(t, r)$ is cut off at the beginning, because of the threshold of sensitivity of the apparatus. This means that in the analysis the quantity determined is $\{\varphi(\omega, r) - \omega\Delta t\}$, i.e., the phase characteristic with an additional linear term $\omega\Delta t$, where Δt is the cutoff time of the signal. However, the value of Δt can be determined from the experimental phase curve. For in the high-frequency part of the range being considered, $d\psi/d\omega \sim 0$; therefore if the experimental curve has a linear behavior in this region, this is caused by the term $\omega\Delta t$. Atmospheric events were studied by the method described in various seasons, at various times of day from 9 AM to 5 PM local time, and at $r \sim 800$ to 3100 km. One example of the treatment of an individual atmospheric event is shown in Fig. 1. In it, a is the phase curve obtained by the harmonic analysis of $E(t, r)$. Subtracting from curve a of the phase characteristics of the receiver and of the standardized source we obtain curve b, while curve c corresponds to the desired function $\psi_i(\omega, r_i)$, obtained after subtracting the straight line $\omega\Delta t$ from b. From the figure it is evident that the theoretical values of $\psi(\omega, r)$ (the continuous curve in Fig. 1c) begin to deviate from the experimental at $f \sim 7$ to 8 kcs.

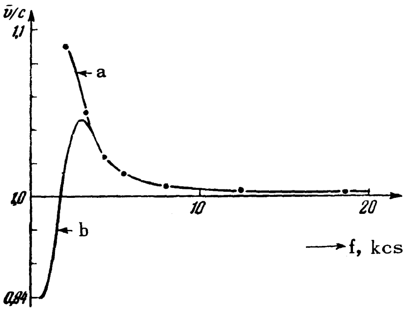

FIG. 2

All the individual signals were treated in similar fashion, and the results were used to plot curves showing the distribution of the quantity $\left(1 + \frac{\psi_i}{\omega} \frac{c}{r_i}\right)$, which determines $\bar{v}/c$, for seven frequency inter-

Most probable values of $\bar{v}/c$

f, kc	1.5—2.5	3—3.5	4—4.5	5—6	7—9	10—14	16—20
v/c, exptl.	1,09	1.05	1.024	1.014	1.006	1.004	1.002
f, kc	2	3		5	7	10	20
$\bar{v}/c$, theoret.	1.014	1.0456		1.017	1.0085	1.0041	1.0015

vals. The distribution curves obtained had pronounced maxima; they give the values of $\bar{v}/c$ shown in Fig. 2 (points) and in the table.

Comparison of the experimental and theoretical[3] values (Fig. 2, curve b) shows that they begin to deviate significantly at $f < 3$ kcs. This deviation is perhaps due to the fact that at such low frequencies the model of the ionosphere chosen in the calculations becomes inappropriate, since the wavelengths are comparable with the thickness of the ionospheric layers.

It should be pointed out that since, at the distances r considered, only the zero-order mode of the spectrum of the received wave plays an important part, therefore the values of $\left(1 + \frac{\psi}{\omega}\frac{c}{r}\right)$ immediately determine the imaginary values S_{01} of the wave numbers; and a comparison of them with the corresponding theoretical values of S_{01} may permit determination of the effective conductivity of the ionosphere with respect to transmission of electromagnetic waves in the range of frequencies investigated.

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SCATTERING OF 333 MEV NEGATIVE π MESONS BY HYDROGEN

V. G. ZINOV and C. M. KORENCHENKO

Joint Institute for Nuclear Research

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THE elastic and exchange scattering of π^- mesons by hydrogen has been studied by us. The beam of π^- mesons was obtained with the use of the synchrocyclotron of the Joint Institute for Nuclear Research. The energy of the beam was found to be 333 ± 9 Mev. Measurements were made with the use of scintillation counters. Liquid hydrogen was used as the target and was contained in a foamed plystyrene container.

Table I presents the measured differential elastic scattering cross-section for π^- mesons after the inclusion of all corrections. Table II presents the corrected differential cross-section for gamma ray emission from the decay of π^0 mesons.

TABLE I

ϑ°_{cms}	$\frac{d\sigma}{d\omega}$, 10^{-27} cm²/sterad
41.9	1.28±0.14
61.3	0.90±0.10
79.2	0.69±0.09
100.8	0.51±0.03
119.7	0.52±0.07
140.6	0.96±0.10
159.2	0.93±0.13

TABLE II

ϑ°_{cms}	$\frac{d\sigma}{d\omega}$, 10^{-27} cm² /sterad
20.8	6.52±1.37
41.0	6.31±1.30
60.0	3.72±0.77
77.7	2.36±0.50
99.0	1.55±0.34
128.8	0.95±0.23
146.9	1.29±0.31
159.7	1.10±0.24

By performing a least squares fit of the function $d\sigma/d\omega = a + b\cos\vartheta + c\cos^2\vartheta$ to the cross-section data (ϑ measured in the center of mass system), one obtains the following results (in units of 10^{-27} cm²/sterad):

a) Elastic scattering of π^-