where the coefficients A depend on the initial conditions.

Equations (10) and (11) can be applied to the description of all relaxation phenomena connected with the corresponding dynamic derivatives. The thermodynamical theory leaves open the question of the temperature dependence of the relaxation times.

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A POSSIBLE TEST OF THE CONSERVATION OF "COMBINED PARITY" IN THE BETA INTERACTION

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It is suggested that the invariance of the β-interaction under time reversal can be examined by observing $\beta-\gamma$ angular correlations in allowed transitions with aligned nuclei. Formulas are obtained for the $\beta-\gamma$ correlation in nuclei oriented by various methods. The general form of the β-interaction is considered with parity nonconservation taken into account. An expression is given for the $\beta-\gamma$ angular and polarization correlations in oriented nuclei for β-transitions of any order of forbiddenness.

EXPERIMENT has at present verified that parity is not conserved in weak interactions, as had been hypothesized in different forms by Lee and Yang[1,2] and by Landau.[3,4] It remains for experiment to test the specific versions of the hypothesis. We shall speak here primarily of a test of the conservation law for "combined parity," which according to Pauli's theorem[5] is equivalent to the invariance under time reversal.

Another important question is whether or not is is possible to describe the neutrino by a two-component equation.[4,2]

In the present work we shall give our main consideration to the first of these problems. We shall do this by investigating the $\beta-\gamma$ angular correlation of oriented nuclei. We shall show that if the nuclei are oriented by the method of Bleaney[6] or Pound[7] (aligned nuclei), then a measurement of the $\beta-\gamma$ correlation for allowed transitions can give important information on the nature of the β-interaction:

(1) If there is no $\beta-\gamma$ angular correlation, then (a) the β-interaction is invariant with respect to

time reversal, and (b) the vector and pseudovector interactions contribute weakly.

(2) If there is a $\beta-\gamma$ angular correlation, but it decreases with an increase in the β energy, then (a) the β-interaction is invariant with respect to time reversal, but (b) either the vector or pseudovector interactions contribute strongly.

(3) An increase in the correlation with an increase in the β energy leads uniquely to the conclusion that the β-interaction is not invariant under time reversal, and "combined parity" is not conserved.

These results are easily understood if one bears in mind the following facts. The expression for the probability of the β-transition must contain the electron momentum $\mathbf{p}(p, \vartheta, \varphi)$, the photon momentum $\mathbf{k}(k, \theta, \phi)$, and the tensor $T_{i\ell}$ which determines the orientation of the aligned nucleus. Since the transition is allowed, $\mathbf{p}$ must enter linearly. Parity is conserved in the γ-transition, so that $\mathbf{k}$ must enter quadratically. With these conditions, it is impossible to construct a scalar quantity of $\mathbf{p}$, $\mathbf{k}$, and $T_{i\ell}$; the only pseudoscalar that can be constructed of these quantities is $[\mathbf{p} \times \mathbf{k}]\mathbf{T}$, where $T_\ell = \sum_i T_{i\ell} k_i$. This pseudoscalar changes sign under time reversal, and therefore cannot enter a theory which is invariant with respect to the transformation $t \rightarrow -t$. The Coulomb interaction of the electron and the nucleus leads to additional terms in the expression for the $\beta-\gamma$ angular correlation. In allowed transitions these terms enter only into the vector and axial vector interactions, and they decrease as the electron energy increases.

Let us choose the most general form of the β-interaction, taking parity nonconservation into account by the method of Lee and Yang.[1] The initial state of the nucleus shall be given by the tensor $P^{j_0}_{g\eta}$, which is related to the density matrix $\rho(\mu_0, \mu_0')$ by

$$P^{j_0}_{g\eta} = \frac{2g+1}{2j_0+1} \sum_{\mu_0, \mu_0'} C^{j_0\mu_0'}_{j_0\mu_0 g\eta}\, \rho(\mu_0, \mu_0'), \tag{1}$$

where μ_0 is the projection of the nuclear spin j_0 on the axis of quantization. If this axis is chosen as the physically defined axis of the most probable nuclear spin orientation (for instance directed along the external orienting field), then the density matrix is diagonal, and the nuclear orientation is given by

$$P^{j_0}_{g0} = \frac{2g+1}{2j_0+1} \sum_{\mu_0} C^{j_0\mu_0}_{j_0\mu_0 g0}\, w(\mu_0), \tag{2}$$

where $w(\mu_0)$ is the probability of finding a nucleus in a state with the z component of the spin equal to μ_0. We choose the initial wave function of the system as

$$\Psi_i = \sum_{\mu_0} a^{\xi}_{\mu_0} \psi_{j_0\mu_0}, \quad \rho(\mu_0, \mu_0') = \langle a^{\xi*}_{\mu_0} a^{\xi}_{\mu_0'} \rangle_{\xi},$$

where $\langle\ \rangle_\xi$ denotes averaging over the elements of the statistical ensemble, and

$$w(\mu_0) = \langle |a^{\xi}_{\mu_0}|^2 \rangle_{\xi}, \quad \sum_{\mu_0} w(\mu_0) = 1.$$

If after β-decay the nucleus undergoes a radiative transition, the polarization tensor of the intermediate state can be expressed in terms of observed quantities which characterize this latter transition. Let $H_{j_1\mu_1}$ be the matrix element for the γ-transition from state $j_1\mu_1$ to state $j_2\mu_2$. Then the polarization tensor for state $j_1\mu_1$ can be written

$$P^{j_1}_{S\sigma} = \frac{2S+1}{2j_1+1} \sum_{\mu_1, \mu_1'} C^{j_1\mu_1}_{j_1\mu_1' S\sigma} H_{j_1\mu_1} H^*_{j_1\mu_1'}. \tag{3}$$

We shall choose the form used previously by the author and others[8,9] for the wave function of the electron outside the nucleus, a function which describes a particle with momentum $\mathbf{p}$ at infinity. We shall define the wave function of the electron within the nucleus as the solution of the Dirac equation for a surface or volume charge distribution. The amplitude of the internal function will be determined from the condition that the internal and external functions join smoothly. The parameters $A_{j\lambda}$, which are adjusted to make these functions joint smoothly, and the wave function phases $\delta_{j\lambda}$ have been previously determined by the author[9-11] and have been tabulated by Sliv and Volchok.[12*] We shall denote the total angular momentum of the electron by j, and its orbital angular momentum by $l_1 = j + \lambda$, where $\lambda = \pm \frac{1}{2}$. The total angular momentum of the neutrino will be denoted by i, and its orbital angular momentum by $l_1 = i + \nu$, where $\nu = \pm \frac{1}{2}$. The matrix elements of the $\beta - \gamma$ transition are calculated in the same way as in the author's previous works.[9,10] As a result we obtain an expression for the angular and polarization correlations of the β particle and the γ ray emitted by the oriented nuclei. This expression is

$$W(\mathrm{p}, \mathrm{k}) = \sum i^{l+l'} (-1)^{j'-j+g} \sqrt{(2l+1)(2l'+1)/(2f+1)}(2j+1)(2j'+1)\, C^{f0}_{l'0l0}\, W(iL'jf;\, j'L)\, W\left(\tfrac{1}{2}\, jl'f;\, lj'\right)$$
$$\times \exp[i(\delta_{j\lambda} - \delta_{j'\lambda'})]\, C^{f\sigma}_{g0S\sigma}\, X(j_1 j_1 S,\, j_0 j_0 g,\, L'L\, f)\, P^{j_0}_{g_0} P^{j_1}_{S\sigma}(\theta\Phi)\, Y^*_{f\sigma}(\vartheta\varphi)\, B^{L\tau}_{\lambda\nu} B^{L'\tau'*}_{\lambda'\nu}. \tag{4}$$

Here $C^{c\gamma}_{a\alpha b\beta}$ are Clebsch-Gordan coefficients, W(abcd; ef) are Racah functions, and X(abc, def, ghi) = X(adg, beh, cfi) are Fano functions. These functions are tabulated.[13-16] Further

$$B^{L\tau}_{\lambda\nu} = (C^{j_0\mu_0}_{L\Lambda j_1\mu_1})^{-1} \sum_{J=0}^{1} \int \psi^*_{j_1\mu_1} S^L_{J\tau}\, Y^{J\tau*}_{L\Lambda} r^{L+\tau} \psi_{j_0\mu_0} d\mathbf{r},$$
$$S^L_{J\tau} = \{[\beta(g_S R_1^- + iG_S R_2^+) + (g_V R_1^+ + iG_V R_2^-)]\,\delta_{J0} + [\beta\sigma(g_T R_1^- + iG_T R_2^+) + \sigma(g_A R_1^+ + iG_A R_2^-)]\,\delta_{J1}\}\,\delta_{L+\tau,N}$$
$$-\{[\gamma_5(ig_A R_2^- + G_A R_1^+) - \beta\gamma_5(ig_P R_2^+ + G_P R_1^-)]\,\delta_{J0} + [\boldsymbol{\alpha}(ig_V R_2^- + G_V R_2^+) - \beta\boldsymbol{\alpha}(ig_T R_2^+ + G_T R_1^-)]\,\delta_{J1}\}\,\delta_{L+\tau,N-1}. \tag{5}$$

The symbols g_S, g_T, g_P, g_V, and g_A denote the coupling constants of the scalar, tensor, pseudoscalar, vector, and pseudovector β-interactions, and G_S, G_T, etc., are analogous coupling constants for terms which can be allowed only in the case of parity nonconservation (that is, those containing an extra $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$). Further, β, γ_5, $\gamma_4 = -\beta$, $\boldsymbol{\alpha}$, and $\boldsymbol{\sigma}$ are the well-known Dirac matrices which act on the nuclear wave functions $\psi_{j_0\mu_0}$ and $\psi_{j_1\mu_1}$, and $Y^{J\tau}_{L\Lambda}$ is a $(J-L)$-vector (with $J = \frac{1}{2}$ this is a spherical spinor). The components of this vector are given by

$$[Y^{J\tau}_{L\Lambda}]_\gamma = (-1)^{J-\gamma}\, C^{L\Lambda}_{f\varphi J-\gamma}\, Y_{f\varphi}, \quad f = L + \tau, \quad -J \leqslant \tau \leqslant J. \tag{6}$$

The spherical function $Y_{f\varphi}$ are given here in Bethe's definition,[17] which differs by a factor of $(-1)^\varphi$ from the definition of Landau and Lifshitz.[18] The quantities $R_1^\pm$ and $R_2^\pm$ arise from the wave functions of the light particles, and are

$$R_1^\pm = \sum_{\omega,\chi} [\delta_{\omega,-\lambda}\delta_{\chi,-\nu} \pm 4\lambda\nu\delta_{\omega,\lambda}\delta_{\chi,\nu}]\, Q_{\omega\chi}, \quad R_2^\pm = \sum_{\omega,\chi} [2\nu\delta_{\omega,-\lambda}\delta_{\chi,\nu} \pm 2\lambda\delta_{\omega,\lambda}\delta_{\chi,-\nu}]\, Q_{\omega\chi},$$
$$Q_{\omega\chi} = A_{j\lambda}\beta^{jiL}_{\omega\chi\tau} \frac{\sqrt{W + 4\omega\lambda}\, \varkappa^{j+\omega} (-q)^{i+\chi}}{(2j+2\omega+1)!!\,(2i+2\chi+1)!!}\, \varphi_\omega \delta_{j+i+\omega+\chi,\, L+\tau}, \tag{7}$$

$\omega = \pm\frac{1}{2}$, $\chi = \pm\frac{1}{2}$, $\mathbf{q}$ is the neutrino momentum, $W = E + V$, $\kappa^2 = W^2 - 1$, E is the total energy of the electron including its rest mass,† $V = \alpha Z/R$ for a surface charge distribution, and $V = 3\alpha Z/2R$ for a uniform volume charge distribution. The quantity φ_ω is equal to unity with an accuracy of $(\alpha Z)^2/5$. In the case of a surface distribution, for instance,

$$\varphi_\omega = 1 + 3(\varkappa R)^2 [10(2j + 2\omega + 3)]^{-1}.$$

In the case of a volume distribution, φ_ω differs from unity by even less.[9,10] Finally, R is the nuclear radius, and $\alpha = 1/137$.

The $\beta^{jiL}_{\omega\chi\tau}$ are the expansion coefficients of the angular parts of the wave functions of the light particles, i.e.,

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*The quantities $a_{j\lambda}$ which are tabulated by Sliv and Volchok[12] differ from the $A_{j\lambda}$ defined by the author[9-11] by the factor

$$a_{j\lambda} = \sqrt{W - 2\lambda}\, \varkappa^j A_{j\lambda} / (2j)!!$$

†We are using units throughout in which $\hbar = c = m = 1$.

$$(Y^{1/2\lambda*}_{j\mu} A_{J\gamma} Y^{1/2\nu}_{in}) = \sum_{L,\tau} \beta^{jiL}_{\lambda\nu\tau} C^{j\mu}_{L\Lambda in} (-1)^{J+\tau+\Lambda} [Y^{J\tau}_{L-\Lambda}]_\gamma,$$

$$\beta^{jiL}_{\lambda\nu\tau} = \sqrt{(2J+1)(2L+1)(2f+1)(2l_1+1)(2i+1)/2\pi}\, C^{l_0}_{l_1 0 f 0} X(1/2\, lj,\ 1/2\, l_1 i,\ JfL), \tag{8}$$

$A_{00} = 1$, $A_{1\gamma} = \sigma_\gamma$ is a Pauli matrix, $\sigma_0 = \sigma_z$, $\sigma_{\pm 1} = \pm(\sigma_x \pm i\sigma_y)/\sqrt{2}$, $\ell = j + \lambda$, $\ell_1 = i + \nu$, and $f = L + \tau$. The explicit form of the $\beta^{jiL}_{\omega\chi\tau}$ is easy to find from the tables of Ref. 16. The sum in (4) is taken over all possible values of all indices. Equation (4) is applicable to β-transitions of all orders of forbiddenness. The index N in Eq. (5) gives the order of forbiddenness of the transition. For allowed transitions $N = 0$, and since $L + \tau \geq 0$, the second curly bracket gives no contribution to $S^L_{J\tau}$. If one treats a transition of a definite order of forbiddenness, only some of the terms remain in the sum in Eq. (4), since the Clebsch-Gordan coefficients, the Racah functions, and the Fano functions in $S^L_{J\tau}$ are different from zero only for certain values of the indices. The number of possible values of g is determined by the character of the nuclear orientation.

For aligned nuclei, g is even. If one measures the angular distribution of the γ-rays, then

$$P^{j_1}_{S\sigma}(\theta,\Phi) = \left[1 - \frac{S(S+1)}{2I(I+1)}\right]\sqrt{(2S+1)(2I+1)(2j_1+1)}\, W(j_2 I j_1 S;\ j_1 I)\, C^{I0}_{I0S0} Y_{S\sigma}(\theta\Phi), \tag{9}$$

where I is the multipole order of the quantum emitted, S is even, and j_1 and j_2 are the nuclear angular momenta before and after the radiative transition. If the nuclei are oriented by the method of Bleaney[6] or Pound[7] then g is even. For this case (4) gives

$$W_2(\mathrm{p},\mathrm{k}) = \sum_{S=0,2,\ldots} \sum_{g=0,2,\ldots} \sum_{\mu_0} \sqrt{2g+1}\, C^{j_0\mu_0}_{j_0\mu_0 g0} w(\mu_0) \Big\{\sqrt{2S+1} \sum_{L=0}^{1} [M_L - \gamma_1 E^{-1} N_L]\, \delta_{S,g}\, U(Lj_0j_1g;\ j_1j_0)\, P_S(\cos\theta)$$

$$-\sqrt{2g+1}\,[p\,\mathrm{Im}\,Q_m - \alpha Z\,\mathrm{Re}\,Q_n]\, E^{-1} U(j_0j_0g1;\ Sj_0)\, \frac{2}{3} iF_{Sg1}(\vartheta\varphi\theta\Phi)\Big\}\left[1 - \frac{S(S+1)}{2I(I+1)}\right] C^{I0}_{I0S0}\, U(j_2Ij_1S;\ j_1I), \tag{10}$$

$$M_0 = (|g_S|^2 + |G_S|^2)|K_S|^2 + (|g_V|^2 + |G_V|^2)|K_V|^2, \quad M_1 = (|g_T|^2 + |G_T|^2)|K_T|^2 + (|g_A|^2 + |G_A|^2)|K_A|^2,$$

$$N_0 = (g_S g^*_V + G_S G^*_V) K_S K^*_V + \text{complex conjugate}, \quad N_1 = (g_T g^*_A + G_T G^*_A) K_T K^*_A + \text{complex conjugate},$$

$$Q_m = (g_T G^*_S + G_T g^*_S) K_T K^*_S - (g_A G^*_V + G_A g^*_V) K_A K^*_V, \quad Q_n = (g_A G^*_S + G_A g^*_S) K_A K^*_S - (g_T G^*_V + G_T g^*_V) K_T K^*_V, \tag{11}$$

$$F_{abc}(\vartheta\varphi\theta\Phi) = 4\pi \sum_\gamma C^{c\gamma}_{b0a\gamma} Y_{a\gamma}(\theta\Phi) Y^*_{c\gamma}(\vartheta\varphi), \quad F_{0bb} = \sqrt{2b+1}\, P_b(\cos\vartheta), \quad F_{a0a} = 4\pi \sum_\gamma Y_{a\gamma}(\theta\Phi) Y^*_{a\gamma}(\vartheta\varphi), \tag{12}$$

$$iF_{221} = -3\sqrt{3/2}\sin\theta\cos\theta\sin\vartheta\sin(\Phi-\varphi), \quad iF_{441} = -(3\sqrt{5/4})(7\cos^2\theta - 3)\cos\theta\sin\theta\sin\vartheta\sin(\Phi-\varphi), \tag{13}$$

$$U(abcd;\ ef) = \sqrt{(2e+1)(2f+1)}\, W(abcd;\ ef), \quad \gamma_1 = \sqrt{1-(\alpha Z)^2},$$

$$K_S = \int \psi^*_{j_1\mu_1} \beta \psi_{j_0\mu_0}\, d\mathbf{r}, \qquad K_T = \sqrt{4\pi}\, [C^{j_0\mu_0}_{1\Lambda j_1\mu_1}]^{-1} \int \psi^*_{j_1\mu_1} \beta\sigma Y^{-1}_{1\Lambda} \psi_{j_0\mu_0}\, d\mathbf{r}.$$

In these equations K_V and K_A differ from K_S and K_T by the absence of β. For allowed transitions, $K_V = -K_S$, and $K_A = -K_T$. Here $Y^{-1}_{1\Lambda}$ is the vector of Eq. (6) with $J = 1$, $L = 1$, and $\tau = -1$. Equation (10) is accurate up to terms of order $(\alpha Z)^2/3$ and αZER compared with unity.* If high accuracy is necessary, $-\gamma_1 E^{-1}$ should be replaced by a^+/a^-, where

$$a^\pm = (W-1) A^2_{1/2\,1/2} \pm (W+1) A^2_{1/2\,-1/2},$$

p should be replaced by c_1/a^-, where

$$c_1 = 2\varkappa A_{1/2\,1/2} A_{1/2\,-1/2} \cos(\delta_{1/2\,-1/2} - \delta_{1/2\,1/2})$$

and αZ should be replaced by c_2/a^-, where

$$c_2 = 2\varkappa A_{1/2\,1/2} A_{1/2\,-1/2} \sin(\delta_{1/2\,-1/2} - \delta_{1/2\,1/2}).$$

*Equation (14) will be given to the same accuracy.

Since it seems that invariance under time reversal is maintained in strong nuclear interactions, we may assert that the phase difference between K_T and K_S^*, as well as that between K_A and K_V^*, etc., is either 0 or π. Whether Q_m and Q_n are real or complex is therefore determined by the coupling constants g and G. If the β-interaction is invariant under the transformation $t \to -t$, all the g and G are real and $\operatorname{Im} Q_m = 0$. In this case the $\beta-\gamma$ correlation is independent of p/E. If this independence were observed, it would be direct proof of the breakdown of invariance under time reversal. The form of allowed β-spectra would seem to indicate that the real part of g_A and G_A is either very small or vanishes identically. There is indirect evidence that the real part of g_V and G_V is also small. If this is true, then Q_n is small. Thus even a rough qualitative study of $\beta-\gamma$ angular correlation in aligned nuclei may answer the question of invariance under time reversal and give indications as to the contribution of the V and A interactions.

If the nuclei are polarized, say by the Gorter-Rose method,[19] there will again be a $\beta-\gamma$ correlation for allowed transitions. Considering both even and odd values of g, we obtain

$$W_1(\mathbf{p},\mathbf{k}) = W_2(\mathbf{p},\mathbf{k}) - \sum_{S=0,2,\dots}\ \sum_{g=1,3,\dots}\ \sum_{\mu_0} C^{j_0\mu_0}_{j_0\mu_0 g0}\, w(\mu_0)\,(2g+1)\sqrt{2S+1}\left\{\frac{2}{3}\left[\frac{p}{E}\operatorname{Re} Q_m + \frac{\alpha Z}{E}\operatorname{Im} Q_n\right]\right.$$

$$\times \sqrt{2j_0+1}\,W(j_0 j_0 g1;\ Sj_0) - \sqrt{\frac{2}{3}}\left[\frac{p}{E}\operatorname{Re} Q_1 + \frac{\alpha Z}{E}\operatorname{Im} Q_1\right]\sqrt{(2j_0+1)(2j_1+1)}\,X(j_1 j_1 S,\ j_0 j_0 g,\ 111)\left.\vphantom{\frac{2}{3}}\right\}$$

$$\times F_{Sg1}(\vartheta\varphi\theta\Phi)\left[1 - \frac{S(S+1)}{2I(I+1)}\right] C^{I0}_{I0S0}\, U(j_2 I j_1 S;\ j_1 I); \tag{14}$$

$$Q_1 = (G_T g_T^* + G_T^* g_T)\,|K_T|^2 - (G_A g_A^* + G_A^* g_A)\,|K_A|^2 - (G_T g_A^* + g_T G_A^*)\,K_T K_A^* + (G_A g_T^* + g_A G_T^*)\,K_A K_T^*; \tag{15}$$

$$F_{211} = -\sqrt{3/2}\,\{(3\cos^2\theta - 1)\cos\vartheta + 3\sin\theta\cos\theta\sin\vartheta\cos(\Phi - \varphi)\},$$

$$F_{231} = (3\sqrt{2}/2\sqrt{7})\,\{(3\cos^2\theta - 1)\cos\vartheta - 2\sin\theta\cos\theta\sin\vartheta\cos(\Phi - \varphi)\},$$

$$F_{431} = -(3/4\sqrt{7})\,\{(35\cos^4\theta - 30\cos^3\theta + 3)\cos\vartheta + 5(7\cos^2\theta - 3)\cos\theta\sin\theta\sin\vartheta\cos(\Phi - \varphi)\}. \tag{16}$$

A measurement of $W_1(\mathbf{p},\mathbf{k})$ may serve as a control experiment in the study of $W_2(\mathbf{p},\mathbf{k})$, and is also interesting in itself for the determination of the g and G coupling constants. The use of the two-component equation[2,4] to describe the neutrino necessitates an experimental proof of the fact that all the coupling constants are related by $g = -G$ or $g = G$. This requires quantitative measurements, so that it is desirable to use the results of independent experiments. In particular, the polarization of electrons emitted by polarized nuclei is of interest. The author has previously[20] derived the required formulas for allowed and forbidden β-transitions. We note that if the nucleus emits β^+ particles, their polarization can be observed not only from scattering asymmetry, but also from the circular polarization of the annihilation radiation. Since the positrons are depolarized by relativistic effects, they will not be completely depolarized in spite of the fact that they are slowed down.

Additional possibilities in the investigation of β processes arise if, in addition to the angular distribution, one observes the angular polarization of the γ-rays emitted after β-decay. It was shown by Shapiro,[21] for instance, that if parity is not conserved there should be a correlation between the direction of the emitted electron and the circular polarization of the γ-ray. The present author[22] has suggested that observations of the circular polarization of x-rays emitted after K-capture be used to determine the sign of g_S/g_T. The circular polarization of the γ-rays gives information on the polarization of the nuclear dipole after β-decay. Thus instead of studying β-transitions in polarized nuclei, one can find the direction of the nuclear spin relative to the direction of the β particle by observing the circular polarization of the subsequent photons. Formally this reduced to the $P^{j_1}_{S\sigma}$ tensor playing the role of $P^{j_0}_{g\eta}$. If S is odd, $P^{j_1}_{S\sigma}$ vanishes on averaging over γ-ray polarizations. If, however, the circular polarization is found from experiment $P^{j_1}_{S\sigma}$ is nonzero for all S, and Eq. (9) becomes

$$P^{j_1}_{S\sigma} = (2S+1)\sum_{M,\,M'} \xi^{(M-M')/2}\, C^{IM}_{IM'SM-M'}\, U(Ij_2Sj_1;\ j_1 I)\, e^{i(M-M')\alpha}\, D^S_{\sigma,M-M'}(\Phi,\theta,0), \tag{17}$$

where the term with $M = M' = 1$ gives $P^{j_1}_{S\sigma}$ for right-circularly polarized photons, the term with $M = M'$

$= -1$ gives it for left-circularly polarized photons, and the sum of terms with $M = 1$, $M' = -1$ and $M = -1$, $M' = 1$ gives this tensor for linearly polarized photons. The variable ξ assumes the values $+1$ for magnetic and -1 for electric transitions. The angle α gives the linear polarization vector ϵ in the coordinate system whose z axis is along $\mathbf{k}$ ($\epsilon_{\pm 1} = \pm e^{\pm i\alpha}/\sqrt{2}$, $\epsilon_0 = 0$). This angle α is the angle between the polarization vector and the x axis in the plane perpendicular to $\mathbf{k}$. Finally, $D^{S}_{\sigma, M-M'}(\phi\theta 0)$ is a matrix of the irreducible representation of the rotation group.[23] It is here defined so that $\psi_{j\mu}(\theta\phi) = \sum_{\mu'} D^{j}_{\mu\mu'}(\phi\theta 0)\psi_{j\mu'}(00)$, and in particular,

$$Y_{lm}(\theta\Phi) = \sqrt{(2l+1)/4\pi}\, D^{l}_{m0}(\Phi\theta 0).$$

Experiments on allowed β-transitions give the most unambiguous information, and are therefore of most interest. In the case of forbidden β-transitions, Eq. (4) depends on more nuclear matrix elements than it does in the case of allowed transitions. These matrix elements must be considered as additional unknown quantities which must be determined by experiment or evaluated roughly with the aid of nuclear models. It is therefore difficult to give unique interpretations to the experimental data. More meaningful conclusions can be reached in studying transitions in which the nuclear angular momentum change is one unit greater than the order of forbiddenness. In this case, however, we obtain no data on the Fermi part of the β-interaction. We shall give here an explicit expression for the $\beta-\gamma$ correlation function for a polarized nucleus undergoing first forbidden transitions, when $j_1 = j_0 \pm 2$. According to Eq. (4) we have

$$W(\mathbf{p}, \mathbf{k}) = \sum_{j, f, g, s} \sum_{l' \geqslant l} \sum_{\mu_0} C^{j_0\mu_0}_{j_0\mu_0 g0}\, w(\mu_0)(-1)^{g} X(j_1 j_1 S, j_0 j_0 g, 22f)(2g+1)$$

$$\times \gamma^{jf}_{ll'} Z^{j}_{ll'} F_{Sgf}(\vartheta\varphi\theta\Phi)\sqrt{2S+1}\left[1 - \frac{S(S+1)}{2I(I+1)}\right] C^{I0}_{I0S0}\, U(j_2 I j_1 S;\ j_1 I), \qquad (18)$$

$$Z^{1/2}_{01} = 2q^2[(p\,\mathrm{Re}\,Q_1 + \alpha Z\,\mathrm{Im}\,Q_1)\,\omega_+ - i(p\,\mathrm{Im}\,Q_1 - \alpha Z\,\mathrm{Re}\,Q_1)\,\omega_-],$$

$$Z^{3/2}_{12} = [p^2 + (\alpha ZE)^2]\left[\left(p\,\mathrm{Re}\,Q_1 + \frac{\alpha Z}{2}\mathrm{Im}\,Q_1\right)\omega_+ - i\left(p\,\mathrm{Im}\,Q_1 - \frac{\alpha Z}{2}\mathrm{Re}\,Q_1\right)\omega_-\right]; \quad Z^{1/2}_{00} = q^2(E+1)(M_1 - N_1),$$

$$Z^{1/2}_{11} = q^2(E-1)(M_1 + N_1), \quad Z^{3/2}_{11} = \frac{p^2 + (\alpha ZE)^2}{2}(E+1)(M_1 - N_1), \quad Z^{3/2}_{22} = \frac{p^2 + (\alpha ZE)^2}{2}(E-1)(M_1 + N_1), \qquad (19)$$

$$j = 1/2,\ 3/2; \quad l \text{ and } l' = j \pm 1/2; \quad 2\omega_{\pm} = 1 \pm (-1)^{f+S+g};$$

$$\gamma^{jf}_{ll'} = (-1)^{l}(2j+1)\sqrt{(2l'+1)/4(2f+1)}\, C^{f0}_{l'0l0}\, U(2-j, 2, j, f;\ j, 2)\, U(1/2\ jl'f;\ lj), \qquad (20)$$

$$\gamma^{1/2\,0}_{ll} = -\sqrt{6}\,\gamma^{1/2\,1}_{01} = \frac{1}{2}\,\gamma^{3/2\,0}_{ll} = 1, \quad \gamma^{3/2\,2}_{11} = \gamma^{3/2\,2}_{22} = \sqrt{7/3}\,\gamma^{3/2\,1}_{12} = -(7/6)\,\gamma^{3/2\,3}_{12} = -\sqrt{14}/5.$$

The quantities M_1, N_1, and Q_1 differ from those in (11) and (15) only in that K_S and K_V are zero, and K_T and K_A are given by

$$K_T = -K_A = [C^{j_0\mu_0}_{2\Lambda j_1\mu_1}]^{-1}\int \psi^{*}_{j_1\mu_1}\beta\sigma \mathbf{Y}^{-1*}_{2\Lambda}\psi_{j_0\mu_0}\,d\mathbf{r}. \qquad (21)$$

All the equations in this article refer to β^--decay. For positron decay, the following substitutions must be made:

$$g_S \to -g_S^{*},\ g_A \to -g_A^{*},\ g_P \to -g_P^{*},\ g_T \to g_T^{*},\ g_V \to g_V^{*},$$

$$G_S \to G_S^{*},\ G_A \to G_A^{*},\ G_P \to G_P^{*}\ G_T \to -G_T^{*},\ G_V \to -G_V^{*},\ \alpha Z \to -\alpha Z.$$

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CONTRIBUTION TO THE DECAY THEORY OF A QUASI-STATIONARY STATE

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Some general problems relating to the decay theory of a quasi-stationary state are considered. Dispersion relations are derived on the basis of the semi-finiteness of the energy distribution density $\omega(E)$. A criterion of physical feasibility in decay theory is formulated and studied on the basis of the Paily and Wiener theorem. It is shown that an exponential decay law cannot hold for all $\Gamma t/\hbar$. Corrections to the exponential decay law are computed under the simplest assumptions. Dispersion relations between the modulus and the phase of the function $p(t)$ are derived and investigated. On the basis of a knowledge of the decay law, these allow us to determine the energy distribution density $\omega(E)$ analytically. The results obtained are derived from the general laws of quantum mechanics and do not depend on the model of the decaying system.

IN the present work we consider several problems of decay theory, in particular, the decay of a quasi-stationary (almost stationary) state.* As is well known, the theory of the decay of a quasi-stationary state has great significance in the investigation of α-decay, in the transmission of particles through a potential barrier, in the theory of the nucleus, in the determination of the distribution of energy levels, etc.[1,2] The basic theorem of decay theory of a quasi-stationary state was obtained by Fock and Krylov in

*A brief account of results that have been obtained is given in Ref. 5.