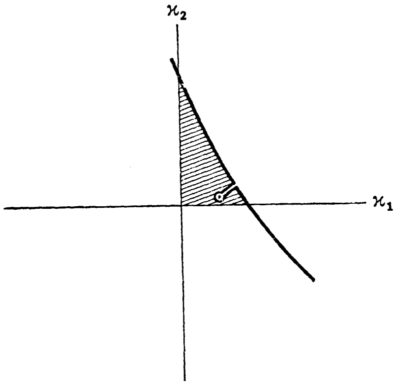


FIG. 4

$$\frac{dW_{\text{trans}}}{d\Omega} = \frac{ce^2 \sin^2\theta \cos^2\theta}{\pi^2 v^2} \frac{\beta^4}{(1-\beta^2\cos^2\theta)^2} \int \left( \frac{\varepsilon + 1}{\varepsilon \cos\theta + \sqrt{\varepsilon - \sin^2\theta}} \right)^2 d\omega. \tag{30}$$

The Poynting vector associated with the Cerenkov radiation also changes but only at the individual points $\omega = \omega'$.

We may note that here, just as in the preceding case, there is a region in which transition-radiation wave field is formed.

In conclusion the author wishes to express his gratitude to I. I. Gol'dman for a number of interesting discussions.


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Translated by H. Lashinsky
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SOVIET PHYSICS JETP VOLUME 6 (33), NUMBER 6 JUNE, 1958

# *EMISSION OF PARTICLES FROM EXCITED NUCLEI*

M. Z. MAKSIMOV

Submitted to JETP editor June 1, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 1411-1416 (December, 1957)

A formula for the dependence of the cross-section of a nuclear reaction involving the emission of several particles on the energy of incident nuclei has been obtained using the statistical theory of the nucleus and Bohr's concept of nuclear reactions. The formula has been used to compute the cross-sections for nuclear reactions involving the emission of 1, 2, 3, or 4 neutrons from $Bi^{209}_{83}$ and $I^{127}_{53}$ when bombarded with protons, deuterons, or $\alpha$ particles. The dependence of the entropy of a nucleus on the excitation energy and mass number has been determined using the gas model of the nucleus. The results of the calculations agree satisfactorily with the experimental data.

IT is known that an excited nucleus emits particles (n, p, $\alpha$, $\gamma$, etc.) in transition to the ground state, the energy distribution of the particles being approximately Maxwellian (as follows, for example, from the evaporation model of the decay[1]). Accordingly, the mean energy carried away by an emitted particle is much smaller than the excitation energy of the nucleus.[2] If the excitation energy is sufficiency large the nucleus cannot, therefore, return to the ground state emitting a single particle.

Both theory[1] and experiment[3,4] show that the probability of emission of a single particle decreases with increasing excitation energy of the nucleus. Consequently, in transition to the ground state such a nucleus will emit several particles, the number of which depends on the initial excitation energy and on the binding energy of the particles in corresponding nuclei.

In the present note we calculate approximately, as a function of the excitation energy, the probability that a certain residual nucleus will be formed after consecutive emission of several (k) particles by the excited nucleus. Since excited nuclei are obtained by bombarding the target nucleus A with another nucleus a, it is easy to find the dependence of the investigated probability on the energies of the incident nuclei.

Consider the following reaction:

$$(A+a)\rightarrow(A+a)^{*}\rightarrow\left(A+a-\sum_{i=1}^{k}b_i\right)+\sum_{i=1}^{k}(b_i), \tag{1}$$

where (X) denotes a nucleus with mass number X. Since, according to Bohr, reaction (1) consists of two independent processes (formation of the compound nucleus $(A + a)^*$, and its decay), the cross-section for such a reaction can be written as[1]

$$\sigma_k=\sigma_c(a)\,\bar{\eta}_k, \tag{2}$$

where $\sigma_c(a)$ is the cross-section for the formation of a compound nucleus by the nuclei A and a, and $\bar{\eta}_k$ is the probability of the decay of the compound nucleus into final products.

The cross-section for the formation of the compound nucleus can be determined by means of the classical formula

$$\sigma_c(a)=\pi R_a^2\begin{cases}0, & E_a<B_a;\\ 1-B_a/E_a, & E_a\geqslant B_a,\end{cases}$$

where $R_a = r_0A^{1/3} + \rho_a$, $E_a$ is the energy of the incident particle in the center-of-mass system of A and a, and $B_a$ is the Coulomb barrier, given by the formula

$$B_a=Z_AZ_ae^2/R_a. \tag{3}$$

Quantum-mechanical calculations show, however, that $\sigma_c(a)$ is different from zero even for $E_a = B_a$, when it equals[5]

$$\sigma_c(a)\approx 0.81g^{-2/3}\pi R_a^2. \tag{4}$$

It is therefore possible to determine the cross-section for the formation of a compound nucleus by charged particles, with sufficient accuracy, in the following way ($B_n = 0$ for neutrons):

$$\sigma_c(a)\approx\pi R_a^2\left(1-(1-\alpha_a)\,B_a/E_a\right),\quad E_a\geqslant B_a; \tag{5}$$

$$\sigma_c(a)\approx\pi R_a^2\alpha_a\exp\{-2g\,f(E_a/B_a)\},\quad E_a<B_a, \tag{6}$$

where

$$f(x)=x^{-1/2}\arccos x^{1/2}-(1-x)^{1/2};\qquad \alpha_a=0{,}81g^{-2/3};\quad g=(2M_aZ_aZ_Ae^2R_a)^{1/2}/\hbar.$$

The cross-sections for reverse reactions, necessary for finding $\eta_k$, will be computed using formula (5) only, which considerably simplifies the calculation.

Formulae for the probability $\eta_k$ of decay of the compound nucleus with emission of 1, 2, 3, etc. particles are given by various authors.[1,6-9] The formulae, however, have been calculated for the case when the compound nucleus, after emitting $k_k$ particles, may be found in any state compatible with the law of the conservation of energy. As a rule, the values of $\eta_k$, calculated according to those expressions, either increase or decrease very slowly with the excitation energy, leading to a large discrepancy between the theoretical and experimental cross-sections.[9] This is connected with the fact that after emitting k particles, the residual nucleus can emit the next (k + 1)-th particle if the excitation energy is larger than the binding energy of any particle in the same nucleus. (Here, as in the following, we neglect the probability of $\gamma$ emission since this process, on one hand, does not change the residual nucleus and, on the other, is very small compared with the probability of particle emission, when the excitation energy is larger than the binding energy.) In the following, therefore, we shall consider only the case when the nucleus, after emitting k particles, is in a state with excitation energy smaller than the binding energy of any particle in it. The probability of such a process is equal to the difference of the total emission probabilities of the k-th and the (k + 1)-th particle, i.e.,

$$\eta_k = I_k\Big(U_a + \sum_{i=1}^{k} \varepsilon_{b_i}\Big) - \sum_{\nu} I_{k+1}\Big(U_a + \sum_{i=1}^{k} \varepsilon_{b_i} + \varepsilon_{b_{k+1,\nu}}\Big), \tag{7}$$

where $\epsilon_{b_i}$ are the binding energies of particles $b_i$ in the corresponding nuclei $A + a - b_1 - \ldots - b_{i-1}$, and $U_a = E_a - \epsilon_a$ is the excitation energy of the nucleus $(A + a)$. Summation over $\nu$ in the second term corresponds to the fact that the $(k + 1)$-th particle may be a neutron, a proton, etc. ($b_{k+1,\,\nu} = n;\ p \ldots$).

If

$$U_a < -\sum_{i=1}^{k} \varepsilon_{b_i} - \varepsilon_{b_{k+1,\,\nu}} \quad (I_{k+1} = 0),$$

then Eq. (7) coincides with the expressions given in Refs. 1, 6 — 9.

Equation (7) determines the probability of formation of the residual nucleus $(A + a - b_1 - \ldots - b_k)$ as the result of consecutive emission of the particles $b_1, b_2, \ldots, b_k$ from the nucleus $(A + a)$ in a given order. The same residual nucleus can be formed by emission of the same k particles in a different order or by emission of entire complexes of nuclei. For the description of a nuclear reaction it is therefore necessary to average Eq. (7) over all permutations of the k particles and then to carry out summation over all possible groupings of the emitted elementary particles, i.e.,

$$\bar{\eta} = \sum_{\text{group}} \frac{1}{k!} \sum_{(k)} (k)\, \eta_k, \tag{8}$$

where (k) denotes permutation; $\sum (k) = k!$

The cross-sections calculated by means of formulae (5), (6), and (8) can be compared with experimental data and used to compute the yield of various isotopes.

As an example, let us consider the following reaction:

$$Cu^{63}_{29} + p \to (Zn^{64}_{30})^* \to Cu^{62}_{29} + \begin{cases} n_1 + p_2 \\ p_1 + n_2 \\ d_1 \end{cases}.$$

Accordingly, the formation probability of the nucleus $Cu^{62}_{29}$ will be

$$\bar{\eta} = [\eta_2(n_1;\, p_2) + \eta_2(p_1;\, n_2)]/2! + \eta_1(d_1)\,/\,1!$$

In order to calculate $I_k(x)$ we make use of the following equation:[9]

$$I_k(x) = \int_0^x f_1(E_{b_1})\, dE_{b_1} \int_0^{x - E_{b_1}} f_2(E_{b_2})\, dE_{b_2} \ldots dE_{b_k} \tag{9}$$

where $f_i(E_{b_i})\,dE_{b_i}$ is the relative differential probability of the i-th emission process, as given by the formula[1]

$$f_i(E_{b_i})\, dE_{b_i} = b_i \gamma_{b_i} \sigma_c(b_i)\, E_{b_i} \omega_i\, dE_{b_i} / \sum_{\nu} \int^{E_{\nu\max}} f_i(E_{b_{i\nu}})\, dE_{b_{i\nu}}, \tag{10}$$

where $b_i$, $\gamma_{b_i}$, and $E_{b_i}$ are, respectively, the mass number, statistical weight, and kinetic energy of the emitted particle; $\sigma_c(b_i)$ is the production cross-section for the formation of a compound nucleus from the nuclei $(A + a - b_1 - \ldots - b_i)$ and $(b_i)$. We shall use the following approximate expression for $\sigma_c(b_i)$ [cf. Eq. (5)]:

$$\sigma_c(b_i) = \pi R^2_{b_i} \begin{cases} 1 - (1 - \alpha_{b_i})\, B_{b_i}/E_{b_i}; & E_{b_i} \geqslant (1 - \alpha_{b_i})\, B_{b_i}; \\ 0 & ;\ E_{b_i} < (1 - \alpha_{b_i})\, B_{b_i}. \end{cases} \tag{11}$$

$\omega_i$ is the level density in the nucleus $(A + a - b_1 - \ldots - b_i)$, given by the following formula:

$$\omega_i \sim (2J_i + 1)\, e^{S_i}, \tag{12}$$

where $J_i$ is the spin of the stable or metastable state of the nucleus $(A + a - b_1 - \ldots - b_i)$, and $S_i$ is its entropy:

$$S_i = S\Big(U_a + \sum_{\rho=1}^{i} (\varepsilon_{b_\rho} - E_{b_\rho});\ A + a - \sum_{\rho-1}^{i} b_\rho\Big). \tag{13}$$

Taking Eqs. (11) and (12) into account and introducing the notation

$$y_i = E_{b_i} - (1 - \alpha_{b_i}) B_{b_i},\ W_{b_i} = -\varepsilon_{b_i} + (1 - \alpha_{b_i}) B_{b_i}, \quad g_i = b_i \gamma_{b_i} R^2_{b_i} (2J_i + 1), \tag{14}$$

we obtain

$$f_i(y_i)\, dy_i = g_i y_i \exp\{S_i\}\, dy_i / \sum_{\nu} \int_0^{y_{\nu\max}} f_{i\nu}(y_i)\, dy_i, \tag{15}$$

where

$$S_i = S\Big(U_a - \sum_{\rho=1}^{i} (W_{b_\rho} + y_\rho);\ A + a - \sum_{\rho=1}^{i} b_\rho\Big); \qquad y^{\nu}_{\max} = U_a - \sum_{\rho=1}^{i-1} (W_{b_\rho} + y_\rho) - W_{b_{i,\nu}}. \tag{16}$$

Consequently, the expression for $\eta_k$ is

$$\eta_k = I_k(U_a - W_k) - \sum_{\nu} I_{k+1}(U_a - W_k - W_{b_{k+1,\nu}}), \tag{17}$$

where $I_n(x)$ is given by (9) and (15); the quantity $W_k = \sum_{i=1}^{k} W_{b_i}$ exceeds the threshold of emission of k particles from the excited nucleus, since we are determining the cross-sections for the reverse reactions induced by the charged particles by Eq. (11). In the case of neutron emission, $W_k$ is exactly equal to the threshold. If both charged particles and neutrons are emitted, $W_k$ represents excitation energy for which the yield of a given reaction becomes significant. In that case, $W_k$ will be called the effective threshold, equal to $W_k + W_{b_{k+1,\nu}}$ for any $(k + 1)$-th emission process.

In order to obtain the dependence of $\eta_k$ on $U_a$, a k-fold integration of Eq. (9) is necessary. It can be seen from Eqs. (9) and (15) that the inner integrals depend also on $U_a$, which makes the computation (numerical integration) difficult. In order to simplify the calculation, it is convenient to make first the following substitution:

$$U_a - \sum_{\rho=1}^{i} (W_{b_\rho} + y_\rho) = t_i. \tag{18}$$

The calculation of $I_k(x)$ presents no difficulties when the dependence of the entropy of the nucleus on the excitation energy and on the mass number is known. If the results are applied to the simplest nuclear reactions, where neutrons are emitted from heavy nuclei $(A > 100)$, it is possible to make several approximations.

First, it is possible to neglect the emission of protons, $\alpha$ particles, etc. We have then, since the last inner integral in the expression for $I_k(x)$ equals unity,

$$\bar{\eta}^n_k \approx I^n_{k-1}(U_a - W_k) - I^n_k(U_a - W_{k+1}), \tag{19}$$

where $W_k$ and $W_{k+1}$ are the thresholds of emission of the k-th and $(k+1)$-th neutron from the excited nucleus.

Taking it into account that $\eta_k \neq 0$ only when $U_a > W_k$ and that

$$\sum_{\rho=1}^{i} W_{b_\rho} < W_k;\ \sum_{\rho=1}^{i} b_\rho < A + a, \tag{20}$$

we obtain the following approximation for $S_i$:

$$S_i \approx S_a - \frac{\partial S_a}{\partial A} \sum_{\rho=1}^{i} b_\rho - \frac{\partial S_a}{\partial U_a} \sum_{\rho=1}^{i} (W_{b_\rho} + y_\rho). \tag{21}$$

Substituting (21) into (15) and then into (9), and taking it into account that $(\partial S_a/\partial U) y_{\nu\,max} \gg 1$, we obtain, after a few transformations,

$$I_k^n(x) \approx P_k\left(\frac{\partial S_a}{\partial U_a} x\right), \quad P_k(y) = \frac{1}{(2k-1)!} \int_0^y t^{2k-1} e^{-t}\, dt. \tag{22}$$

The functions $P_k(y)$ are tabulated[10] for various values of k and y. Graphs of the functions $P_k(y)$ for four values of k (indicated on the curves) and for $0 \leq y \leq 15$ are given in Fig. 1.

The dependence of the entropy on the excitation energy and mass number can be obtained by assuming a specific nuclear model. Using the results of the gas model, we have[6]

$$S_a \approx \pi \sqrt{(A+a)\,U_a/33}, \qquad U_a = E_a - \varepsilon_a. \tag{23}$$

The cross-section for the reaction $(A + a) \rightarrow (A + a - kn) + k(n)$ is, therefore,

$$\sigma_{kn}(E_a) \approx \sigma_c(a)\,[P_{k-1}(y_k) - P_k(y_{k+1})], \tag{24}$$

where

$$y_k = \frac{\pi}{2} \sqrt{\frac{A+a}{33}} \left(E_a - \varepsilon_a - W_k\right) \Big/ \sqrt{E_a - \varepsilon_a},$$

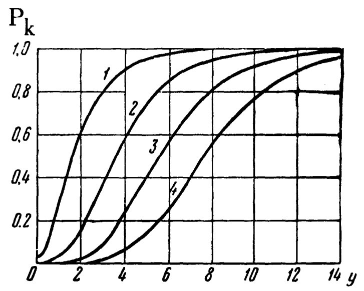


**FIG. 1.** Graphs of the functions $P_k(y)$.

$E_a$ is the energy of the incident particle in the center-of-mass system, and $\epsilon_a$ is the binding energy of this particle in the nucleus $(A + a)$.

The cross-sections for reactions accompanied by neutron emission have been calculated according to the formulae (24), (5), and (6), for incident protons, deuterons, and $\alpha$ particles. Certain thresholds $W_k$ necessary for the calculation of theoretical curves have been determined by comparison with experimental cross-sections, mainly from their ratios, in order to exclude the cross-section for the compound nucleus formation. A direct determination of the thresholds from mass values is difficult, especially in the case of emission of two, three, etc. neutrons, since the masses of corresponding isotopes are known to an accuracy of 1 — 2 Mev. In addition, the following radii $R_a$ have been assumed for the interactions between the incident particles and the nuclei:[1]

$$R_d \approx R_p = 1.5 A^{1/3} \cdot 10^{-13}\ \text{cm}; \quad R_\alpha = (1.5 A^{1/3} + 1.2) \cdot 10^{-13}\ \text{cm}.$$

The interaction radii are rather arbitrary and, as a rule, yield an overvalued geometrical cross-section (by ~ 10%). However, since the production cross-section for a nucleus as calculated from Eqs. (5) and (6) is only approximate, it is not necessary to know $R_a$ very accurately. Moreover, the dependence of the reaction cross-section on the energy of the incident particles will not change significantly since all cross-sections are calculated for $E_a \gtrsim 0.5\,B_a$, in which region the cross-section of compound nucleus formation depends relatively weakly on $E_a$ and is not very sensitive to small variations in $R_a$.

The results of the calculations are shown in Figs. 2 and 3. It can be seen that the cross-sections obtained with Eq. (24) are in good agreement with the experimental data. The cross-section for the (d, n) reaction has not been calculated, since this process is mainly due to deuteron stripping and to splitting of the latter in the Coulomb field of the nucleus. The cross-section for the $(\alpha, n)$ reaction is not shown, in view of its smallness.

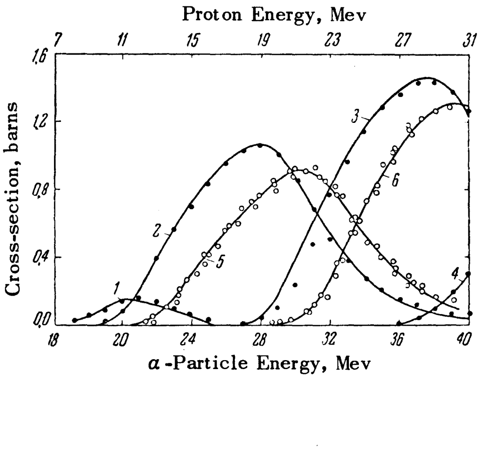


FIG. 2. Dependence of the $Bi^{209}_{83}$ reaction cross-section on the energy of incident particles: 1 — (p, n); 2 — (p, 2n); 3 — (p, 3n); 4 — (p, 4n); 5 — $(\alpha, 2n)$; 6 — $(\alpha, 3n)$. The solid curves are drawn according to Eq. (24); ● — data of Ref. 4; ○ — data of Ref. 3.

In conclusion, the author wishes to express his gratitude to M. M. Agrest for valuable advice and discussion.

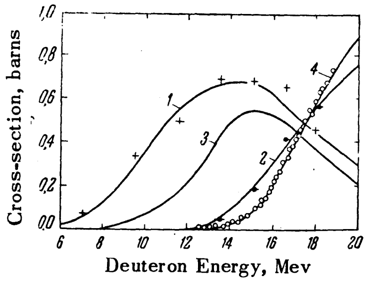


FIG. 3. Dependence of the $Bi^{209}_{83}$ reaction cross-section on the deuteron energy: 1 — (d, 2n); 2 — (d, 3n); and of the $I^{127}_{53}$ cross-section: 3 — (d, 2n); 4 — (d, 3n). Solid curves drawn according to Eq. (24); +, ● — data of Ref. 11; ○ — data of Ref. 3.

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SOVIET PHYSICS JETP VOLUME 6 (33), NUMBER 6 JUNE, 1958

# *SOME PROBLEMS OF MAGNETOGASDYNAMICS WITH ACCOUNT OF FINITE CONDUCTIVITY*

G. S. GOLITSYN and K. P. STANIUKOVICH

Moscow State University

Submitted to JETP editor June 6, 1957; resubmitted September 4, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 1417-1427 (December, 1957)

It is shown that if finite conductivity is taken into account, the equations of magnetogasdynamics become parabolically degenerate. The set of equations is replaced by an approximate but completely hyperbolic set, for which the characteristics are found. It is shown that the equations of a stationary one-dimensional flow have a singularity where the flow velocity is equal to the local sound velocity. Conditions of the transition of the flow velocity through this critical value under the action of a magnetic field have been studied. Small oscillations in a conducting medium, shock waves, and the structure of the shock are investigated.

THE magnetogasdynamics of an ideally conducting medium have been sufficiently studied. Types of vibration,[1,2] shock waves,[3,4] and their structure[5-7] have been investigated; one-dimensional motions have been studied,[8] where the characteristics were found for a system of equations and the particular (Reinmann) solution was found for arbitrary isentropy.

Taking the finite conductivity into account greatly complicates the equation by introducing new nonlinearities, raising the order of the system, and changing its character. As will be shown below, the