ANGULAR DISTRIBUTION AND POLARIZATION OF ELECTRONS FROM THE BETA-DECAY OF ORIENTED NUCLEI

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The angular distributions and polarizations of electrons (or positrons) from the β-decay of oriented nuclei are calculated for allowed transitions with inclusion of effects of the nonconservation of parity.

RECENTLY conducted experiments[1] have confirmed the hypothesis of Lee and Yang[2] that parity is not conserved in weak interactions. In a paper of Landau[3] the idea has been put forward that the Hamiltonian of the weak interactions is invariant with respect to combined inversion, i.e., with respect to two operations carried out in succession — inversion of space and charge conjugation (despite the possible non-invariance of the Hamiltonian in question with respect to each of these operations individually). The invariance of the theory with respect to combined inversion leads in particular to the reality of all the constants in the interaction of β-decay.

In the present paper we determine for allowed transitions the β-neutrino correlation, the angular distribution, and the polarization of the electrons (or positrons) produced in the β-decay of oriented nuclei. In doing this we make no preliminary assumptions about the real or imaginary nature of the different interaction constants. Therefore the formulas obtained may turn out to be useful for clearing up the question of the existence of invariance with respect to the combined inversion.

The most general expression for the Hamiltonian of the β-interaction in the case of nonconservation of parity has been given in Ref. 2. It has the form

$$H = (\psi_p^+\gamma_4\psi_n)(C_S\psi_e^+\gamma_4\psi_\nu + C_S'\psi_e^+\gamma_4\gamma_5\psi_\nu)$$
$$+ (\psi_p^+\gamma_4\gamma_\mu\psi_n)(C_V\psi_e^+\gamma_4\gamma_\mu\psi_\nu + C_V'\psi_e^+\gamma_4\gamma_\mu\gamma_5\psi_\nu) + {}^1/_2(\psi_p^+\gamma_4\sigma_{\lambda\mu}\psi_n)(C_T\psi_e^+\gamma_4\sigma_{\lambda\mu}\psi_\nu + C_T'\psi_e^+\gamma_4\sigma_{\lambda\mu}\gamma_5\psi_\nu)$$
$$+ (\psi_p^+\gamma_4\gamma_\mu\gamma_5\psi_n)(-C_A\psi_e^+\gamma_4\gamma_\mu\gamma_5\psi_\nu - C_A'\psi_e^+\gamma_4\gamma_\mu\psi_\nu) + (\psi_p^+\gamma_4\gamma_5\psi_n)(C_P\psi_e^+\gamma_4\gamma_5\psi_\nu + C_P'\psi_e^+\gamma_4\psi_\nu) + \text{Hermitian conj.} \quad (1)$$

Since in the following we consider only allowed transitions, in the calculation of the matrix elements we can neglect the pseudoscalar interaction and replace the wave functions of the electron (or positron) and antineutrino (or neutrino) by their values for $r \to 0$.

For definiteness let us first consider electron emission. The wave function of an electron moving in the Coulomb field of the nucleus and having at infinity the momentum $\mathbf{p}$ and the polarization μ is described by the wave function

$$\psi_{\mathbf{p}\mu} = 8\pi\sqrt{\frac{m}{2\varepsilon}}\sum_{jlM}(v_\mu, \Omega_{jlM}^*(\mathbf{p}))\, i^{l-1}e^{-i\delta_{jl}}\frac{e^{\pi\nu/2}\,|\Gamma(\gamma_{jl}+i\nu)|}{\Gamma(2\gamma_{jl}+1)}(2pr)^{\gamma_{jl}-1}$$
$$\times\begin{pmatrix} i\sqrt{\frac{\varepsilon}{m}+1}\,\mathrm{Re}\left[e^{-i(pr-\eta_{jl})}(\gamma_{jl}+i\nu)F(\gamma_{jl}+1+i\nu, 2\gamma_{jl}+1; 2ipr)\right]\Omega_{jlM}(\mathbf{r}) \\ \sqrt{\frac{\varepsilon}{m}-1}\,\mathrm{Im}\left[e^{-i(pr-\eta_{jl})}(\gamma_{jl}+i\nu)F(\gamma_{jl}+1+i\nu, 2\gamma_{jl}+1; 2ipr)\right]\Omega_{jl'M}(\mathbf{r}) \end{pmatrix} \quad (2)$$

Here we have used the notations adopted in the book of Akhiezer and Berestetskii[4] [in this usage v_μ is a normalized two-component spinor, $(v_\mu^*, v_\mu) = 1$]. At infinity the function $\psi_{\mathbf{p}\mu}$ has the form of the superposition of a plane wave and a converging spherical wave and is normalized to unit amplitude of the plane wave. For $r \to 0$ one keeps in the sum over j and l in Eq. (2) only the two terms with $j = \frac{1}{2}$ and $l = 0, 1$, and sets $r = 0$ everywhere except in the factor $(2pr)^{\gamma_0 - 1}$, where $\gamma_0 = (1 - Z^2e^2)^{1/2}$. Then we get

$$\psi_{\substack{p\mu \\ r\to 0}} = \frac{\Gamma(\gamma_0+i\nu)\, e^{\pi\nu/2} e^{i\pi(\gamma_0-1)/2}}{\sqrt{2}\,\Gamma(2\gamma_0+1)} (2pr)^{\gamma_0-1} \begin{pmatrix} \left\{\left[(1+\gamma_0)\sqrt{1+\frac{m}{\varepsilon}} + iZe^2\sqrt{1-\frac{m}{\varepsilon}}\right] + \right. \\ \left. +\left[(\gamma_0-1)\sqrt{1+\frac{m}{\varepsilon}} + iZe^2\sqrt{1-\frac{m}{\varepsilon}}\right](\sigma \mathbf{n}_r)(\sigma \mathbf{n}_p)\right\} v_\mu(\xi) \\ \left\{\left[(\gamma_0-1)\sqrt{1-\frac{m}{\varepsilon}} + iZe^2\sqrt{1+\frac{m}{\varepsilon}}\right](\sigma \mathbf{n}_r) + \right. \\ \left. +\left[(\gamma_0+1)\sqrt{1-\frac{m}{\varepsilon}} + iZe^2\sqrt{1+\frac{m}{\varepsilon}}\right](\sigma \mathbf{n}_p)\right\} v_\mu(\xi), \end{pmatrix}, \tag{3}$$

where $\mathbf{n}_r$ and $\mathbf{n}_p$ are unit vectors in the directions of $\mathbf{r}$ and $\mathbf{p}$. In allowed transitions there is no change of the parity of the nuclear state, and therefore those terms in Eq. (3) which contain the factor $\mathbf{n}_r$ contribute nothing to the matrix elements of the transitions in question. Dropping them and making the usual replacement of $(2pr)^{\gamma_0-1}$ by $(2pR)^{\gamma_0-1}$ (R is the radius of the nucleus), we can rewrite Eq. (3) in the following way:

$$\psi^0_{p\mu} = f u^c_{p\mu}(\xi), \tag{4}$$

where

$$f = \frac{\sqrt{2(1+\gamma_0)}\,\Gamma(\gamma_0+i\nu)\, e^{\pi\nu/2} e^{i\pi(\gamma_0-1)/2}}{\Gamma(2\gamma_0+1)} (2pR)^{\gamma_0-1}, \tag{5}$$

and $u^c_{p\mu}(\xi)$ is a spin function (ξ is the spin variable) which has the form

$$u^c_{p\mu}(\xi) = \begin{vmatrix} a v_\mu \\ b(\sigma \mathbf{n}_p)\cdot v_\mu \end{vmatrix}, \tag{6}$$

$$a = \frac{1}{2\sqrt{1+\gamma_0}}\left[(\gamma_0+1)\sqrt{1+\frac{m}{\varepsilon}} + iZe^2\sqrt{1-\frac{m}{\varepsilon}}\right], \quad b = \frac{1}{2\sqrt{1+\gamma_0}}\left[(\gamma_0-1)\sqrt{1-\frac{m}{\varepsilon}} + iZe^2\sqrt{1+\frac{m}{\varepsilon}}\right]. \tag{7}$$

For the wave function of a free electron with momentum $\mathbf{p}$ and polarization μ we have at $r = 0$ the expression

$$u^0_{p\mu}(\xi) = \begin{vmatrix} a_0 v_\mu \\ b_0(\sigma \mathbf{n}_p) v_\mu \end{vmatrix}, \tag{8}$$

$$a_0 = \sqrt{1/2(1+m/\varepsilon)}, \quad b_0 = \sqrt{1/2(1-m/\varepsilon)}. \tag{9}$$

In analogy with Eq. (8), the function ψ_ν corresponding to the production of an antineutrino with momentum $\mathbf{q}$ (and mass zero) has at $r = 0$ the form

$$u_{q\nu} = \frac{1}{\sqrt{2}}\begin{vmatrix} v_\nu \\ ((\sigma \mathbf{q})/q) v_\nu \end{vmatrix}. \tag{10}$$

The β-decay matrix element in which we are interested can now be written in the following way:

$$M = f\,(u^{c*}_{p\mu} O u_{q\nu}), \tag{11}$$

where

$$O = A[\gamma_4\gamma(C_S + C'_S\gamma_5) + (C_v + C'_v\gamma_5)] - i\mathbf{B}[\gamma(C_T\gamma_5 + C'_T) + \gamma_4\gamma(C_A\gamma_5 + C'_A)], \tag{12}$$

$$A = \int \Psi^*_f \Psi_i\, d\tau, \quad \mathbf{B} = \int \Psi^*_f \sigma \Psi_i\, d\tau, \tag{13}$$

and Ψ_i and Ψ_f are the initial and final wave functions of the nucleus.

To calculate the probability of the decay we must sum the square of the absolute value of the expression (11) over the spins of the electron and antineutrino. This summation is easily carried out by means of the matrices N and P,

$$N_{\eta\eta'} = \sum_\nu u_{q\nu}(\eta)\, u^*_{q\nu}(\eta'), \quad P_{\xi\xi'} = \sum_\mu u^c_{p\mu}(\xi)\, u^{c*}_{p\mu}(\xi') \tag{14}$$

and gives

$$\sum_{\mu\nu} |M|^2 = |f|^2 \sum_{\mu\nu} (u_{p\mu}^{c*} O u_{q\nu}) (u_{q\nu}^{*} O^{+} u_{p\mu}^{c}) = |f|^2 \operatorname{Sp}(ONO^{+}P); \tag{15}$$

N is the usual projection operator

$$N(\mathbf{q}) = -i\hat{q}\gamma_4 / 2q. \tag{16}$$

By means of Eqs. (6) and (7) it is not hard to obtain also the explicit expression for the matrix P:

$$P = \begin{vmatrix} |a|^2 & ab^{*}\sigma\mathbf{n}_p \\ a^{*}b\sigma\mathbf{n}_p & |b|^2 \end{vmatrix} = \frac{m - i\hat{p}}{2\varepsilon}\gamma_4 + \frac{m}{2\varepsilon}[(\gamma_0 - 1) + Ze^2\mathbf{n}_p\gamma]. \tag{17}$$

We finally get for the differential probability of the β-decay (in units $mc^2/\hbar$) the expression

$$N\left(\mathbf{p}, \frac{\mathbf{q}}{q}\right)\frac{d\varepsilon}{m}d\Omega_e d\Omega_\nu = F\left(Z, \frac{\varepsilon}{m}\right)\overline{\operatorname{Sp}(ONO^{+}P)}(2\pi m)^{-5}p\varepsilon(\varepsilon_0 - \varepsilon)^2 d\varepsilon d\Omega_e d\Omega_\nu, \tag{18}$$

where the bar over the factor denotes averaging with respect to M and summing over M′, these being the magnetic quantum numbers of the initial and final states of the nucleus, respectively; ϵ_0 is the maximum energy of the electrons; and $F(Z, \epsilon/m)$ is the usual notation for the function which includes the influence of the Coulomb field of the nucleus:[5]

$$F(Z, \varepsilon/m) = |f|^2. \tag{19}$$

To calculate the polarization of the emitted electrons we introduce the density matrix

$$\rho(\xi, \xi') = Q\sum_{\mu\lambda\mu}\overline{u_\mu^0(\xi)(u_\mu^{c*}(\eta)O_{\eta\eta'}u_\nu(\eta'))(u_\nu^{*}(\zeta)O_{\zeta\zeta'}^{+}u_\lambda^{c}(\zeta'))u_\lambda^{0*}(\xi')}. \tag{20}$$

Q is a constant to be determined from the normalization condition $\operatorname{Sp}\rho = 1$. We further define the matrix R:

$$R_{\xi\eta} = \sum_\mu u_\mu^0(\xi)u_\mu^{c*}(\eta). \tag{21}$$

By the use of Eqs. (6) — (9) we find

$$R = \sqrt{\frac{\gamma_0 + 1}{2}}\frac{(m - i\hat{p})\gamma_4}{2\varepsilon}\left[1 - i\frac{Ze^2}{(\gamma_0 + 1)p}(\varepsilon - m\gamma_4)\right]. \tag{22}$$

It is not hard to verify that $R^{+}R = P$, and therefore the correctly normalized matrix ρ can be written in the following form:

$$\rho = RONO^{+}R^{+} / \overline{\operatorname{Sp}(ONO^{+}P)}. \tag{23}$$

If we do not prescribe the direction of emission of the neutrino, then we must proceed further to average the numerator and the denominator in Eq. (23) over the direction of **q**. We then get for the density matrix

$$\rho = \int (RONO^{+}R^{+})d\Omega_\nu \Big/ \int \overline{\operatorname{Sp}(ONO^{+}P)}d\Omega_\nu. \tag{24}$$

On the other hand, the general expression for the density matrix ρ describing the polarization properties of the electrons moving with the momentum **p** has the form[6]

$$\rho = (1 + i\gamma_5\hat{\zeta})(m - i\hat{p})\gamma_4 / 4\varepsilon. \tag{25}$$

The four-vector $\zeta_\mu = (\boldsymbol{\zeta}, \zeta_0)$ satisfies the condition $\zeta_\mu p_\mu = 0$. Therefore in a system of coordinates in which $\mathbf{p} = 0$, we have $\zeta_0 = 0$, i.e., $\zeta_\mu^0 = (\boldsymbol{\zeta}^0, 0)$. Consequently, the polarization properties can be defined, as is usually done, by means of the three-dimensional vector $\boldsymbol{\zeta}^0$ connected with $\boldsymbol{\zeta}$ by the relation

$$\zeta^0 = \zeta_t + (m/\varepsilon)\zeta_l, \tag{26}$$

where ζ_t and ζ_l are the transverse and longitudinal components of the vector ζ. Equations (24) and (25) give

$$\zeta_\mu = \frac{i\varepsilon}{m}\,\mathrm{Sp}\,(\rho\gamma_4\gamma_5\gamma_\mu) = \frac{i\varepsilon}{m}\int \mathrm{Sp}\,(\overline{RONO^+R^+\gamma_4\gamma_5\gamma_\mu})\,d\Omega_\nu \Big/ \int \overline{\mathrm{Sp}\,(ONO^+P)}\,d\Omega_\nu. \qquad (27)$$

The further calculation of the angular distributions by means of Eqs. (12), (13), and (16) — (18) and of the polarization of the electrons by means of these same equations and Eqs. (26) and (27) is carried out in the usual way. Therefore we at once give below the results of these calculations for the three types of allowed transitions, $J' = J,\ J \pm 1$ for both the electron and the positron decays.* Here we have introduced, in addition to the usually accepted notations,[2,5] also the following: $\mathbf{n}_\nu = \mathbf{q}/q$, and $\mathbf{j}$ is the unit vector in the direction of orientation of the nuclei. In what follows the index zero is omitted from the quantity $\gamma_0 = (1 - Z^2e^2)^{1/2}$.

I. Correlation of directions of the β particle and the neutrino

$$N(\mathbf{p}, \mathbf{n}_\nu)\,dW\,d\Omega_e\,d\Omega_\nu = (2\pi)^{-5}\xi F\,(\pm Z, W)\,(W^2-1)^{1/2}\,W\,(W_0-W)^2\,dW\,d\Omega_e\,d\Omega_\nu$$
$$\times\Big\{1 + \Big(a_1 + \frac{Ze^2}{Wv_e}a_2\Big)(\mathbf{v}_e\mathbf{n}_\nu) + \frac{b}{W} + \frac{\langle J_z\rangle}{J}\Big[\Big(c_1 + \frac{Ze^2}{Wv_e}c_2\Big)(\mathbf{j}\mathbf{v}_e)$$
$$+ \Big(d_1 + \frac{\gamma}{W}d_2\Big)(\mathbf{j}\mathbf{n}_\nu) + \Big(g_1 + g_2\frac{Ze^2}{Wv_e}\Big)(\mathbf{j}\,[\mathbf{n}_\nu\times\mathbf{v}_e])\Big] + \Big(f_1 + \frac{Ze^2}{Wv_e}f_2\Big)\Big(\frac{1}{3}\,\mathbf{v}_e\mathbf{n}_\nu - (\mathbf{v}_e\mathbf{j})(\mathbf{n}_\nu\mathbf{j})\Big)\Big(1 - \frac{3\langle J_z^2\rangle}{J(J+1)}\Big)\Big\}. \qquad (28)$$

Here

$$\xi = (|C_T|^2 + |C'_T|^2 + |C_A|^2 + |C'_A|^2)\,|M_{\mathrm{GT}}|^2 + (|C_S|^2 + |C'_S|^2 + |C_V|^2 + |C'_V|^2)\,|M_{\mathrm{F}}|^2,$$

$$a_1\xi = \frac{1}{3}(|C_T|^2 + |C'_T|^2 - |C_A|^2 - |C'_A|^2)\,|M_{\mathrm{GT}}|^2 - (|C_S|^2 + |C'_S|^2 - |C_V|^2 - |C'_V|^2)\,|M_{\mathrm{F}}|^2,$$

$$a_2\xi = \pm\Big[-\frac{2}{3}\,\mathrm{Im}\,(C_AC_T^* + C'_AC'^*_T)\,|M_{\mathrm{GT}}|^2 + 2\,\mathrm{Im}\,(C_VC_S^* + C'_VC'^*_S)\,|M_{\mathrm{F}}|^2\Big],$$

$$b\xi = \pm 2\gamma\,[\mathrm{Re}\,(C_TC_A^* + C'_TC'^*_A)\,|M_{\mathrm{GT}}|^2 + \mathrm{Re}\,(C_SC_V^* + C'_SC'^*_V)\,|M_{\mathrm{F}}|^2],$$

$$c_1\xi = \pm 2\,\mathrm{Re}\,(C_TC'^*_T - C_AC'^*_A)\,|M_{\mathrm{GT}}|^2\varepsilon_{JJ'} + 2\sqrt{\frac{J}{J+1}}\,\mathrm{Re}\,[(C_TC'^*_S + C'_TC_S^* - C_AC'^*_V - C'_AC_V^*)\,M_{\mathrm{F}}^*M_{\mathrm{GT}}],$$

$$c_2\xi = 2\,\mathrm{Im}\,(C_TC'^*_A + C'_TC_A^*)\,|M_{\mathrm{GT}}|^2\varepsilon_{JJ'} \pm 2\sqrt{\frac{J}{J+1}}\,\mathrm{Im}\,[(C_TC'^*_V + C'_TC_V^* - C_AC'^*_S - C'_AC_S^*)\,M_{\mathrm{F}}^*M_{\mathrm{GT}}],$$

$$d_1\xi = \pm 2\,\mathrm{Re}\,(C_TC'^*_T + C_AC'^*_A)\,|M_{\mathrm{GT}}|^2\varepsilon_{JJ'} - 2\sqrt{\frac{J}{J+1}}\,\mathrm{Re}\,[C_TC'^*_S + C'_TC_S^* + C_AC'^*_V + C'_AC_V^*)\,M_{\mathrm{F}}^*M_{\mathrm{GT}}],$$

$$d_2\xi = \pm 2\,\mathrm{Re}\,(C_TC'^*_A + C'_TC_A^*)\,|M_{\mathrm{GT}}|^2\varepsilon_{JJ'} - 2\sqrt{\frac{J}{J+1}}\,\mathrm{Re}\,[(C_TC'^*_V + C'_TC_V^* + C_AC'^*_S + C'_AC_S^*)\,M_{\mathrm{F}}^*M_{\mathrm{GT}}],$$

$$g_1\xi = 2\sqrt{\frac{J}{J+1}}\,\mathrm{Im}\,[(C_TC_S^* + C'_TC'^*_S - C_AC_V^* - C'_AC'^*_V)\,M_{\mathrm{F}}^*M_{\mathrm{GT}}],$$

$$g_2\xi = \mp 2\sqrt{\frac{J}{J+1}}\,\mathrm{Re}\,[(C_TC_V^* + C'_TC'^*_V - C_AC_S^* - C'_AC'^*_S)\,M_{\mathrm{F}}^*M_{\mathrm{GT}}].$$

$$f_1\xi = (|C_T|^2 + |C'_T|^2 - |C_A|^2 - |C'_A|^2)\,|M_{\mathrm{GT}}|^2\Delta_{JJ'}, \quad f_2\xi = \mp 2\,\mathrm{Im}\,(C_AC_T^* + C'_AC'^*_T)\,|M_{\mathrm{GT}}|^2\Delta_{JJ'}.$$

The upper signs refer everywhere to the electron decay, and the lower signs to the positron decay. The functions $\epsilon_{JJ'}$ and $\Delta_{JJ'}$ are defined by the relations

$$\varepsilon_{JJ'} = \begin{cases} \dfrac{1}{J+1} & \text{if } J' = J, \\ 1, & \text{if } J' = J-1, \\ -\dfrac{J}{J+1}, & \text{if } J' = J+1, \end{cases} \qquad \Delta_{JJ'} = \begin{cases} -1, & \text{if } J' = J, \\ \dfrac{J+1}{2J-1}, & \text{if } J' = J-1, \\ \dfrac{J}{2J+3}, & \text{if } J' = J+1. \end{cases} \qquad (29)$$

*The formulas for the positron decay are obtained from the corresponding formulas for the electron decay if one makes in the latter the following replacements: (1) C_S, C_A, C_P, C'_V, C'_T by C_S^*, C_A^*, C_P^*, C'^*_V, C'^*_T; (2) C'_S, C'_A, C'_P, C_V, C_T by $-C'^*_S$, $-C'^*_A$, $-C'^*_P$, $-C_V^*$, $-C_T^*$; and (3) e^2 by $-e^2$.

For unoriented nuclei $\langle J'_z\rangle = 0$ and $1 - \frac{3\langle J_z^2\rangle}{J(J+1)} = 0$.

In virtue of the invariance of the Hamiltonian for the strong interactions with respect to time reversal, the product $M_F^* M_{GT}$ is real,[7] i.e.,

$$M_F^* \cdot M_{GT} = \pm |M_F| \cdot |M_{GT}|. \tag{30}$$

It must be pointed out that the term $(Ze^2/Wv_e)\, a_2(\mathbf{v}_e\mathbf{n}_\nu)$ in the curly brackets of Eq. (28) is omitted through an error from Eq. (A.2) of Lee and Yang's paper.[2]

II. Integration of Eq. (28) over all angles of emission of the neutrino gives the angular distribution of the electrons (or positrons)

$$N(\mathrm{p})\, dW d\Omega_e = \frac{\xi}{8\pi^4}\left(1 + \frac{b}{W}\right) F(\pm Z,\ W)(W^2 - 1)^{1/2}\, W(W_0 - W)^2 dW \cdot (1 + \alpha\cos\vartheta), \tag{31}$$

ϑ is the angle between $\mathbf{v}_e$ and $\mathbf{j}$, and

$$\alpha = \frac{\langle J_z\rangle}{J}\,\frac{c_1 + c_2 Ze^2/Wv_e}{1 + b/W}\, v_e. \tag{32}$$

For the transitions $J \to J - 1$ and $J \to J + J$ Eq. (32) gives results identical with those obtained in Eqs. (A.6) and (A.7) of Ref. 2.

III. The general expression for the polarization of the electrons (or positrons) of energy W emitted at the angle ϑ with the direction of orientation of the nuclei has the following form:

$$\zeta^0 = \left\{\xi\left[1 + \frac{b}{W} + \frac{\langle J_z\rangle}{J}\left(c_1 + c_2\frac{Ze^2}{Wv_e}\right) v_e\cos\vartheta\right]\right\}^{-1}\left\{\left(\alpha_1 + \alpha_2\frac{Ze^2}{Wv_e}\right.\right.$$

$$\left.+\ \alpha_3\,\frac{1 - (\gamma/W)\frac{\langle J_z\rangle}{J}\cos\vartheta)}{v_e}\,\mathbf{v}_e + \left(\beta_1 + \frac{\beta_2}{W}\right)\frac{\langle J_z\rangle}{J}\,\mathbf{j} + \left(\gamma_1 + \gamma_2\frac{Ze^2}{Wv_e}\right)\frac{\langle J_z\rangle}{J}[\mathbf{j},\ \mathbf{v}_e]\right\}, \tag{33}$$

$$\alpha_1 = \pm 2\,[\mathrm{Re}\,(C_T C_T'^* - C_A C_A'^*)\,|M_{GT}|^2 + \mathrm{Re}\,(C_S C_S'^* - C_V C_V'^*)\,|M_F|^2],$$

$$\alpha_2 = 2\,[\mathrm{Im}\,(C_T C_A'^* + C_T' C_A^*)\,|M_{GT}|^2 + \mathrm{Im}\,(C_S C_V'^* + C_S' C_V^*)\,|M_F|^2],$$

$$\alpha_3 = [|C_T|^2 + |C_T'|^2 + |C_A|^2 + |C_A'|^2 \mp 2\,\mathrm{Re}\,(C_T C_A^* + C_T' C_A'^*)]\,|M_{GT}|^2 \varepsilon_{JJ'} + 2\sqrt{\frac{J}{J+1}}\{\pm\mathrm{Re}\,[(C_T C_S^*$$

$$+ C_T' C_S'^* + C_A' C_V'^* + C_A C_V^*)\, M_F^* M_{GT}] - \mathrm{Re}\,[(C_T C_V^* + C_T' C_V'^* + C_A C_S^* + C_A' C_S'^*)\, M_F^* M_{GT}]\},$$

$$\beta_1 = \pm 2\,\mathrm{Re}\,(C_T C_A^* + C_T' C_A'^*)\,|M_{GT}|^2 \varepsilon_{JJ'} + 2\sqrt{\frac{J}{J+1}}\,\mathrm{Re}\,[(C_T C_V^* + C_T' C_V'^* + C_A C_S^* + C_A' C_S'^*)\, M_F^* M_{GT}],$$

$$\beta_2 = \gamma\left\{(|C_T|^2 + |C_T'|^2 + |C_A|^2 + |C_A'|^2)\,|M_{GT}|^2 \varepsilon_{JJ'} \pm 2\sqrt{\frac{J}{J+1}}\ \mathrm{Re}\,[(C_T C_S^* + C_T' C_S'^* + C_A' C_V'^* + C_A C_V^*)\, M_F^* M_{GT}]\right\},$$

$$\gamma_1 = \pm 2\,\mathrm{Im}\,(C_T C_A'^* + C_T' C_A^*)\,|M_{GT}|^2 \varepsilon_{JJ'} + 2\sqrt{\frac{J}{J+1}}\,\mathrm{Im}\,[(C_T C_V'^* + C_T' C_V^* - C_A C_S'^* - C_A' C_S^*)\, M_F^* M_{GT}],$$

$$\gamma_2 = -2\,\mathrm{Re}\,(C_T C_T'^* - C_A C_A'^*)\,|M_{GT}|^2 \varepsilon_{JJ'} \mp 2\sqrt{\frac{J}{J+1}}\,\mathrm{Re}\,[(C_T C_S'^* + C_T' C_S^* - C_A C_V'^* - C_A' C_V^*)\, M_F^* M_{GT}].$$

If the neutrino is a longitudinal particle,[8,9] then we should put $C' = -C$ in all the formulas.

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CONNECTION BETWEEN THE HULTHÉN AND KOHN METHODS IN COLLISION THEORY

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Various possible direct variational methods for the determination of the phase shifts of the radial wave function are considered. It is shown that the most natural criterion of the quality of the trial function is the condition of the consistency of the equations. The comparison of the phase-shift results obtained by the Hulthén and Kohn methods, and the verification of whether the integral identity is satisfied, are not independent criteria and in fact reduce to the consistency condition. It is also shown how the correct value can be chosen from two phase-shift values obtained by the Hulthén method, without resorting to comparison with the results of other methods.

WE consider the equation for the phase shift in collision theory:

$$\psi''(r) + (k^2 - V)\psi(r) = 0; \tag{1}$$

$$\psi(0) = 0, \quad \psi|_{r\to\infty} \sim A\sin(kr + \eta). \tag{2}$$

The variational principle for this problem can be written in the form[1]

$$\delta J = \delta\int_0^\infty \psi(r)\left(\frac{d^2}{dr^2} + k^2 - V\right)\psi(r)\,dr = -A^2 k\,\delta\eta. \tag{3}$$

If we substitute in this functional an approximate function $\tilde{\psi}(r)$ which satisfies the conditions (2) and which depends on the parameters α_i, we can obtain equations for these parameters from the variational principle. It is well known that, in contrast to the problem of the discrete spectrum, the set of equations can be formed in this case, in a non-unique fashion.

We consider the very simple but also very important case in which a linear combination of n functions $\varphi_i(r)$ are substituted in the functional J:

$$\tilde{\psi}(r) = \sum_{i=1}^{n} \alpha_i \varphi_i(r). \tag{4}$$

In this case, let