INTERPRETATION OF AN "ANOMALOUS" REPRESENTATION OF THE INVERSION GROUP

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THE purpose of this paper is to obtain a simple interpretation of an "anomalous" representation of the group of inversions of four-dimensional space-time. Gel'fand and Tsetlin[1] gave the name "anomalous" to a representation using operators which satisfy the condition

$$[T_{i'k'}T_{i''k''}]_+ = 0, \quad i, k = 0, 1, \quad i \neq k, \tag{1}$$

The inversion operators themselves of course commute.

Gel'fand and Tsetlin showed that, in the case of a scalar representation of the full Lorentz group, the anomalous representation is equivalent to the concept of parity doublets.[2] Our purpose here is to interpret the anomalous representation of the group of inversions of four-dimensional space-time in terms of a representation of the five-dimensional proper orthogonal group with pseudo-Euclidean metric.

The time-inversion T_{10} can be interpreted by means of a rotation operator H_{54} connecting the two half-spinors of four-dimensional space which represent a five-dimensional spinor. We may write

$$H_{54} = \exp\left(\frac{i\pi}{2} I_{54}\right), \tag{2}$$

where I_{54} is an infinitesimal operator of the representation O_5. It is easy to see that

$$T_{10} = H_{54} = \begin{pmatrix} 0' & i(-1)^{k+1}I \\ i(-1)^k I & 0' \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{3}$$

We introduce a five-dimensional vector corresponding to the five-dimensional spinor. The vector is determined by a 4×4 matrix X which satisfies

$$X\psi = 0$$

and is defined by

$$X = \gamma_i x_i \ (i = 1, 2, \ldots, 5), \quad X^2 = x_5^2 + x_4^2 - x_1^2 - x_2^2 - x_3^2.$$

If

$$H_{54}^* X H_{54} = X', \tag{5}$$

then X' is defined by the coordinates $(-x_5, -x_4, x_1, x_2, x_3)$. Thus H_{54} corresponds correctly to the operation of time-inversion.

In the same way one can show that T_{11} defined by

$$T_{11} = H_{43}H_{21}, \quad H_{21} = \exp(i\pi I_{21}); \quad H_{43} = \exp(\pi I_{43})$$

corresponds to inversion of both space and time. Thus

$$T_{11} = \begin{pmatrix} i(-1)^k & 0' \\ 0' & i(-1)^{k+1} \end{pmatrix}. \tag{6}$$

Also

$$T_{01} = T_{11}T_{10}, \tag{6'}$$

defines T_{01} as the operator of space inversion $x_i' = -x_i \ (i = 1, 2, 3)$. There exists also an alternative anomalous representation in which T_{10} and T_{01} are respectively the space and time inversions. In this case Eq. (3), (6), and (6') coincide with formulae given by Gel'fand and Tsetlin.[1]

The foregoing results are easily extended to any finite-dimensional representation of the full Lorentz group. One has only to use the fact that every representation of the rotation group is a direct product of an even or odd number of spinor representations.

We consider the connection of the anomalous representation with the four-dimensional Dirac equation.

We choose the anomalous representation in which T_{10} and T_{01} are respectively space and time inversions. Then T_{10}, T_{01}, T_{11} leave invariant only an extended 8-component Dirac equation

$$\Gamma_i \frac{\partial\psi}{\partial x_i} + m\psi = 0, \quad \Gamma_i = \begin{pmatrix} \gamma_i & 0 \\ 0 & -\gamma_i \end{pmatrix}, \tag{7}$$

whereas the second variant of the anomalous representation leaves invariant a 4-component Dirac equation. The first variant of the anomalous representation also leaves invariant the Dirac equation for 4-spinors of the second kind[2]

$$\gamma_i \partial\psi / \partial x_i + m\gamma_5 \psi = 0. \tag{8}$$

Fermions with $m \neq 0$ which are described by the anomalous representation are in some ways reminiscent of the longitudinal neutrino which obeys the 2-component Dirac equation.[3] According to the choice of representation, they satisfy either a 4-component or an 8-component equation, and in the latter case T_{11} plays the role of γ_5 in the 2-component theory. It is possible that the eight components of the Dirac equation which describes "anomalous" Fermions may be connected with the existence of isotopic spin.

In conclusion we observe that Schwinger[4] proposed a connection between the Pauli exclusion principle and the lack of invariance of the Dirac equation under time-inversion. Ordinary time-inversion interchanges the two equations

$$\gamma_i \partial\psi / \partial x_i + m\psi = 0 \text{ and } \gamma_i \partial\psi / \partial x_i - m\psi = 0,$$

In the case of spinor particles described by the anomalous representation of the full Lorentz group (using the second variant, with T_{10} and T_{01} for time and space inversion respectively) the Lagrangian of the system is a true scalar under T_{10}. In other words, changing the order of factors in it does not change its sign. The requirement that the Lagrangian be invariant under the inversion T_{10} does therefore not lead to any additional restriction on ψ, and consequently does not imply the Pauli principle.

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[1] I. M. Gel'fand and M. L. Tsetlin, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 1107 (1956), Soviet Phys. JETP **4**, 947 (1957).

[2] É. Cartan, "Leçons sur la Théorie des Spineurs" (2 vol., Hermann, Paris, 1938).

[3] L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 59 (1957), Soviet Phys. JETP **5**, 101 (1957).

[4] J. Schwinger, Phys. Rev. **82**, 914 (1951).

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GALVANOMAGNETIC PROPERTIES OF MANGANESE FERRITE

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DATA on the galvanomagnetic properties of the ferromagnetic semiconductors — ferrites — are still quite scant. Yet a study of these properties is of interest from the point of view of clarifying the nature of the conductivity of materials of this type, and also from the point of view of establishing a connection between the electric properties and their antiferromagnetic state, since ferrites can be classified as "uncompensated" antiferromagnets.

We measured the temperature dependence of the galvanomagnetic effect in a ferrite consisting of 50% (mol.) MnO and 50% (mol.) Fe_2O_3. The resistance of this ferrite is not too high and the effect can therefore be measured with direct current over the range from room temperature to 350°C. The ferrite was