SCATTERING OF LIGHT IN A FERMI LIQUID

A. A. ABRIKOSOV and I. M. KHALATNIKOV

Institute for Physical Problems, Academy of Sciences, U.S.S.R.

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The scattering of light in a Fermi liquid is studied. An expression is obtained for the distribution of the scattered light with respect to angle and frequency, and estimates are carried out for liquid He^3.

LANDAU, in one of his papers,[1] has shown that it is possible for a special type of oscillation called "zero sound" to be propagated in Fermi liquids at sufficiently low temperatures. One of the conditions for the possibility of propagating "zero sound" is, in particular, the inequality

$$\omega_s \tau \gg 1, \qquad (1)$$

where τ is a relaxation time which, for He^3, is of the order of $10^{-12}T^{-2}$ sec.[2] If, for example, the temperature where $\sim 0.01°$ K, then a frequency of more than 10^8 cycles per second would be required to observe zero sound, so that such an experiment would be very difficult to perform.

In place of this method, an indirect method can be proposed which consists in observing Rayleigh scattering of light in liquid He^3.* As is well known, in Rayleigh scattering there arise, in addition to the principal line, satellite lines differing from it in frequency by

$$\Delta\omega = \pm 2 \frac{u}{c} \omega \sin\frac{\theta}{2},$$

where u is the velocity of sound and θ is the scattering angle. The velocity of zero sound in He^3 is of the order of 2×10^4 cm/sec, so that $\Delta\omega \sim 10^{-6}\omega$.[2] Thus, in principle, the velocity of zero sound can be measured by observing the frequency distribution of the scattered light. The requirement (1) can be satisfied because of the high frequency corresponding to visible light.

Apart from the above considerations, the scattering of light in a Fermi liquid at sufficiently low temperatures has several specific features, which make it of interest to carry out a theoretical study of this phenomenon, particularly the distribution of intensity with respect to frequency.*

As is well known, the frequency dependence and the angular distribution for Rayleigh scattering of unpolarized light are given by the equation†

$$dh = \frac{\omega^4}{6\pi c^4}\frac{1}{2\pi V}\overline{\left|\int \delta D_{\Delta\omega}(\mathbf{r})\, e^{-i\mathbf{q}\mathbf{r}}\, dV\right|^2}\,\frac{3}{4}(1+\cos^2\theta)\frac{d\Omega}{4\pi}\, d\Delta\omega, \qquad (2)$$

where ω is the frequency of the incident light, θ is the scattering angle, $\mathbf{q}$ is the change in the wave vector of the light, equal in absolute magnitude to $(2\omega/c)\sin(\theta/2)$, and $\delta D_{\Delta\omega}$ is the Fourier component of the fluctuation of the dielectric permeability $\delta D(t)$:

$$\delta D_{\Delta\omega} = \frac{1}{\sqrt{t_0}}\int_0^{t_0} \delta D(t)\, e^{i\Delta\omega t}\, dt, \qquad (3)$$

where t_0 is a certain large quantity which, in the final equation, will go to infinity.

The bar in equation (2) denotes averaging over the fluctuations. In what follows we shall, for simplicity, set the volume of the system equal to unity.

In view of the very small polarizability of helium atoms, it is possible to consider that the change in the dielectric permeability is due to density fluctuations, i.e., $\delta D = (\partial D/\partial N)\,\delta N$, where N is the number of particles per unit volume. However, according to the general theory of Fermi

*The idea of using Rayleigh scattering was first suggested by S. P. Kapitza.

*We note that at high temperatures, where $\omega_s \tau \ll 1$, the scattering of light will be described by the usual equations.[3]

†Here dh is the so-called differential coefficient of extinction. The integral over dh with respect to $d\Omega$ and $d\Delta\omega$ gives the total coefficient of extinction h, representing the logarithmic decrement of the attenuation of the photon flux density in the medium.

liquids set forth by Landau,[4] the number of excitations is equal to the number of atoms of the liquid. Consequently it is possible to write

$$\int \delta D_{\Delta\omega}(\mathbf{r})\, e^{-i\mathbf{qr}}\, dV = \frac{\partial D}{\partial N}\int \delta n_{\mathbf{q},\Delta\omega}(\mathbf{p})\, d\tau_p, \qquad (4)$$
$$d\tau_p = 2dp_x\, dp_y\, dp_z / (2\pi\hbar)^3,$$

where $\delta n_{\mathbf{q},\Delta\omega}(\mathbf{p})$ is the Fourier component with respect to $\mathbf{r}$ and t [the latter in the sense of Eq. (3)] of the fluctuation of the excitation distribution function.

Before proceeding with further calculations, we point out one important circumstance. In Eq. (2) the average is taken over all possible fluctuations. In the region of temperatures and frequencies where $\hbar\Delta\omega \geq kT$, it is necessary to take account of quantum effects in the averaging process. This can be done satisfactorily, if one knows the result for the purely classical case ($kT \gg \hbar\Delta\omega$), by introducing a certain correction factor. For scattering in which there is an increase of the frequency by $\Delta\omega$ (anti-Stokes scattering), the factor is $(\hbar\Delta\omega/kT)\,N(\Delta\omega)$; for scattering in which the frequency decreases (Stokes scattering), the factor is $(\hbar\Delta\omega/kT)[N(\Delta\omega)+1]$, where $N(\Delta\omega)$ is the Bose distribution function. If a negative $\Delta\omega$ is used to describe Stokes scattering, it turns out that, because of the relation $N(-\Delta\omega)+1 = -N(\Delta\omega)$, the correction factor for both cases has the form

$$\frac{\hbar\Delta\omega}{kT}\,[e^{\hbar\Delta\omega/kT} - 1]^{-1}. \qquad (5)$$

We shall suppose thus that $kT \gg \hbar\omega$. To find the fluctuations of the distribution function we make use of the method suggested by Rytov[5] and by Landau and Lifshitz[6] for calculating fluctuations in electrodynamics and hydrodynamics.* In using this method we find the fluctuations of a "stray force" entering into the kinetic equation, whereupon by solving this equation we also obtain the fluctuations of the distribution function.

For the case of a Fermi liquid we will proceed from the kinetic equation, which we write in the form

$$\frac{\partial\delta n}{\partial t} + \frac{\partial\delta n}{\partial\mathbf{r}}\frac{\partial\varepsilon}{\partial\mathbf{p}} - \frac{\partial n_0}{\partial\mathbf{p}}\int f(\mathbf{p},\mathbf{p}')\,\frac{\partial\delta n(\mathbf{p}')}{\partial\mathbf{r}}\,d\tau_{p'}$$
$$= I(\delta n) + y(\mathbf{p},\mathbf{r},t). \qquad (6)$$

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After the elimination of the "stray force" $y(\mathbf{p},\mathbf{r},t)$, this equation represents, in the approximation linear in δn, the kinetic equation for Fermi liquids found by Landau; here ϵ is the energy of an excitation, n_0 the equilibrium distribution function, and $f(\mathbf{p},\mathbf{p}')$ the function introduced in Ref. 4.

In what follows we shall be interested only in frequencies and temperatures for which Eq. (1) is satisfied, i.e., for which it is possible to ignore collisions. The detailed form of the collision integral will not be essential to us, since it plays the role of an auxiliary quantity in the calculations and in the final results can be set equal to zero. In view of this we set

$$I(\delta n) = -\delta n/\tau, \qquad (7)$$

where τ is a large quantity. It is next necessary to find the rate of change of the entropy. Remembering that the number of particles and the total energy are fixed and making use of the equation determining $f(\mathbf{p},\mathbf{p}')$,

$$\delta\varepsilon(\mathbf{p}) = \int f(\mathbf{p},\mathbf{p}')\,\delta n(\mathbf{p}')\,d\tau_{p'}, \qquad (8)$$

we find that

$$\dot{S} = -k\left\{\int\frac{\delta n\,[I(\delta n)+y]}{n_0(1-n_0)}\,d\tau_p\,dV\right.$$
$$\left. + \frac{1}{kT}\int f(\mathbf{p},\mathbf{p}')\,\delta(\mathbf{r}-\mathbf{r}')\,\delta n I(\delta n')\,d\tau_p\,dV\,d\tau_{p'}\,dV'\right\}. \qquad (9)$$

If we remember also that $n_0(1-n_0) \approx kT\delta(\epsilon-\mu)$, where μ is the chemical potential, it is not difficult to see that $\delta n(\mathbf{p})$ should have the form

$$\delta n(\mathbf{p}) = \delta n^{\varepsilon}(\vartheta,\varphi)\,\delta(\varepsilon-\mu), \qquad (10)$$

where ϑ and φ are the polar angles of the vector $\mathbf{p}$. This equation indicates that fluctuations of the distribution function take place only in the region of the Fermi surface.

It is natural to take the same form for y:

$$y(\mathbf{p}) = y^{\varepsilon}(\vartheta,\varphi)\,\delta(\varepsilon-\mu). \qquad (11)$$

We now introduce the notation

$$F(\gamma) = \left[f(\mathbf{p},\mathbf{p}')\frac{d\tau_{p'}}{d\varepsilon'}\right]_{\varepsilon=\varepsilon'=\mu},$$

where γ is the angle between $\mathbf{p}$ and $\mathbf{p}'$, and we expand δn, y, and F in spherical harmonics:

$$\delta n^{\varepsilon}(\vartheta,\varphi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} A_n^m P_n^m(\cos\vartheta)\,e^{im\varphi},$$
$$y^{\varepsilon}(\vartheta,\varphi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} y_n^m P_n^m(\cos\vartheta)\,e^{im\varphi}, \qquad (12)$$
$$F(\gamma) = \sum_{n=0}^{\infty} F_n P_n(\cos\gamma).$$

Since δn^{ϵ} and y^{ϵ} are real quantities,

$$A_n^m = (A_n^{-m})^*, \quad y_n^m = (y_n^{-m})^*.$$

Making use of Eq. (7) for the collision integral, we obtain the following equation for the rate of change of the entropy:

$$\dot{S} = \int dV \left(\frac{d\tau_p}{d\varepsilon}\right)_{\varepsilon=\mu} \frac{1}{T} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left(\frac{F_n}{2n+1} + 1\right) \frac{1}{2n+1} \frac{(n+|m|)!}{(n-|m|)!} \left(\frac{A_n^m}{\tau} - y_n^m\right) A_n^{-m}. \quad (13)$$

We now introduce the notation

$$\dot{x}_n^m = -A_n^m/\tau + y_n^m. \quad (14)$$

Then if Eq. (13) is to have the form[6]

$$\dot{S} = -\sum_i X_i x_i$$

we have to take as the generalized force X_i the expression

$$X_n^m = \frac{1}{T}\left(\frac{d\tau_p}{d\varepsilon}\right)_{\varepsilon=\mu} \left(\frac{F_n}{2n+1} + 1\right) \frac{1}{2n+1} \frac{(n+|m|)!}{(n-|m|)!} A_n^{-m}. \quad (15)$$

In Eq. (14) the quantity y_n^m plays the role of the "stray force." Writing this expression in the form

$$\dot{x}_n^m = -\sum_{n',\,m'} \gamma_{n,\,n'}^{m,\,m'} X_{n'}^{m'} + y_n^m,$$

where the coefficients γ are determined without difficulty from (15), we have, according to the general theory of fluctuations,

$$\begin{aligned} \overline{y_n^m(\mathbf{r},t)\, y_{n'}^{m'}(\mathbf{r}',t')} &= k(\gamma_{n,\,n'}^{m,\,m'} + \gamma_{n',\,n}^{m',m})\,\delta(t-t')\,\delta(\mathbf{r}-\mathbf{r}') \\ &= \frac{2}{\tau}\delta_{n,\,n'}\delta_{m,-m'}\,\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t') \quad (16) \\ &\times\left[kT\left(\frac{d\tau_p}{d\varepsilon}\right)_{\varepsilon=\mu}\left(\frac{F_n}{2n+1}+1\right)\frac{1}{2n+1}\frac{(n+|m|)!}{(n-|m|)!}\right]^{-1} \end{aligned}$$

Finally, having made use of (12) and the relation

$$\sum_n (2n+1)\, P_n(\cos\gamma) = 2\delta(\cos\gamma - 1),$$

we obtain after certain transformations the general expression

$$\begin{aligned} \overline{y(\mathbf{p},\mathbf{r},t)\, y(\mathbf{p}',\mathbf{r}',t')} &= \frac{2kT}{\tau}\,\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t') \\ &\times\left\{\frac{(2\pi\hbar)^3}{2}\,\delta(\mathbf{p}-\mathbf{p}')\,\delta(\varepsilon-\mu) \right. \quad (17) \\ &\left. + \delta(\varepsilon-\mu)\,\delta(\varepsilon'-\mu)\left(\frac{d\varepsilon}{d\tau_p}\right)\sum_{n=0}^{\infty}\frac{F_n P_n(\cos\gamma)}{1+F_n/(2n+1)}\right\}. \end{aligned}$$

With the aid of this expression and the kinetic equation (6) we can calculate those fluctuations of the distribution function which are of interest to us. Since for the general case of an arbitrary function f this is a rather complex procedure, we restrict ourselves to the case f = const.*

Making use of the fact that the fluctuations occur only on the Fermi surface, and using Eq. (6), we express $\delta n^{\epsilon}_{\mathbf{q},\Delta\omega}(\vartheta,\varphi)$ in terms of the corresponding Fourier components of $y^{\epsilon}(\vartheta,\varphi)$. This gives

$$\begin{aligned} &\int \delta n^{\varepsilon}_{\mathbf{q},\,\Delta\omega}(\vartheta,\varphi)\frac{d\Omega}{4\pi} \\ &= \int \frac{y^{\varepsilon}_{\mathbf{q},\,\Delta\omega}(\vartheta,\varphi)\,(d\Omega/4\pi)}{-i\omega+1/\tau+i\mathbf{qv}} \Big/ \left(1 + F_0\int\frac{i(\mathbf{qv})\,(d\Omega/4\pi)}{-i\omega+1/\tau+i\mathbf{qv}}\right), \quad (18) \end{aligned}$$

where $\mathbf{v} = (\partial\epsilon/\partial\mathbf{p})_{\epsilon=\mu}$. Averaging the square of the absolute value of this expression with the help of Eq. (17) [keeping in mind that the Fourier component with respect to the time is derived according to Eq. (3)], we find

$$\begin{aligned} &\overline{\left|\int \delta n^{\varepsilon}_{\mathbf{q},\,\Delta\omega}(\vartheta,\varphi)\frac{d\Omega}{4\pi}\right|^2} = 2\left(\frac{d\varepsilon}{d\tau_p}\right)_{\varepsilon=\mu}\frac{kT}{\tau}\left[\frac{1}{2}\int_{-1}^{1}\frac{dx}{|qvx-\omega+i/\tau|^2}\right. \quad (19) \\ &\left.-\frac{F_0}{1+F_0}\left|\frac{1}{2}\int_{-1}^{1}\frac{dx}{qvx-\omega+i/\tau}\right|^2\right]\Big/\left|1+\frac{F_0}{2}\int_{-1}^{1}\frac{qvxdx}{qvx-\omega+i/\tau}\right|^2 \end{aligned}$$

We are interested in the limiting value of this expression when $\tau \to \infty$. When $qv > |\Delta\omega|$, the denominator has no poles, and what turns out to be important is the residue of the integral in the numerator. For this we obtain

$$\begin{aligned} &\frac{1}{2\pi}\,\overline{|\delta n_{\mathbf{q}\Delta\omega}(\mathbf{p})\, d\tau_p|} \quad (20) \\ &= kT\left(\frac{d\tau_p}{d\varepsilon}\right)\frac{1}{qv}\left\{\left[1+F_0\left(1-\frac{\Delta\omega}{2qv}\ln\frac{qv+\Delta\omega}{qv-\Delta\omega}\right)\right]^2+\left(\frac{F_0\Delta\omega\pi}{2qv}\right)^2\right\}^{-1} \end{aligned}$$

In the opposite case, i.e., when $qv < |\Delta\omega|$, it is the pole in the denominator of (19) which is important. It is not difficult to see that such a pole arises if $F_0 > 0$ and the equality $\Delta\omega = \pm\eta qv$ holds, where η satisfies the equation

*Actually, for He^3, the function f is not constant. It is possible to find the first two terms of the series (12) from the experimental data,[2] which give $F(\gamma) = 5.2 + 1.3\cos\gamma$.

$$1 + F_0\left[1 - \frac{\eta}{2}\ln\left(\frac{\eta+1}{\eta-1}\right)\right] = 0. \tag{21}$$

This expression agrees with Eq. (14) of Ref. 1 for the velocity of zero sound. Using the relation

$$\frac{1}{\pi}\lim_{\tau\to\infty}\frac{1/\tau}{(\omega-\omega_0)^2+\tau^{-2}} = \delta(\omega-\omega_0),$$

we obtain without difficulty

$$\frac{1}{2\pi}\left|\int \delta n_{\mathbf{q},\Delta\omega}(\mathbf{p})\,d\tau_p\right|^2$$
$$= kT\left(\frac{d\tau_p}{d\varepsilon}\right)_{\varepsilon=\mu}\frac{2(\eta^2-1)}{F_0(1+F_0-\eta^2)}\left[\delta(\Delta\omega-\eta qv)+\delta(\Delta\omega+\eta qv)\right]. \tag{22}$$

Thus the angular distribution and the frequency dependence of the scattered light are expressed by the following equation [in which the quantum factor (5) has been introduced]:

$$dh = \frac{\omega^4}{4\pi c^4}\left(\frac{\partial D}{\partial N}\right)^2\left(\frac{d\tau_p}{d\varepsilon}\right)_{\varepsilon=\mu}(1+\cos^2\theta)\frac{\hbar\Delta\omega}{e^{\hbar\Delta\omega/kT}-1}$$
$$\times\left[\frac{\theta(qv-|\Delta\omega|)}{2qv}\Big/\left\{\left[1+F_0\left(1-\frac{\Delta\omega}{2qv}\ln\frac{qv+\Delta\omega}{qv-\Delta\omega}\right)\right]^2+\left(\frac{F_0\Delta\omega\pi}{2qv}\right)^2\right\}\right.$$
$$\left.+\frac{\eta^2-1}{F_0(1+F_0-\eta^2)}\left[\delta(\Delta\omega-\eta qv)+\delta(\Delta\omega+\eta qv)\right]\right]\frac{d\Omega}{4\pi}d\Delta\omega, \tag{23}$$

where

$$\theta(y) = \begin{cases} 1 & y>0 \\ 0 & y<0 \end{cases}$$

The result obtained has a simple physical significance. As can be easily seen, the frequency spectrum consists of a central part $-qv < \Delta\omega < qv$ and two sharp lines at $\Delta\omega = \pm\eta qv$. The central part corresponds to the Doppler width of the principal line. A comparison of Eq. (1) with Eq. (14) of Ref. 1, concerning the oscillations of a Fermi liquid, shows that the secondary lines appear as satellites in Rayleigh scattering, arising in connection with the possibility of propagating zero sound ($\eta v = u$). The relation between the intensities of the central part and of the secondary satellites depends in general upon the scattering angle. In the limiting cases of high temperatures ($kT \gg \hbar\omega u/c$) and low temperatures ($kT \ll \hbar\omega \times u/c$), however, this expression does not depend on angle. It is possible to carry out a numerical estimate for He^3, using the well known parameters.[2] It turns out that, for high temperatures, the central part has about 20% of the total intensity, and the secondary lines about 40% each. At low temperatures the distribution will be cut off on the side of positive $\Delta\omega$ because of the quantum factor. In particular, of the two satellites there will remain only the Stokes line for $\Delta\omega = -uq$, with 90% of the intensity. Only 10% of the total intensity will appear in the central line.

The total scattering intensity is obtained by integrating (23) with respect to $d\Delta\omega$ and $d\Omega$. For high temperatures ($kT \gg \hbar\Delta\omega \sim \hbar\omega u/c$) it is equal to

$$h = \frac{\omega^4 kT}{6\pi c^4}\left(\frac{\partial D}{\partial N}\right)^2\left(\frac{d\tau_p}{d\varepsilon}\right)_{\varepsilon=\mu}J_1, \tag{24}$$

where J_1 is a numerical integral, equal to about 0.5 for He^3. In the quantum limit ($kT \ll \hbar\omega u/c$) we have

$$h = \frac{\hbar\omega^5 v}{6\pi c^5}\left(\frac{\partial D}{\partial N}\right)^2\left(\frac{d\tau_p}{d\varepsilon}\right)_{\varepsilon=\mu}J_2. \tag{25}$$

Here J_2 is a numerical integral which is equal to about 0.2 for He^3. In order to make a quantitative estimate of the above expressions for He^3, it is necessary to know the quantity $\partial D/\partial N$. There have been no measurements of this quantity; consequently we set $D - 1$ proportional to N, and evaluate the coefficient of proportionality from data on liquid He^4. For a wavelength $\lambda = 5461$ A the index of refraction of liquid He^4 is equal to 1.027. This gives $\partial D/\partial N = 2.5 \times 10^{-24}$. Substitution in Eqs. (24) and (25) gives

$$\begin{aligned} &h(He^3) \sim 10^{-69}\omega^4 T\ \text{cm}^{-1} \text{ for } \omega \ll 2\cdot10^{17}T\ \text{sec}^{-1}, \\ &h(He^3) \sim 10^{-87}\omega^5\ \text{cm}^{-1} \text{ for } \omega \gg 2\cdot10^{17}T\ \text{sec}^{-1}. \end{aligned} \tag{26}$$

It is necessary to keep in mind that the frequency has to satisfy Eq. (1), i.e., $\Delta\omega \gg 1/\tau$, or

$$\omega \gg 10^{18}T^2\ \text{sec}^{-1}. \tag{27}$$

If this condition were not satisfied, the line width would be too great. Thus for the visible region of frequencies, temperatures below 0.05° K are necessary. It is not difficult to see that to temperatures of the order of 0.01° K in the visible range of frequencies there will correspond a value of $\hbar(He^3) \sim 10^{-9}$ cm^{-1}, which certainly is too small for the effect to be measured.* Because of the fact that ω enters into the expression for h to a very high power, however, it is possible that the scattering can be successfully measured in the ultraviolet region.

In conclusion the authors express their gratitude to Academician L. D. Landau for his interest in the work.

*For liquid He^4 in the visible region, $h \sim 10^{-8}$ cm^{-1} (Ref. 7). (Approximately the same result ought to be obtained for He^3 for $\Delta\omega\tau \ll 1$.) Measurements of h have been made for He^4 (Refs. 8, 9), but they were at the limit of the experimental capabilities.

[1] L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 59 (1957), Soviet Phys. JETP **5**, 101 (1957).

[2] I. M. Khalatnikov and A. A. Abrikosov, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 110 (1957), Soviet Phys. JETP **6**, 84 (1958).

[3] L. D. Landau and E. M. Lifshitz, Электродинамика сплошных сред (Electrodynamics of Continuous Media), GITTL, Moscow, 1957.

[4] L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 1058 (1956), Soviet Phys. JETP **3**, 920 (1956-57).

[5] S. M. Rytov, Теория электрических флуктуаций и теплового излучения (Theory of Electrical Fluctuations and Thermal Radiation) Acad. Sci. Press, 1953.

[6] L. D. Landau and E. M. Lifshitz, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 618 (1957), Soviet Phys. JETP **5**, 512 (1957).

[7] V. L. Ginzburg, J. Exptl. Theoret. Phys. (U.S.S.R.) **13**, 243 (1943).

[8] I. A. Jaklovlev, J. Phys. (U.S.S.R.) 7, 307 (1943).

[9] A. W. Lawson and L. Meyer, Phys. Rev. **93**, 259 (1954).

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THEORY OF DIFFUSE SCATTERING OF X-RAYS AND THERMAL NEUTRONS IN SOLID SOLUTIONS. III. ACCOUNT OF GEOMETRICAL DISTORTIONS OF THE LATTICE

M. A. KRIVOGLAZ

Institute of Metal Physics, Academy of Sciences, Ukrainian S.S.R.

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A method for treatment of various problems connected with static distortions of crystal lattices is proposed, in which the distortions are related to fluctuation waves of the composition and of the internal parameters. Scattering of x-rays and thermal neutrons in binary solutions of arbitrary composition and with arbitrary values of the short and long range order parameters is considered. Anisotropy of the crystal and its atomic structure are taken into account explicitly. The scattering intensity can be expressed in terms of the thermodynamical characteristics of the solution (or correlation parameters), elastic moduli (or interatomic interaction constants), and also in terms of parameters characterizing the dependence on the concentration of the cell shape and dimensions. The particular cases of ideal, dilute, almost completely ordered solutions and also of solutions located near the critical point on the decay curve or near phase-transition points of the second kind are investigated. The diffuse scattering intensity distribution in a Cu_3Au solution, calculated without making use of the theoretical parameters, agrees satisfactorily with the experimental distribution.

In earlier papers[1,2] (quoted in the following as I and II) the diffuse scattering of x-rays and neutrons in solid solutions was investigated within the framework of phenomenological[1] and microscopic[2] theories. In the course of these investigations it was assumed that the sole cause giving rise to diffuse scattering was the random distribution of the atoms among the lattice points of a geometrically ideal lattice. In this article we shall investigate the influence of geometrical distortions of the lattice, associated with the difference in the size of atoms of different kinds, on the scattering. During the last few years an intensive experimental study was begun of the diffuse scattering due to the above cause and of the weakening of the lines in an x-ray photograph (see, for example, Ref. 3 where references to other work are given). These problems have been studied theoretically for several particular cases in a number of papers.[4-9]

In this article we investigate the general case