For $C_1 = 0$ Eq. (5) is the same as Eq. (33) of Ref. 1 with $A = 0$. Thus the two ellipsoids[3] tangent at the k_z axis are the limiting case of the equipotential surfaces for the whole sequence of space groups $D_{2d}^1 - D_{2d}^{12}$, $C_{3v}^1 - C_{3v}^{12}$, $C_{4v}^1 - C_{4v}^{12}$, $D_4^1 - D_4^{10}$, and this can hold for tellurium type lattices if the relation $p_{1,2}^z = 0$ is fulfilled.

In conclusion I take occasion to thank E. I. Rashba, who called my attention to the illegitimacy of the conclusions drawn in Ref. 1 from the relations (22).

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Translated by W. H. Furry
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POTENTIAL WELLS FOR CHARGED PARTICLES IN A HIGH-FREQUENCY ELECTROMAGNETIC FIELD

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As is well known, in source-free regions of an electrostatic field there can be no absolute maxima or minima of the potential; this fact excludes the possibility of maintaining a charged particle in a state of stable equilibrium (Earnshaw's theorem). This same situation also excludes the possibility of localizing particles, if by localization we mean a state in which a particle with energy smaller than some given magnitude cannot go beyond the limits of a bounded region, no matter what the initial conditions.

The above statement does not apply in the case of a high-frequency electromagnetic field where, as we shall show below, localization of particles can be accomplished.

Consider a particle of charge e and mass m moving in an external electromagnetic field $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r})e^{i\omega t}$, $\mathbf{H}(\mathbf{r}, t) = \mathbf{H}(\mathbf{r})e^{i\omega t}$. In the nonrelativistic approximation the equation of motion is

$$\ddot{\mathbf{r}} = \eta \mathbf{E}(\mathbf{r}, t) + (\eta/c)\dot{\mathbf{r}} \times \mathbf{H}(\mathbf{r}, t), \qquad (1)$$

where $\eta = e/m$. If the frequency of the external field ω is sufficiently high, the solution of Eq. (1) can be written as a sum of a slowly varying (in terms of the oscillation period of the external field) function $\mathbf{r}_0(t)$ and an oscillating function $\mathbf{r}_1(t)$ (frequency ω). Assuming that $\mathbf{r}_1(t)$ is much smaller than the distance L over which the amplitude of the external field changes markedly,

$$|\mathbf{r}_1(t)| \ll L, \qquad (2)$$

and neglecting terms of order $|\mathbf{r}_1/L|$ and $|\dot{\mathbf{r}}_0/L|$, averaging Eq. (1) over the period of the high-frequency field we obtain an equation for $\mathbf{r}_0(t)$:

$$\ddot{\mathbf{r}}_0(t) = -\nabla\Phi, \quad \Phi = (\eta/2\omega)^2 |\mathbf{E}|^2 \qquad (3)$$

Thus, the time average of the force acting on the particle is derivable from a potential; the potential is proportional to the square of the modulus of the electric intensity and is independent of the sign of the charge.

There are an unlimited number of possibilities for creating potential wells $\Phi(\mathbf{r})$. The simplest of these are realized in quasi-electrostatic multipole fields

$$\mathbf{E}(\mathbf{r}, t) = \nabla\{r^n P_n^m(\cos\theta)\cos m\varphi\} e^{i\omega t},$$

where r, θ, and φ are the spherical ordinates and the P_n^m are the associated Legendre polynomials. For example, the potential Φ in the field of a quasi-static axial quadrupole ($m = 0$, $n = 2$) is of the form $\Phi = \text{const}\, r^2(1 + 3\cos^2\theta)$, i.e., there is an absolute minimum at the origin.*

To determine the motion of the particle inside the potential well we consider the first integral of Eq. (3):

$$\frac{m}{2}\left(|\dot{\mathbf{r}}_0|^2 + \frac{\eta^2}{2\omega^2}|\mathbf{E}|^2\right) = \frac{m}{2}\left(|\dot{\mathbf{r}}_0|^2 + \frac{1}{2}|\dot{\mathbf{r}}_1|^2\right) = \text{const} = |e|V_0. \qquad (4)$$

The left-hand part of Eq. (4) is equal to the time average of the kinetic energy of the particle, where the kinetic energy of the oscillatory (with frequency ω) motion plays the role of the potential energy.

If the $\mathbf{E} = 0$ at the center of the potential well, particles with energy less than or equal to V_0, are localized within a region at the boundaries of which the following conditions are satisfied

*It is interesting to note that in an axially-symmetric quadrupole field the original equation (1) in Cartesian coordinates leads to three Mathieu equations; this allows us to analyze the properties of the solution without the limitation imposed by (2).

$$L\omega^2/|\eta| \gg |E| > 2\omega(V_0/|\eta|)^{1/2}.$$

Finally, we may note in forming potential wells it is possible to make simultaneous use of fields at different frequencies;[1] then, in averaging Eq. (1) over a sufficiently long time interval the potential Φ in Eq. (3) will be of the form

$$\Phi = (\eta^2/4)\sum_n (|E_n|/\omega_n)^2.$$

Thus it is possible to create three-dimensional potential wells from one-dimensional and two-dimensional wells. This possibility is of interest, in itself, as a means of focussing rectilinear beams of charged particles.[2,3]

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ON THE THEORY OF THE DE HAAS – VAN ALPHEN EFFECT FOR OPEN ISOENERGETIC SURFACES

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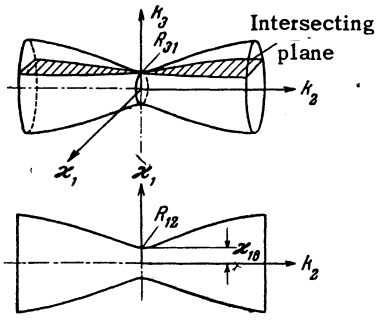
THE theory of electron motion with an arbitrary dispersion law $E(\mathbf{k})$ within a crystal located in a uniform magnetic field was developed in Refs. 1 – 3. The magnetic susceptibility χ of an aggregate of such electrons oscillates with changes of the magnetic field strength H (the de Haas – van Alphen effect). When the isoenergetic surface in $\mathbf{k}$-space that is determined by the equation $E(\mathbf{k}) = \text{const}$ is closed, the period of these oscillations is determined[1,2] by the magnitude of the extremal section S_m of the isoenergetic surface by a plane perpendicular to $\mathbf{H}$. When the surface $E(\mathbf{k}) = \text{const}$ is an open surface such as a corrugated cylinder with the magnetic axis perpendicular to the cylinder axis, the oscillations are generally determined not by extremal sections but by the "boundary" sections which are discussed below.

The figure shows part of an open surface and its intersection with the plane $k_3 = \text{const}$ ($H = H_z$). When k_3 is greater than some value k_b the tra-



jectories (the curves which bound the section) are closed, and when $|k_3| < k_b$ they are open. We use the term "boundary section" for that obtained with the plane $k_3 = k_b$. The energy spectrum of electrons in closed trajectories, i.e., with $|k_3| > k_b$, is almost discrete (slightly broadened discrete levels), and when $|k_3| < k_b$ it is almost continuous (small gaps in a continuous spectrum[3]). Close to the boundary section the width of the gaps is of the same order as the width of the allowed bands (formed from discrete levels[3]).

Because of the (exponential) smallness of the discontinuities (gaps) when $|k_3| < k_b$, the extremal section makes a very small contribution to the oscillating part of the state sum (1). The principal contribution comes from the boundary section.

The number of electron states with energies from zero to E will be[1-3]

$$Z_{osc}(E, H) = \frac{1}{2\pi^2 a_2}\sum_\alpha \sum_{p=-\infty}^{+\infty}{}' \frac{1}{2\pi i p} \times \int_{-k_{3\max}}^{k_{3\max}} dk_3 \int_0^{a_1/2\alpha_0^2} \exp\{-2\pi i p n(k_1, k_3, E)\}\, dk_1. \quad (1)$$

Here a_2 is the lattice constant, $\alpha_0 = \sqrt{\hbar c/eH}$, k_1 is a continuous parameter describing the broadening, and α is the spin quantum number. For sections S (in a single cell of $\mathbf{k}$-space) close to the boundary section the following dispersion law was obtained in Ref. 3:

$$n = \frac{\alpha_0^2}{2\pi} S - \frac{1}{2} - \frac{1}{\pi} \arcsin \frac{\cos(2\pi\alpha_0^2 k_1/a_2)}{\sqrt{1+e^{-\pi\lambda}}}, \quad (2)$$