$$L\omega^2/|\eta| \gg |E| > 2\omega(V_0/|\eta|)^{1/2}.$$

Finally, we may note in forming potential wells it is possible to make simultaneous use of fields at different frequencies;[1] then, in averaging Eq. (1) over a sufficiently long time interval the potential Φ in Eq. (3) will be of the form

$$\Phi = (\eta^2/4)\sum_n (|E_n|/\omega_n)^2.$$

Thus it is possible to create three-dimensional potential wells from one-dimensional and two-dimensional wells. This possibility is of interest, in itself, as a means of focussing rectilinear beams of charged particles.[2,3]

[1] F. B. Knox, Australian Journal of Physics **10**, 221 (1957).

[2] V. S. Tkalich, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 625 (1957), Soviet Phys. JETP **5**, 518 (1957).

[3] M. A. Miller, Communication at the Second All-Union Conference MVO on Radioelectronics, Saratov, 1957.

Translated by H. Lashinsky
44

ON THE THEORY OF THE DE HAAS – VAN ALPHEN EFFECT FOR OPEN ISOENERGETIC SURFACES

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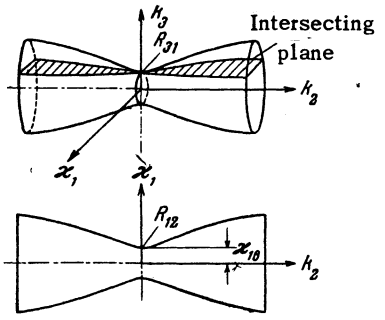
Submitted to JETP editor October 18, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 243-245 (January, 1958)

THE theory of electron motion with an arbitrary dispersion law $E(\mathbf{k})$ within a crystal located in a uniform magnetic field was developed in Refs. 1 – 3. The magnetic susceptibility χ of an aggregate of such electrons oscillates with changes of the magnetic field strength H (the de Haas – van Alphen effect). When the isoenergetic surface in $\mathbf{k}$-space that is determined by the equation $E(\mathbf{k}) = \text{const}$ is closed, the period of these oscillations is determined[1,2] by the magnitude of the extremal section S_m of the isoenergetic surface by a plane perpendicular to $\mathbf{H}$. When the surface $E(\mathbf{k}) = \text{const}$ is an open surface such as a corrugated cylinder with the magnetic axis perpendicular to the cylinder axis, the oscillations are generally determined not by extremal sections but by the "boundary" sections which are discussed below.

The figure shows part of an open surface and its intersection with the plane $k_3 = \text{const}$ ($H = H_z$). When k_3 is greater than some value k_b the tra-



jectories (the curves which bound the section) are closed, and when $|k_3| < k_b$ they are open. We use the term "boundary section" for that obtained with the plane $k_3 = k_b$. The energy spectrum of electrons in closed trajectories, i.e., with $|k_3| > k_b$, is almost discrete (slightly broadened discrete levels), and when $|k_3| < k_b$ it is almost continuous (small gaps in a continuous spectrum[3]). Close to the boundary section the width of the gaps is of the same order as the width of the allowed bands (formed from discrete levels[3]).

Because of the (exponential) smallness of the discontinuities (gaps) when $|k_3| < k_b$, the extremal section makes a very small contribution to the oscillating part of the state sum (1). The principal contribution comes from the boundary section.

The number of electron states with energies from zero to E will be[1-3]

$$Z_{osc}(E, H) = \frac{1}{2\pi^2 a_2}\sum_\alpha \sum_{p=-\infty}^{+\infty}{}' \frac{1}{2\pi i p} \times \int_{-k_{3\max}}^{k_{3\max}} dk_3 \int_0^{a_1/2\alpha_0^2} \exp\{-2\pi i p n(k_1, k_3, E)\}\, dk_1. \quad (1)$$

Here a_2 is the lattice constant, $\alpha_0 = \sqrt{\hbar c/eH}$, k_1 is a continuous parameter describing the broadening, and α is the spin quantum number. For sections S (in a single cell of $\mathbf{k}$-space) close to the boundary section the following dispersion law was obtained in Ref. 3:

$$n = \frac{\alpha_0^2}{2\pi} S - \frac{1}{2} - \frac{1}{\pi} \arcsin \frac{\cos(2\pi\alpha_0^2 k_1/a_2)}{\sqrt{1+e^{-\pi\lambda}}}, \quad (2)$$

$$\lambda = -2a_1^2\varepsilon^{-2}(k_3 - k_b)R_{31}\sqrt{R_{12}/\varkappa_{1b}},$$
$$\varepsilon = \sqrt{a_1a_2/\alpha_0} \quad (\varepsilon \ll 1), \qquad (3)$$
$$R_{31}^{-1} = -\partial^2 k_b/\partial \varkappa_{1b}^2, \quad R_{21}^{-1} = -\partial^2 \varkappa_{1b}/\partial k_2^2.$$

Here κ_{1b} is the value of κ_1 on the boundary of cells in k-space (for the boundary section κ_{1b} and R_{12} vanish but their ratio remains constant). Substituting (2) in (1), integrating over k_1 and dropping small quantities, we obtain for the integral in (1)

$$\frac{a_2}{2\alpha_0^2}\int_{-k_{3\max}}^{k_{3\max}} e^{-ip\alpha_0^2}\,[e^{(k_b-k_3)/k_0}+1]^{-1}\,dk_3, \qquad (4)$$

where

$$k_0 = (\varepsilon^2/2\pi a_1^2 R_{31})\sqrt{\varkappa_{1b}/R_{12}}. \qquad (5)$$

For the calculation of (4) it is necessary to know the dependence of the area of a section S on $x = (k_b - k_3)/k_0$. Assuming that in the vicinity of the "isthmus" shown in the figure the surface (a corrugated cylinder) can be replaced by a one-sheet hyperboloid, we obtain this dependence in the form

$$S = S_b - (1/\pi\alpha_0^2)\,x\ln|ax|;$$
$$a = (\varepsilon^2/4\pi^3)(a_2/a_1)^2\sqrt{R_{12}/\varkappa_{1b}}. \qquad (6)$$

The integral (4) with the single parameter $|\ln a|$ was calculated numerically, assuming $|\ln a| \sim 1 - 10$.

Passing from the state sum to the thermodynamic potential Ω, we obtain

$$\Omega_{\text{osc}} = \frac{a_2 kT}{\pi a_1 \alpha_0^4 R_{31}}\sqrt{\frac{\varkappa_{1b}}{R_{12}}}\,\frac{1}{(\ln a)^2}\left(1+\frac{3\pi}{|\ln a|}\right)$$
$$\times\sum_{p=1}^{\infty}\frac{\cos(p\alpha_0^2 S_b(\zeta)-\gamma)\cos\left(\frac{1}{2}\mu_0 H\frac{\partial S_b}{\partial\zeta}p\alpha_0^2\right)}{p^3\sinh\left(\pi p\alpha_0^2\frac{\partial S_b}{\partial\zeta}\frac{kT}{}\right)}, \qquad (7)$$
$$\gamma = \arctan\frac{3p|\ln a|+9\pi}{p|\ln a|+6\pi} - \frac{\pi}{2},$$

which differs from the usual potential (for closed surfaces) as follows: (1) The period of the oscillations is determined not by S_m but by S_b and the phase γ is weakly dependent on the magnetic field; (2) the factor $|\partial^2 S_m/\partial k_3^2|^{-1/2}$ of the amplitude is replaced by

$$\frac{1}{\alpha_0 R_{31}}\frac{1}{(\ln a)^2}\left(1+\frac{3\pi}{|\ln a|}\right)\sqrt{\frac{\varkappa_{1b}}{R_{12}}}.$$

This denotes multiplication of the amplitude by $H^{1/2}$, so that the oscillating terms in χ will contain H^{-1} instead of $H^{-3/2}$. In absolute magnitude the amplitude is generally smaller by the factor ϵ^{-1} than for electrons in closed trajectories. But when, for example, we have a field $H \sim 10^4$ oersted and thus $\epsilon \sim 10^{-2}$, the reduction by the factor ϵ^{-1} can be concealed by other factors.

Other oscillatory effects such as oscillations of the resistance (the Shubnikov – de Haas effect), oscillations of the thermoelectric power or of the Hall field, etc. will also be determined in the present case by boundary and not by extremal sections.

The author takes this opportunity to thank I. M. Lifshitz for discussion of this work.

[1] I. M. Lifshitz and A. M. Kosevich, Dokl. Akad. Nauk SSSR **96**, 963 (1954); J. Exptl. Theoret. Phys. (U.S.S.R.) **29**, 730 (1955), Soviet Phys. JETP **2**, 636 (1956).

[2] G. E. Zil'berman, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 296 (1957), Soviet Phys. JETP **5**, 208 (1957).

[3] G. E. Zil'berman, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 387 (1957), Soviet Phys. JETP **6**, 299 (1958).

Translated by I. Emin
45

CONCERNING THE THEORY OF RAYLEIGH SCATTERING OF LIGHT IN LIQUIDS

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Submitted to JETP editor October 19, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 246-247 (January, 1958)

IN a recently published article, S. M. Rytov[1] develops a theory of scattering of light in liquids under the assumption that this scattering is due entirely to fluctuation deformations $u_{\alpha\beta}$ and to fluctuations of the temperature Θ. Actually, only the fluctuations of $u_{\alpha\beta}$ and Θ are considered in Ref. 1, when it is indicated at the same time that the theory includes all internal processes in the medium and describes the entire spectrum of the scattered light. Next, it is emphasized, in the remarks on p. 518 of Ref. 1 [p. 404 in the translation] that the spectral amplitudes of the fluctuations of any internal parameter can be expressed linearly in terms of $u_{\alpha\beta}$ and Θ, and there is no scattering by isotropic fluctuations which cannot

up the crystals as infinitely long unidimensional or two-dimensional atom complexes, bound together by "small" forces of one nature, whereas in the complex itself the atoms are bound by "big" forces of another nature.

6. The difference between the typical molecular crystals (e.g., the CH_4 or C_6H_6 crystals) and the heteropolar molecular crystals (such as the NaCl, $HgCl_2$ or PbS crystals) lies: (1) in the degree of molecularity β; (2) in the nature of the forces in the molecules; (3) in the nature of intermolecular forces. The quantity β is defined as the ratio of the intramolecular energy $U^a \cong D$ (D is the energy of dissociation of the diatomic molecule into ions) to the intermolecular energy U^e per bond. For the substances for which β is given below, it is possible to take $U^e \approx 2S/l$. Example:

$\beta = 300(CH_4)$, $200(HCl)$, $22(HgCl_2)$, $10(NaCl)$ taking $l = 12$ in all four cases.

Translated by I. Polidi
334

ERRATA

Volume 5

Page	Line	Reads	Should Read
1043	Eq. (4)		$W = y^2 a_{14}^2 \sin 2\phi/2\rho\,(a_{11}a_{44} - \alpha_{14}^2 \sin^2 3\phi)$ The coefficient k_2 equals $0.185 \times 10^{-3}\,cm^{-1}$.
1044	3 from bottom (l.h.)	$\Delta y = 2.87 \times 10^{-3}$ cm	$\Delta y = 3.18 \times 10^{-3}$ cm
	4 from top (r.h.)	$\Delta\vartheta_\Sigma = 7.2 \times 10^{-5}$ radians	$\Delta\vartheta_\Sigma = 5.9 \times 10^{-5}$ radians

Volume 6

1090	4 and 5 from top	2—(d, 3n); and of the I_{53}^{127} cross section, 3—(d, 2n); 4—(d, 3n)	2—(d, 3n) on I_{53}^{127} and 3—(d, 3n); 4—(d, 3n) on Bi_{83}^{209}
1091	6 from bottom expression for determinant C(y)	ρ, γp, h, $1/\rho$	ρy_2, $\gamma p y_2$, $h y_2$, y_2/ρ
1094	7 from bottom	For $\gamma = 5/3$, μ has . . .	Here μ has . . .

Volume 7

55	16 from bottom	Correct submittal date is April 5, 1957	
169	17 from bottom	Delete "Joint Institute for Nuclear Research"	
215	Table	Add: Note. Columns 2—9 give the number of counts per 10^6 monitor counts	
215	Table, column headings	1, 2, 3, 4-7, 8	1, 2, 3, 4, 8-7
312	Eq. (8)	. . . $(1 \pm \mu/2M)^2$	. . . $(1 \mp \mu/2M)^2$
313	2, r.h. col.	$\alpha_{33} = 0.235$	$a_{33} = 0.235$
692	Eq. (5)	$m_B/M_B = \ldots \mp [1 + \ldots]$	$m_B/M_B = \mp [1 + \ldots]$
461	Title	. . . Elastically Conducting	. . . Electrically Conducting