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ON THE THEORY OF MAGNETIC SUSCEPTIBILITY OF METALS

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RECENTLY, the magnetic susceptibility of an electron gas was calculated by several authors,[1-3] taking into account long-range Coulomb correlation. However, only the susceptibility due to the Fermi branch of the excitation spectrum was taken into account there. We would like to direct attention to the fact that the (Bose) plasma-oscillation quanta also make a definite contribution to the susceptibility. Actually, although these excitations are neutral and give no contribution to the current, their energy depends on the magnetic field intensity H and consequently the plasma quanta are "carriers of magnetism." At ordinary temperatures, real plasma quanta in a metal are practically not excited; their zero-point energy, however, also depends on H. This leads, as we shall show, to plasma diamagnetism comparable to the Landau diamagnetism.

As is known (see, for example, Ref. 4), a separation of plasma oscillations into longitudinal and transverse is still possible in a weak magnetic field. For our problem, only the former are of interest; the frequency of a longitudinal plasma quantum (in the frame of an isotropic model) is

$$\omega^2 = \omega_L^2 + \omega_H \sin^2\alpha + O(k^2), \qquad (1)$$

where **k** is the wave vector of a plasmon, α is the angle between **k** and H, $\omega_L^2 = 4\pi ne^2/m$, $\omega_H = eH/mc$, and n and m are the concentration and effective mass of the electrons ($\omega_H \ll \omega_L$). We shall disregard terms of order k^2 in Eq. (1) (apparently, they are small in comparison with ω_L^2 for all k up to the limiting wave number k_0).

The magnetic susceptibility per unit volume, due to the dependence of the zero-point energy of the plasma on the magnetic field, is:

$$\chi = -\frac{1}{2}\frac{\hbar}{(2\pi)^3}\frac{\partial^2}{\partial H^2}\int\limits_{k\leqslant k_0} d\mathbf{k}\,\omega(\mathbf{k}). \qquad (2)$$

By virtue of Eq. (1), this yields

$$\chi = -(1/18\pi^2)(e^2/mc^2)(\hbar/m)k_0^3/\omega_L. \qquad (3)$$

The quantity k_0 in our approximation (small H) can be regarded as independent of the magnetic field. Setting $\hbar k_0 = \beta p_F$, where p_F is the Fermi boundary momentum and β is a dimensionless parameter (which may depend on n) we obtain

$$\chi = -(\beta^3/12\sqrt{\pi})(\hbar/mc)(ne^2/mc^2)^{1/2}$$
$$= -0.96\cdot 10^{-18}(m_0/m)^{3/2}\beta^3\sqrt{n} \qquad (4)$$

(m_0 is the mass of a free electron). Inasmuch as $\beta < 1$, but evidently $\beta > \frac{1}{2}$,[5] $|\chi| \sim 10^{-6} - 10^{-7}$. This quantity can be fully comparable with the result of Pines, obtained in disregarding the zero-point energy of the plasma. Hence it is clear that this neglect, generally speaking, is by no means justified and the quantiative comparison of Pines' theory with experiment must be reviewed.

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THE PROPERTIES OF THE GREEN FUNCTION FOR PARTICLES IN STATISTICS

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IN an attempt to apply the methods which have recently been developed in quantum electrodynamics to statistics, one is faced with the problem of the properties that will be exhibited in that case by the fundamental quantities entering into these methods. We shall show here that some very general relations can be derived for the particle Green function G, which is defined in the well known manner as

$$G = -i\langle T\psi_1\psi_2^+\rangle, \quad (1)$$

where the indices 1 and 2 indicate that the ψ operator must be taken at t_1 or t_2. T is the symbol for the chronologically ordered product, while the averaging is taken over the actual state of the given macroscopic system. It is well known that chronological ordering means that

$$G = -i\langle\psi_1\psi_2^+\rangle \text{ for } t_1 > t_2, \quad G = \mp i\langle\psi_2^+\psi_1\rangle \text{ for } t_1 < t_2, \quad (2)$$

where the upper and lower signs refer to the Bose and Fermi statistics respectively.

The space-time dependence of the matrix elements of the operator ψ is given by equations of the form

$$\psi_{nm}(t, \mathbf{r}) = \psi_{nm}^{(0)} \exp\{i(\omega_{nm}t - \mathbf{k}_{nm}\mathbf{r})\},$$
$$\omega_{nm} = (E_n - E_m)/\hbar, \quad \mathbf{k}_{nm} = (\mathbf{P}_n - \mathbf{P}_m)/\hbar,$$

where the indices n, m refer to the states of a closed system with total energy E and total momentum **P**. From the definition of the adjoint operator, we have $(\psi^+)_{mn} = (\psi_{nm})^*$.

Using these matrix elements we can write (2) in the form

$$\begin{aligned}&\text{for } t_1 > t_2\text{: } G = -i\sum_m |\psi_{nm}^{(0)}|^2 \exp\{i\omega_{nm}t - i\mathbf{k}_{nm}\mathbf{r}\},\\ &\text{for } t_1 < t_2\text{: } G = \mp i\sum_m |\psi_{mn}^{(0)}|^2 \exp\{i\omega_{mn}t - i\mathbf{k}_{mn}\mathbf{r}\},\end{aligned} \quad (3)$$

where we use the notation $t = t_1 - t_2$, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and where the index n refers to the given state over which the averaging of Eqs. (1) or (2) is performed.

We shall now proceed to average expressions (3) over a Gibbsian ensemble. According to the basic principles of statistics, this operation means that we express the quantity G as a function of the temperature T and the chemical potential μ instead of as a function of the energy E and the number of particles in the system N. We have for $t > 0$,

$$G = -i\sum_{n,m} \exp\{(\Omega + \mu N_n - E_n)/T\} \times |\psi_{nm}^{(0)}|^2 \exp\{i\omega_{nm}t - i\mathbf{k}_{nm}\mathbf{r}\}$$

where the temperature is measured in energy units. Since we sum now over the two indices n and m we can interchange the indices in such a sum. We use this possibility in the expression for G for $t_1 < t_2$ and write for $t < 0$,

$$\begin{aligned}G &= \mp i\sum_{n,m} \exp\{(\Omega + \mu N_m - E_m)/T\} \times |\psi_{nm}^{(0)}|^2 \exp\{i\omega_{nm}t - i\mathbf{k}_{nm}\mathbf{r}\}\\ &= \mp i\sum_{m,n} \exp\{(\Omega + \mu N_n - E_n)/T\}\\ &\times \exp\{(\hbar\omega_{nm} + \mu)/T\} |\psi_{nm}^{(0)}|^2 \exp\{(i\omega_{nm}t - i\mathbf{k}_{nm}\mathbf{r}\}.\end{aligned}$$

In the last transformation we have used the fact that the matrix elements $\psi_{nm}^{(0)}$ are different from zero only if $N_m = N_n + 1$.

We now go over from the space-time representation of the Green function to its Fourier coefficients,

$$G(\omega, \mathbf{k}) = \iint G(t, \mathbf{r}) e^{i(\omega t - \mathbf{kr})} dt\, d^3\mathbf{r}.$$

Integration over space gives a delta function of $\mathbf{k} - \mathbf{k}_{mn}$. The integration over dt must be performed separately over the interval from $-\infty$ to 0 and from 0 to $+\infty$, using the well known formula

$$\int_0^\infty e^{i\alpha x} dx = \pi\delta(\alpha) + \frac{i}{\alpha}.$$

We get as a result

$$\begin{aligned}G(\omega, \mathbf{k}) = &-(2\pi)^3 \sum_{n,m} \exp\{(\Omega + \mu N_n - E_n)/T\}\\ &\times |\psi_{nm}^{(0)}|^2 \delta(\mathbf{k} - \mathbf{k}_{mn}) \{i\pi\delta(\omega - \omega_{mn})[1 \pm e^{(\mu - \hbar\omega_{mn})/T}]\\ &+ \frac{1}{\omega_{mn} - \omega}[1 \mp e^{(\mu - \hbar\omega_{mn})/T}]\}.\end{aligned} \quad (4)$$

Comparing the two terms within the braces we see that there exists a certain relation between the real (G') and the imaginary (G'') part of the Green function. In the case of Bose statistics this relation is

$$G'(\omega, \mathbf{k}) = \frac{1}{\pi}\int_{-\infty}^{\infty} \tanh\frac{\hbar x - \mu}{2T} \cdot \frac{G''(x, \mathbf{k})}{x - \omega} dx, \quad (5)$$

where we take the principal value of the integral. We have always

$$G''(\omega, \mathbf{k}) < 0. \quad (6)$$

as follows from Eq. (4).

In the case of Fermi statistics we have

$$G'(\omega, \mathbf{k}) = \frac{1}{\pi}\int_{-\infty}^{\infty} \coth\frac{\hbar x - \mu}{2T} \cdot \frac{G''(x, \mathbf{k})}{x - \omega} dx, \quad (7)$$

where the sign of $G''(\omega, \mathbf{k})$ is the opposite of the sign of the difference $\hbar\omega - \mu$ and both these quantities go through zero at the same time,

$$G''(\omega, \mathbf{k})/(\hbar\omega - \mu) < 0. \quad (8)$$

At the absolute zero of temperature both (5) and (7) go over into

$$G'(\omega, \mathbf{k}) = \pm \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{G''(x, \mathbf{k})}{x - \omega} dx, \quad (9)$$

where the plus sign refers to $\hbar x > \mu$ and the minus sign to $\hbar x < \mu$.

It is interesting to note that these formulae show that the function G is not an analytical function of the variable ω. We can construct two analytical functions (which have no singularities in the upper half-plane) as follows

$$G' + i \tanh \frac{\hbar\omega - \mu}{2T} \cdot G'' \text{ and } G' + i \coth \frac{\hbar\omega - \mu}{2T} \cdot G''.$$

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