FISSION OF ROTATING NUCLEI

G. A. PIK-PICHAK

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An expression is obtained for the fission barrier of a rotating nucleus. The fission cross section is estimated for the reaction of N^{14} ions on heavy nuclei.

DRUIN, Polikanov, and Flerov[1] studied fission induced by heavy particles. Ions of N^{14} with energies ~ 100 Mev were captured by nuclei of U, Bi, Au, and Re, producing a nucleus with high excitation energy and high angular momentum. The maximum angular momentum given to the nucleus is

$$M_{\max} = \sqrt{2\mu (R^* + R_1)^2 (E - B)},$$

where R_1 and R^* are the radii of the ion and the target nucleus, μ is the reduced mass, E is the energy of the ion in the center-of-mass system, and B is the Coulomb barrier. For nuclei with A ~ 200 and ion energies ~ 100 — 200 Mev, the maximum angular momentum is ~ 50 — 120ħ. At such high excitation energy, this angular momentum is almost entirely associated with rotation of the nucleus as a whole. The fraction of the angular momentum associated with the spin of the nucleons is easily estimated for the case of a Fermi gas.[2] The spin of the nucleus turns out to be $\hbar s = M\hbar^2/\Delta I$, where Δ is the spacing between single particle levels ~ 0.1 Mev, and I is the moment of inertia of the nucleus. For excitation energies of the size we are considering, where shell effects can be neglected, the moment of inertia of the nucleus is assumed to be equal to that of a rigid body[3] (for the case of a sphere, $I = I_0 = \frac{2}{5} AmR^2$, where Am and R are the mass and radius of the compound nucleus). On this assumption, $\hbar^2/I \sim 1$ kev and $\hbar s/M \ll 1$.

For $M \sim 50 - 120\hbar$, the rotational energy $E_{rot} = M^2/2I$ is comparable with the height of the fission barrier ($E_{rot} \sim 5 - 20$ Mev), so in this case the rotation of the nucleus must be taken into account in calculating the fission barrier.

The total change in energy of the rotating nucleus upon deformation is

$$\Delta E = \Delta E_s + \Delta E_q + \Delta E_{\text{rot}}, \tag{1}$$

where ΔE_s, ΔE_q, and ΔE_{rot} are the changes in the surface, Coulomb, and rotational energies. The last change is caused by a change in the momentum of inertia (as a result of deformation of the nucleus) while the angular momentum is kept fixed. The stable shape during rotation of the nucleus is, naturally, that of an axially symmetric oblate ellipsoid with its axis of symmetry along the direction of the angular momentum. Deformations of different types will either favor or hinder fission. For example, deformations corresponding to an elongated ellipsoid with its symmetry axis perpendicular to the angular momentum will favor fission, while the same ellipsoid with its axis along the angular momentum will hinder fission (because of the effect of the centripetal forces).

The transition from the shape of an oblate ellipsoid to that of a prolate ellipsoid with its symmetry axis perpendicular to the angular momentum is accomplished by means of a deformation which is not axially symmetric.

The shape of the nuclear surface is given by the radius vector

$$r(\theta, \varphi) = R_0 \left(1 + \sum_{l,m} \alpha_{lm} D^l_{m0}(\theta, \varphi, 0)\right), \tag{2}$$

where R_0 is the radius of the sphere of equal volume, $D^{\ell}_{m0}(\theta, \varphi, 0) = D^{m}_{\ell}(\theta, \varphi)$ are the spherical functions with the normalization

$$D^l_m(0, 0) = 1.$$

If we consider a nucleus rotating about one of its axes of inertia and limit ourselves to $\ell = 2$, it is convenient to introduce deformation coordinates in the form[4]

$$\alpha_{21} = \alpha_{2-1} = 0; \ \alpha_{20} = \alpha \cos\gamma; \ \alpha_{22} = \alpha_{2-2} = \frac{\alpha}{\sqrt{2}} \sin\gamma.$$

Then the change in the semiaxes of the ellipsoid will be:

$$\delta R_x = \alpha R_0 \cos\left(\gamma - \frac{2\pi}{3}\right); \ \delta R_y = \alpha R_0 \cos\left(\gamma - \frac{4\pi}{3}\right);$$

$$\delta R_z = \alpha R_0 \cos\gamma.$$

In these variables, we have (dropping terms $\sim \alpha^4$):

$$\Delta E_s + \Delta E_q = 4\pi R^2 O \left\{\frac{2}{5} z\alpha^2 - \frac{4}{105}\alpha^3 \cos^3 \gamma\right\}, \qquad z = 1 - x = 1 - (Z^2/A)/(Z^2/A)_{cr}, \tag{3}$$

where $R = r_0 A^{1/3}$; $4\pi r_0^2 O = 15$ Mev is the magnitude of the surface tension, and A and Z are the mass and charge of the nucleus. The value of (3) is triply degenerate in γ, since there is no preferred direction in space, so that it makes no difference along which axis the fission occurs.

But if the nucleus is rotating and the z axis is directed along the angular momentum, the moment of inertia with respect to the z axis is

$$I_z = I_0\left(1 - \alpha_{20} + \frac{3}{7}\alpha_{20}^2 + \frac{22}{7}\alpha_{22}^2 + \ldots\right). \tag{4}$$

Let us calculate the change in energy for a rotating nucleus, limiting ourselves for simplicity to terms of first degree in y in the rotational energy:

$$\Delta E = 4\pi R^2 O \left\{y\alpha_{20} + \frac{2}{5} z\alpha_{20}^2 + \frac{4}{5} z\alpha_{22}^2 - \frac{4}{35}\alpha_{20}^3 + \frac{24}{35}\alpha_{20}\alpha_{22}^2\right\}, \quad y = (M^2/2I_0)/4\pi R^2 O. \tag{5}$$

The condition for an extremal gives four solutions

$$\alpha_{22} = 0, \ \frac{12}{35}\alpha_{20}^2 - \frac{4}{5} z\alpha_{20} - y = 0, \tag{6}$$

$$\alpha_{20} = -\frac{7}{6} z, \ \alpha_{22}^2 = \left(\frac{7}{6}\right)^2 \frac{3}{2}\left(z^2 - \frac{5}{7} y\right). \tag{7}$$

Solution (7) gives two saddle points located symmetrically around $\gamma = 0$. At the saddle points the nucleus has a shape close to that of an axially symmetric elongated ellipsoid with its axis of symmetry perpendicular to the angular momentum.

Solution (6) corresponds to an axially symmetric shape of the nucleus with its symmetry axis along the angular momentum. The first solution of (6) with $\alpha_{20} > 0$ gives a saddle shape, while the second solution of (6) with $\alpha_{20} < 0$ requires additional investigation. In the neighborhood of the second solution (6), the energy has the form:

$$\Delta E = \text{const} + \frac{2}{5} z\sqrt{1 + 15y/7z^2}\,\Delta^2 + \frac{4}{5} z\left(2 - \sqrt{1 + 15y/7z^2}\right)\gamma^2,$$

where Δ and γ are small deviations from the values of α_{20} and α_{22}, respectively, in (6). It is easy to see that for $y < y_{cr} = 7z^2/5$, the solution gives an absolute minimum, while for $y > y_{cr}$ we get a saddle, since the coefficient of γ^2 becomes negative. [We also note that for $y = y_{cr}$ the solutions (7) and the solution of (6) for $\alpha_{20} < 0$ coincide.] Thus for $y \geq y_{cr}$ there is no stable state of the nucleus and, consequently, the fission barrier for such a nucleus is equal to zero.

Inclusion of quadratic terms in α in ΔE_{rot} does not change the qualitative results, but for a more exact calculation these terms cannot be neglected. This gives an essential change not only in the value of y_{cr}, but also in the dependence of the fission barrier on angular momentum. Inclusion of further terms ($\sim \alpha^4$ and $\alpha_{\ell m}$ with $\ell > 2$) produces little change in the result. Naturally these terms become important for y close to y_{cr}. If we limit ourselves to 15% accuracy in the dependence of the fission barrier on angular momentum for $z \leq 0.3$, we may drop terms $\sim \alpha^4$ and $\alpha_{\ell m}$ with $\ell > 2$, and neglect terms of third degree in α in ΔE_{rot}. As a result of long-winded but straightforward computations, the height of the fission barrier E_f, corresponding to fission through the saddles (7) is equal in this approximation to

$$E_f = 4\pi R^2 O\left[0.73 z^3 - (1.2 z + 5.6 z^2) y + (4.6 + 11 z) y^2\right] = 4\pi R^2 O f(z, y), \tag{8}$$

and the quantity y_{cr} (neglecting y^2) is

$$y_{cr} = 7z^2/5(1 + 6z). \tag{9}$$

The deformation at the position of the minimum (6) is

$$\alpha_{20} = -1.25y/z + (0.58 + 1.8z)y^2/z^3; \ \alpha_{22} = 0, \tag{10}$$

and the energy corresponding to (10),

$$\Delta E_{min} = -0.625y^2/z + \cdots.$$

Strictly speaking, the fission barrier is the quantity $E_f - \Delta E_{min}$, but since for $y < y_{cr}$, $\Delta E_{min} \ll E_f$, and $|\Delta E_{min}| \sim |E_f|$ only for $y \approx y_{cr}$, we can neglect ΔE_{min} in this case. The quantity f(x, y) is shown in Fig. 1. The solid curves give f(x, y) according to (8).

We note that an axially symmetric deformation with symmetry axis perpendicular to the angular momentum gives a position of the saddle only slightly different from (8). The barrier height for such a "relative" saddle actually is the same as for (8). The value of f(x, y) for axially symmetric deformations is shown by the dashed line in Fig. 1.

Now we find the magnitude of the fission cross section. For high excitation energy, two main processes are possible: neutron evaporation and fission of the nucleus. We use the statistical formulas for the neutron and fission widths[5]*

$$\Gamma_n = (T^2 A^{2/3}/\pi K)\exp(-E_n/T),$$
$$\Gamma_f = (T/2\pi)\exp(-E_f/T), \tag{11}$$

where $T = (10U/A)^{1/2} \approx 2$ Mev for $A \sim 200$ and $U \sim 100$ Mev is the excitation energy of the nucleus, $K = 10$ Mev, E_n is the neutron binding energy.

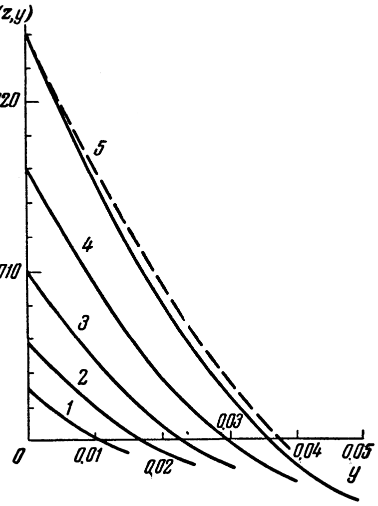

FIG. 1. 1 – z = 0.16; 2 – z = 0.20; 3 – z = 0.24; 4 – z = 0.28; 5 – z = 0.32.

We shall assume that the temperature and angular momentum of the compound nucleus do not change when a neutron is boiled off (this is valid, at least, for the first stage of the evaporation process, in which half the number of neutrons emerge of those which can be boiled off at the given excitation energy). In this case the probability of fission with emission of a number of neutrons equal to or less than m is

$$W_f(y) = 1 - (\Gamma_n/\Gamma)^{m+1}, \tag{12}$$

where $\Gamma = \Gamma_n + \Gamma_f$. The total fission cross section for $M_{max}/\hbar \gg 1$ is

$$\sigma_f = \pi (R^* + R_1)^2 \left(1 - \frac{B}{E}\right) \frac{1}{y_{max}} \int_0^{y_{max}} W_f(y)\,dy,$$
$$y_{max} = \frac{\mu (R^* + R_1)^2}{I_0} \frac{E - B}{4\pi R^2 O}. \tag{13}$$

*Γ_f depends on the angular momentum M not only via the dependence of the fission barrier E_f on M, but also through the M dependence of the factor in front of the exponential, which comes from integration of the angular distribution of the fission fragments. (For $M \neq 0$, the fission probability depends on the direction of fission.) But because of the weak dependence of this factor on M, we shall use formula (11) which was derived for the case of isotropic fission.

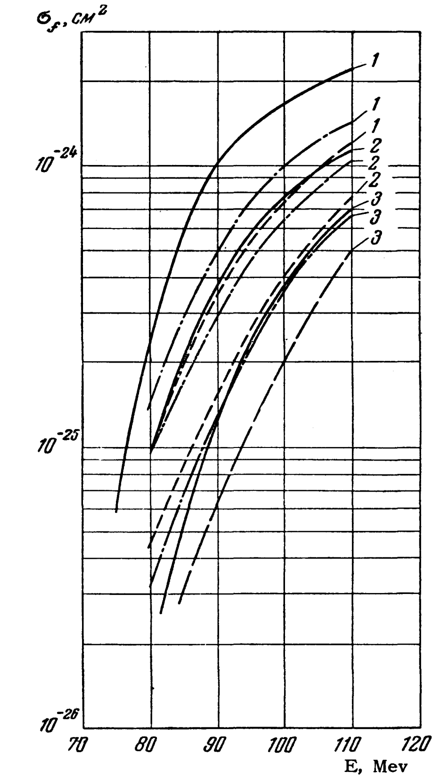

FIG. 2. Curve 1 – Bi, 2 – Au, 3 – Re.

The number of emerging neutrons as a function of energy of the ions was determined[6] for the reaction on Au^{197}. The same dependence is assumed for the reactions on Bi^{209} and Re^{187}. The solid curves of Fig. 2 show the experimental curves for the fission cross section as a function of energy of incident ions in the laboratory system. The behavior of the fission cross section calculated from (13) with $r_0 = 1.5 \times 10^{-13}$ cm [the dashed curve is for $(Z^2/A)_{cr} = 52$, the dot-dash curve for $(Z^2/A)_{cr} = 51$], are in qualitatively good agreement with experiment. The best agreement is obtained for $(Z^2/A)_{cr} = 51$. This number is somewhat greater than that found from the value of the fission barrier in U^{238} (Ref. 7). This difference is apparently explained by the fact that, first of all, the value of the fission barrier for Au and Re in the approximate expression (8) is lowered relative to the exact calculation[8] for zero angular momentum (and also for angular momenta different from zero) and secondly, the fission barrier of U^{238} apparently depends essentially on shell effects. [Thus, for example, from the fission barrier of Am^{242}, $E_f \cong 6.1$ Mev,[9,10] we get $(Z^2/A)_{cr} = 49$.]

For higher energies of the ions than those used in Refs. 1 and 6, the fission cross section on nuclei lighter than Au will increase, while for energies for which $y_{max} \sim y_{cr}$, the fission cross section will actually coincide with the total reaction cross section. Since the value of z will be greater than 0.3 for target nuclei lighter than Au, expression (8) will contain a large error, so we can only approximately give the region of ion energies for which the fission cross section will equal the total cross section. Thus, for example, for Yb and Dy, this region is around 150 Mev. The fission cross section, naturally, also increases with increasing mass of the bombarding particles.

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ON THE THEORY OF THERMAL EXCITATION OF POLARONS

V. A. MOSKALENKO

Kishinev State University

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The probability of a quantum transition of a polaron from the 1s to the 2p state caused by thermal vibrations of the lattice is computed. The adiabatic form of perturbation theory is used in the calculations. At room and higher temperatures transition to the 2p state occurs during $10^{-8} - 10^{-9}$ sec.

1. INTRODUCTION

POLARONS are the principal carriers of current in ionic crystals.[1] As is well known, in crystals which have large cohesive energies polarons are characterized by larged effective masses, and also by the existence of a fluctuation movement of the electron with respect to the center of gravity of the polaron. There exists a series of bound states, between which quantum transitions are possible. In this paper we shall consider such a transition between a 1s ground state and a 2p final state.

During the transition, the momentum of the polaron is conserved and changes in its kinetic energy occur at the expense of changes in its effective mass. The process under consideration turns out to be a multiphonon process. Frenkel[2] was the first to show that such transitions are possible in crystals. He pointed out that the equilibrium configuration of the field oscillators changes during such a transition. A quantitative theory of nonradiative transitions at F centers, based on this idea, was presented by a number of authors.[3-6] Another mechanism for thermal transitions was suggested by Kubo.[7]

In this paper the basic idea and method of the