ser[14] for the fission U^{235} by thermal neutrons, was extended to include the entire nucleus under consideration. The calculated values of E_n were found to be in good agreement with measured values of $d\nu/dE_x = 1/E_n$ (Refs. 15 — 17).

The mean energy E_γ of the instantaneous γ rays was assumed constant, 8 Mev, in the calculations, as confirmed by experiments on the study of the prompt-fission γ rays from U^{235}(n, f) and Cf^{252} (Refs. 15, 18).

For comparison with the calculations, all the experimental values of γ, for the fission induced by neutrons, were reduced to values of ν for spontaneous fission of the corresponding compound nuclei, using the formula $d\nu/dE_x = 1/E_n$. The validity of this operation was confirmed by direct comparison of ν for the processes Pu^{235}(n, f) and Pu^{241}(n, f) (Ref. 5) with ν for spontaneous fission of Pu^{240} and Pu^{242} (Refs. 1 — 3).

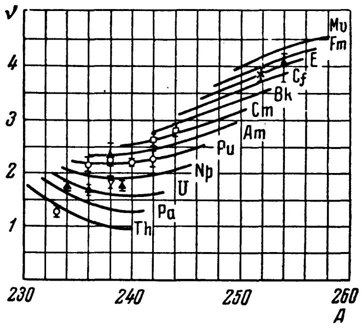

Fig. 1. Dependence of the average number of prompt fission neutrons on A, for various Z. Lower point ○ – Th, ▲– U, ▽ –Np, ○ – Pu, □ – Cn, × – Cf, ● – Fm.

The diagram shows a family of curves of ν as functions of A for various Z. Most experimental data are in satisfactory agreement with the calculations. An exception is the value of ν for the spontaneous fission of U^{238} (Ref. 7). The value of ν for the spontaneous fission of Th^{232} was not compared with the results of these calculations, for, according to the latest measurements for the period of spontaneous fission of Th^{232} (Ref. 19), there is a certain doubt concerning the value $\nu_{Th}/\nu_U = 1.07 \pm 0.1$ (Ref. 6).

The non-monotonic course of $\nu(A)$ is due to the shell structure of the fragments. Without taking the shells into account, $d\nu/dA > 0$. On nuclei with $A < 240$, i.e., with $A_\ell < 100$, the nearness of a shell with 50 neutrons manifests itself — the excitation energy increases with diminishing A, and the sign of $d\nu/dA$ is reversed.

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ON THE DOUBLE BETA-DECAY OF Ca^{48}

V. B. BELIAEV and B. N. ZAKHAR'EV

Joint Institute of Nuclear Studies

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A number of experimental researches have shown[1,2] that the period of double β-decay is greater than 10^{18} years. In this connection, the necessity has arisen of re-examining the estimates that have been given in the literature for the probability of β-decay with the emission of two neutrinos. Goeppert-Mayer[3] estimated the probability of two-neutrino decay, but did not calculate the nuclear matrix element, which led to disagreement with experiment. We have carried out this calculation in the approximation of the shell model.

We consider the case most convenient for calculation, the decay $Ca^{48} - Ti^{48}$. The maximum decay energy, the identical structures of the parent and product nuclei, and the fully occupied shells of definite spins J distinguish this reaction among all the cases for which double β-decay is energetically possible.

It is assumed that the transition occurs by way of the virtual intermediate state Sc^{48}. All three of the nuclei in question (Ca^{48}, Sc^{48}, Ti^{48}) have the same core Ca^{40}, which plays no part in the process. Therefore we shall be interested in only 8 nucleons in each nucleus. We note further that the radial functions of the nuclei in question are identical, and the corresponding integrals are not involved in our considerations. The construction of the functions is facilitated by the convenient structure of the nuclei chosen. Their filled shells and almost filled shells do not require the rather complicated apparatus of fractional parentage coefficients.

The functions for the initial and final states have been given by Maksimov and Smorodinskii.[4] The function for the intermediate state is constructed analogously. Its spin-orbit corresponds to the Young schemes [2111111] for T = 3 and [11111111] for T = 4. In the first case the intermediate function reduces, from the point of view of spatial symmetry, to a function of s = 2 nucleons. Thus we have

$$\Phi_p^M = \frac{1}{\sqrt{8}} \sum \pm \chi(i)\, \Psi_p^M(i),$$

where, for example, for the case in which the first nucleon is a proton

$$\Psi_p^M(1) = \frac{r}{8!\,27\cdot 7} \hat{A}_7 \begin{bmatrix} 3 \\ 4 \end{bmatrix} \left(\begin{bmatrix} 5 \\ 6 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \end{bmatrix} - \begin{bmatrix} 5 \\ 7 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \end{bmatrix} - \begin{bmatrix} 5 \\ 8 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \end{bmatrix} \right) \sum_m ({}^7/_2\, {}^7/_2\, M - m\, m \mid {}^7/_2\, {}^7/_2\, 1 M)\, \varphi_{M-m}^{(1)} \varphi_m^{(2)}$$

Here $(j_1 j_2 m_1 m_2 \mid j_1 j_2 J M)$ is a Clebsch-Gordan coefficient, M is the z component of the angular momentum J, and $\hat{A}_7$ denotes antisymmetrization with respect to seven particles (without the first). The square of the matrix element (with tensor interaction) for the transition $Ca^{48} - Sc^{48}$ $(T = 3) - Ti^{48}$ is $M^2 = 0.006$ (for the transition through Sc^{48} with T = 4 it is an order of magnitude smaller). The half-value period is

$$T = \frac{0.693\cdot 6\cdot 7\cdot 15\, |\Gamma(3+2s)|^4 h^{13}}{4^6 \pi^8 \gamma^2 m^{11} c^{10} g^4} \times \left(\frac{4\pi m c \rho}{h}\right)^{-4s} \frac{m^2 c^4}{M^2} \cdot 10^{-9} \text{ years (Ref. 3).}$$

Taking it into account that the decay can also go through excited intermediate states, we get for the half-life a value of about 10^{19} years. As has been shown by Bohr and Mottelson,[5] a correction factor must be applied to the probability of decay calculated by the shell model; we take it from the data for nuclei in the neighborhood of Ca^{48}; $\lambda \sim 0.01$. In our case, however, the transition is between even-even nuclei, and therefore it can be hoped that less of a correction to the shell model may turn out to be needed ($\lambda > 0.01$). It would therefore be interesting to check how reliable the shell model is for Ca^{48}.

We take occasion to express our gratitude to Professor Ia. A. Smorodinskii and to L. A. Maksimov for valuable advice.

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METHOD OF MEASURING PARTICLE ENERGIES ABOVE 10^{11} ev

N. L. GRIGOROV, V. S. NURZIN, and I. D. RAPOPORT

Moscow State University

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MORE than two years ago one of us (Grigorov) proposed a method for determining the energy of a separate nuclear-active particle, based on the measurement of the total energy liberated in dense matter by all the secondary particles produced