mates that have been given in the literature for the probability of β-decay with the emission of two neutrinos. Goeppert-Mayer[3] estimated the probability of two-neutrino decay, but did not calculate the nuclear matrix element, which led to disagreement with experiment. We have carried out this calculation in the approximation of the shell model.

We consider the case most convenient for calculation, the decay $Ca^{48} - Ti^{48}$. The maximum decay energy, the identical structures of the parent and product nuclei, and the fully occupied shells of definite spins J distinguish this reaction among all the cases for which double β-decay is energetically possible.

It is assumed that the transition occurs by way of the virtual intermediate state Sc^{48}. All three of the nuclei in question (Ca^{48}, Sc^{48}, Ti^{48}) have the same core Ca^{40}, which plays no part in the process. Therefore we shall be interested in only 8 nucleons in each nucleus. We note further that the radial functions of the nuclei in question are identical, and the corresponding integrals are not involved in our considerations. The construction of the functions is facilitated by the convenient structure of the nuclei chosen. Their filled shells and almost filled shells do not require the rather complicated apparatus of fractional parentage coefficients.

The functions for the initial and final states have been given by Maksimov and Smorodinskii.[4] The function for the intermediate state is constructed analogously. Its spin-orbit corresponds to the Young schemes [2111111] for T = 3 and [11111111] for T = 4. In the first case the intermediate function reduces, from the point of view of spatial symmetry, to a function of s = 2 nucleons. Thus we have

$$\Phi_p^M = \frac{1}{\sqrt{8}} \sum \pm \chi(i) \Psi_p^M(i),$$

where, for example, for the case in which the first nucleon is a proton

$$\Psi_p^M(1) = \frac{r}{8!\,27 \cdot 7} \hat{A}_7 \begin{bmatrix} 3 \\ 4 \end{bmatrix} \left(\begin{bmatrix} 5 \\ 6 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \end{bmatrix} - \begin{bmatrix} 5 \\ 7 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \end{bmatrix} - \begin{bmatrix} 5 \\ 8 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \end{bmatrix} \right) \sum_m (7/2\, 7/2\, M-m\, m \mid 7/2\, 7/2\, 1M)\, \varphi_{M-m}^{(1)} \varphi_m^{(2)}$$

Here $(j_1 j_2 m_1 m_2 \mid j_1 j_2 JM)$ is a Clebsch-Gordan coefficient, M is the z component of the angular momentum J, and $\hat{A}_7$ denotes antisymmetrization with respect to seven particles (without the first). The square of the matrix element (with tensor interaction) for the transition $Ca^{48} - Sc^{48}$ $(T = 3) - Ti^{48}$ is $M^2 = 0.006$ (for the transition through Sc^{48} with T = 4 it is an order of magnitude smaller). The half-value period is

$$T = \frac{0.693 \cdot 6 \cdot 7 \cdot 15 \,|\Gamma(3+2s)|^4 h^{13}}{4^6 \pi^8 \gamma^2 m^{11} c^{10} g^4} \times \left(\frac{4\pi mc\rho}{h}\right)^{-4s} \frac{m^2 c^4}{M^2} \cdot 10^{-9} \text{ years (Ref. 3).}$$

Taking it into account that the decay can also go through excited intermediate states, we get for the half-life a value of about 10^{19} years. As has been shown by Bohr and Mottelson,[5] a correction factor must be applied to the probability of decay calculated by the shell model; we take it from the data for nuclei in the neighborhood of Ca^{48}; $\lambda \sim 0.01$. In our case, however, the transition is between even-even nuclei, and therefore it can be hoped that less of a correction to the shell model may turn out to be needed ($\lambda > 0.01$). It would therefore be interesting to check how reliable the shell model is for Ca^{48}.

We take occasion to express our gratitude to Professor Ia. A. Smorodinskii and to L. A. Maksimov for valuable advice.

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METHOD OF MEASURING PARTICLE ENERGIES ABOVE 10^{11} ev

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MORE than two years ago one of us (Grigorov) proposed a method for determining the energy of a separate nuclear-active particle, based on the measurement of the total energy liberated in dense matter by all the secondary particles produced when the primary particle passes through a thick layer of matter.

If we know the ionization I(x) produced by the "primary" particle and all its descendents at each point x of the absorber, then the energy E_0 of the "primary" particle will be

$$E_0 = \varepsilon \int_0^\infty I(x)\,dx,$$

disregarding the decay processes involving neutrino production (in dense matter this process can be neglected); ϵ is the mean energy for the production of one pair of ions in the material of the absorber.

We recently constructed an instrument based on the above principle, and obtained experimental results at 3860 m above sea level.

The instrument is a truncated pyramid 170 cm high, the upper cross-section of which is approximately 0.6 m^2, and the lower one approximately 0.8 m^2 (Fig. 1). The pyramid contains eight layers of iron with a total thickness of 85 cm.

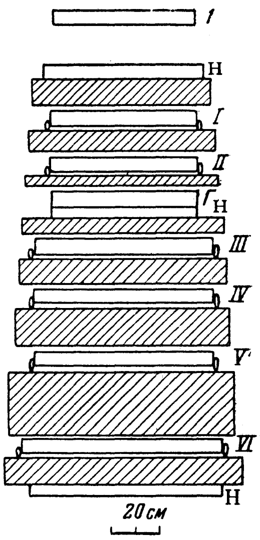

Fig. 1. Schematic cross-section through the setup. I–VI – rows of ionization chambers; 1, 2 – rows of telescopic counters; H – boxes with hodoscopic counters.

When chosing the absorber material it was necessary to compromise between the following requirements: (a) the range of the electron-photon cascade should be greater than the range for the nuclear interaction; (b) the absorber matter should be sufficiently dense to absorb all the primary and all the secondary particles at sufficiently small thickness, ~ 1 m.

The ionization was measured in our instrument by means of cylindrical pulse ionization chambers, made of iron and brass (to eliminate transient effects), with walls 1 mm thick. The chambers cover an area 0.36 m^2 in the upper section of the pyramid and 0.6 m^2 in the lower one. The diameter of each chamber is 4 cm, and the length fluctuates from 65 to 75 cm. The chambers are filled with pure argon to a pressure of 5.5 atmos.

The ionization chambers are placed in the instrument between layers of iron in six rows (I — VI in Fig. 1). The upper row contains 15 chambers, the lower 21. The total number of chambers in the instrument is 105. Each three chambers is connected to one amplifier with a dynamic range of approximately 800, whose output is connected with an individual cathode ray tube 8 cm in diameter. The electric pulses produced in the ionization chambers are recorded by photographing the screens of all tubes, which are mounted in a single block. This block, which we call a multi-channel oscillograph, contains 49 cathode ray tubes. In addition to ionization chambers, the instrument contains a counter telescope (row 1 and 2) that restricts the solid angle for "primary" particles, and several boxes with hodoscopic counters.

The instrument is controlled as follows. The ionization pulse from each row of chambers, regardless of the chamber in which the pulse originates is received by a special selector, which operates if the pulse exceeds a certain threshold value V_{thr} simultaneously in any n rows of chambers (both n and V_{thr} can be varied over a wide range). If the selector operates simultaneously with the discharges in the telescopic counters, the resultant electric pulse triggers the beams of all the multi-channel oscillograph tubes and the amplitude of the pulses from the ionization chambers are photographed. The minimum recordable ionization corresponds to a simultaneous passage of 5 — 10 relativistic particles through the center line of the chamber.

The fact that our instrument records ionization with the aid of a large number of independently-operating chambers, makes it possible to trace

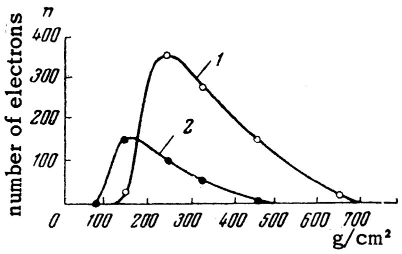

Fig. 2. Examples of events of electron-nuclear cascades: 1 – energy of "primary" particle 1.5 × 10^{11} ev, 2 – energy of "primary" particle 0.4 × 10^{11} ev.

the energy losses of an individual high-energy particle in a dense substance.

Certain examples of recorded events are shown in Fig. 2. In the future we plan to place a cloud chamber over the energy detector to permit study of an elementary interaction event between nuclear-active particles of known energy and nuclei of specified atomic numbers.

A detailed description of the experimental data obtained with the above instrument will be published.

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COLLECTIVE MOTIONS IN A SYSTEM OF QUASI-PARTICLES

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IN the self-consistent field approximation the consideration of a strong interaction between particles leads to a dependence of the energy of a separate particle (which is considered as a quasi particle) on the state of motion of the other particles in the system. Even in the case of a spatially uniform distribution of the particles in the system, a consideration of the interaction leads to a complicated dependence of the particle energy on its momentum.*

The Hamiltonian function of a quasi particle in the case of a non-uniform distribution in space can be taken in the form

$$\varepsilon_j(\mathbf{p}) + \int G(|\mathbf{r}-\mathbf{r}'|)\sum_i |\psi_i(\mathbf{r}')|^2 d\mathbf{r}', \quad (1)$$

where the first term takes into account correlations at small distances apart and the kinetic energy, while the second term refers to long-range interactions (Hartree field). The equation of motion for ψ_j is of the form

$$\varepsilon_j(\hat{\mathbf{p}})\varphi_j + \int G(|\mathbf{r}-\mathbf{r}'|)\sum_i |\psi_i(\mathbf{r}')|^2 d\mathbf{r}'\psi_j = \varepsilon_j\psi_j. \quad (2)$$

We shall restrict our considerations to those states of the system which are close to a spatially uniform distribution of the quasi particles as far as the coordinates are concerned, and to a random distribution as far as the velocities are concerned, i.e., to states near the ground state. In that case the momenta of the collective motions (motions of a hydrodynamical character) will be small and the operator $\epsilon_j(\hat{\mathbf{p}})$ can be written in the form $\epsilon_j(\hat{\mathbf{p}}_0+\hat{\mathbf{p}})$, where $\hat{\mathbf{p}}$ is the operator of the momentum of the collective motion (a small quantity) and $\hat{\mathbf{p}}_0$ the operator of the momentum of the random motion. Expanding ϵ_j in powers of $\hat{\mathbf{p}}$ and limiting ourselves to terms quadratic in $\hat{\mathbf{p}}$, we obtain

$$\varepsilon_j(\hat{\mathbf{p}}_0+\hat{\mathbf{p}}) = \varepsilon_j(\hat{\mathbf{p}}_0) + \frac{1}{2}[\nabla_{\mathbf{p}_0}\varepsilon_j(\hat{\mathbf{p}}_0), \mathbf{p}]_+ + \frac{1}{4}[\Delta_{\mathbf{p}_0}\varepsilon_j(\hat{\mathbf{p}}_0), \hat{\mathbf{p}}^2]_+ + \cdots, \quad (3)$$

where $[\,,\,]_+$ denotes an anticommutator.

We introduce into the wave function the new variable $\mathbf{r}_0$, canonically conjugate to $\hat{\mathbf{p}}_0$. If we assume that the commutator $[\hat{\mathbf{p}}_0, \hat{\mathbf{p}}]_- = 0$, then $\psi_j = \psi_{0j}(\mathbf{r}_0)\Phi_j(\mathbf{r})$. Substituting (3) into (2) and averaging over the functions $\psi_{0j}(\mathbf{r}_0)$ which satisfy the equation

$$\varepsilon_j(\hat{\mathbf{p}}_0)\psi_{0j} = \varepsilon_{0j}\psi_{0j}, \quad (4)$$

we obtain the equation

$$-\frac{\hbar^2}{2m_j^*}\Delta\Phi_j - i\hbar(\mathbf{v}_j^0\nabla)\Phi_j + \int G(|\mathbf{r}-\mathbf{r}'|)\sum_i|\Phi_i(\mathbf{r}')|^2 d\mathbf{r}'\Phi_j = (\varepsilon_j - \varepsilon_{0j})\Phi_j, \quad (5)$$

in which we have used the notation

$$1/m_j^* = \int \psi_{0j}^*\Delta_{\mathbf{p}_0}\varepsilon_j(\hat{\mathbf{p}}_0)\psi_{0j}d\mathbf{r}',$$

$$\mathbf{v}_j^0 = \int \psi_{0j}^*\nabla_{\mathbf{p}_0}\varepsilon_j(\hat{\mathbf{p}}_0)\psi_{0j}d\mathbf{r}', \quad \varepsilon_{0j} = \int \psi_{0j}^*\varepsilon_j(\hat{\mathbf{p}}_0)\psi_{0j}d\mathbf{r}'.$$

Equation (5) obtained in this way describes only the collective motions in a system of quasi particles, but its coefficients depend on the characteristics of the random motion of the quasi particles in the ground state determined by Eq. (4).

Substituting into (5)

$$(\varepsilon_j - \varepsilon_{0j}) \to i\hbar\partial/\partial t, \quad \Phi_j = (1+\rho_j)^{1/2}e^{iS_j/\hbar}$$

(Φ_j is normalized in a unit volume), and retain-

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*Taking exchange interaction into account gives, for instance, in the case of Coulomb forces,

$$\varepsilon(p) = \frac{e^2}{\pi\hbar}p_0\left(2 + \frac{p_0-p}{p_0 p}\ln\frac{p_0+p}{p_0-p}\right)$$

(p_0 is the momentum at the Fermi surface); taking force correlations into account leads to a more complicated dependence $\varepsilon(p)$.