This result is in good agreement with the data given by Moliere[6] for $rE/E_k \ll 1$.

By way of an example it is easy to calculate that for $E = 10^8$ ev, at a distance of 1 m from the axis of the shower and at a primary-electron energy of $E_0 = 10^{14}$ ev, we have $N_1/N \approx 7-8$. This effect can be explained by the fact that the high-energy electrons located near the shower axis are accompanied by a greater number of photons. For distances $r \geq E_k/E$ it is necessary to take ionization losses into account.

In conclusion I express my gratitude to I. L. Rozental' for advice and aid in this work, and also to I. P. Ivanenko for useful discussions.

[1] I. Ia. Pomeranchuk, J. Exptl. Theoret. Phys. (U.S.S.R.) **14**, 1252 (1944).

[2] A. B. Migdal, J. Exptl. Theoret. Phys. (U.S.S.R.) **15**, 313 (1945).

[3] S. Z. Belen'kii, Лавинные процессы в космических лучах (Cascade Processes in Cosmic Rays), Gostekhizdat, 1948.

[4] J. Nishimura and K. Kamata, Progr. Theor. Phys. **5**, 899 (1950).

[5] L. D. Landau, J. of Phys. (U.S.S.R.) **3**, 235 (1940).

[6] G. Moliere, Phys. Rev. **77**, 715 (1950).

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CONTRIBUTIONS TO THE THEORY OF DISPERSION RELATIONS

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It was shown by Bogoliubov, Logunov, and the author[1] that for those processes in which μ, e, and ν particles participate on par with strongly interacting particles, the anti-Hermitian part of the amplitudes expressed through the action of such weakly-interacting particles is equal to zero to first order in the weak-interaction constant C. This simplifies considerably the consequences of the dispersion relations as well as their form. Namely, the dispersion relations lead in this case to the statement that the amplitude for the process depends polynomially upon the sum of the 4-momenta of the weakly interacting particles (for decay, upon the difference), while the polynomial coefficients depend only upon the momenta of the strong-interacting particles. If as usual, the interaction Lagrangian does not contain derivatives of the fields, then the amplitudes are independent of the momenta of the weakly-interacting particles. It is easy to consider similarly processes in which only weakly-interacting particles participate (for example, the decay of the μ meson). The causality principle leads in this case to a Lagrangian which is local in all the fields, and the dispersion relations lead to the statement that the amplitude depends only polynomially upon the momenta.

It is worth noting that the weak interaction cases allow simple analyses on the basis of dispersion theory. For example, the wide spread opinion that in order to obtain the dispersion relations it suffices to apply the principle of causality formulated through the vanishing of the probability current commutator of space-like points, is easily seen to be incorrect.

Indded, to first order in C, the probability current commutator for weakly-interacting particles is zero over all space for any Lagrangian including a non-local one. Generally, non-local Lagrangians do not lead to polynomial dependence. Consider, for example, the Lagrangian:

$$L(x) = \int K(\xi^2)\,\varphi(x+\xi)\,\varphi(x-\xi)\,\psi(x+\xi)\,\psi(x-\xi)\,d\xi.$$

Applying to it perturbation theory, we obtain the following expression for the scattering amplitude:

$$S(p,q;\,p',q') = \delta(p+q-p'-q')\,\overline{K}((q+p)^2),$$

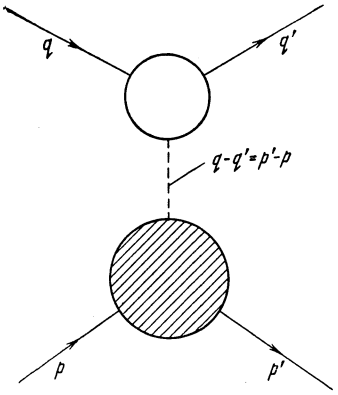
where q, q′ are the momenta of the scattered particles, and p, p′ are the momenta of the scatterers.

In this fashion, the dependence of the amplitude upon the momenta is determined from the kernel of the interaction Lagrangian and, in general, is not polynomial. Note that in deriving the dispersion relations, Goldberger et al.[2] also make use of time-ordered operators, in addition to the causality principle in the form of a commutator. These two conditions are combined in the generalized formulation of the principle of causality, and the formulation now proves sufficient to obtain the dispersion relations. From the example of weak interactions, it is easy to verify that the dispersion relations may similarly arise in certain non-local interactions. Consider for example the

process indicated on the figure.

The shaded area refers to strongly interacting particles. The momentum transfer is $q' - q = p - p'$, and therefore the amplitude depends only upon the squares of the momentum transfer $(p - p')^2$ (relativistic invariance), even though the interaction of scattered particles is non-local.

Thus if the scattering amplitude satisfies the dispersion relations, it still does not generally follow that the causality principle is satisfied.



One can only assert that if the dispersion relations are violated, then so is the principle of causality. A similar example was suggested by Lehmann.[4]

In conclusion I consider it my pleasant duty to thank N. N. Bogoliubov for his guidance and A. A. Logunov for discussion of the subjects presented here.

[1] Bogoliubov, Bilen'kii, and Logunov, Dokl. Akad. Nauk SSSR **115**, 891 (1957), Soviet Phys. "Doklady" **2**, 381 (1957).

[2] Проблемы современной физики (Probl. of Contemp. Physics) **2** (1957), Dispersion relations.

[3] N. N. Bogoliubov, Izv. Akad. Nauk SSSR, ser. fiz. **19**, 237 (1955).

[4] Proceedings of Seventh Rochester Conference, IV, 33 (1957).

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ADHESION OF SLOW ELECTRONS TO SF_6 *AND* CCl_4 *MOLECULES*

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FOX[1] has shown recently that the process $SF_6 + e \rightarrow SF_6^-$ has a resonant character and occurs in an energy range on the order of 0.05 ev at electron energies less than 0.1 ev. Carbon tetrachloride also captures electrons with energies close to zero, dissociating thereby into CCl_3 and Cl^- (Refs. 2 — 4), but in this case the energy and width of the capture region were determined with low accuracy.

We determined the energy and resonant capture cross-section of slow electrons in SF_6 and CCl_4.

The measurements were carried out in a setup similar to that of Lozier.[5] The electron beam was collimated by a cellular diaphragm and a magnetic field (15 — 20 oersted), and was passed through a diaphragm with variable potential and a screening grid into an equipotential region, and then to a collector. The ions produced in the equipotential region were gathered on a cylindrical collector, screened by the grid. Equality of the potential was insured by thorough compensation for the contact potentials.

To obtain monochromatic electrons, use was made of the so-called "quasi-monochromatization,"[6] which consists of passing a beam of electrons through a diaphragm that transmit only electrons with energies exceeding the diaphragm potential. The diaphragm potential was periodically varied by ΔV, and the increment in ion current due to this variation was measured. Obviously this increment is due to electrons with a distribution of width ΔV. It was possible to obtain by this method electrons with an exponential energy distribution 0.2 — 0.3 ev wide (the vertical and the right-hand solid curves of Figs. 1 and 2). The

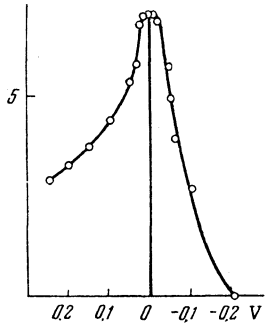

Fig. 1. o — ion current in SF_6.

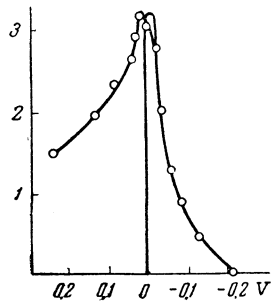

Fig. 2. o — ion current in CCl_4.