PHASE INDETERMINACIES IN NUCLEON-NUCLEON SCATTERING

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THE cross section for scattering of mesons by nucleons remains invariant under the phase substitution indicated by Minami.[1] The two sets of phase shifts, obtained from one another through this substitution, can only be distinguished either by means of polarization experiments,[2-3] or by analyzing the energy dependence of the cross section at low energies. We obtain below a similar transformation for the case of nucleon-nucleon scattering.

The elastic scattering of nucleons against nucleons is completely described by the scattering matrix $M(\mathbf{k}, \mathbf{k}_0; \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2)$, which determines the amplitude χ_f of the scattered wave in terms of the initial spin state χ_i:

$$\chi_f = M(\mathbf{k},\ \mathbf{k}_0;\ \boldsymbol{\sigma}_1,\ \boldsymbol{\sigma}_2)\,\chi_i. \tag{1}$$

Here $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the Pauli matrices for the two nucleons, and $\mathbf{k}_0$ and $\mathbf{k}$ denote unit vectors along the incident and scattered nucleon directions.

In order to obtain the transformation of interest, note that the scattering cross section for unpolarized nucleons $\sigma_0 = \frac{1}{4}\,\mathrm{Sp}\, MM^+$, is invariant to an exchange of $M(\mathbf{k}, \mathbf{k}_0; \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2)$ with one of the following matrices

$$M_1 = (\boldsymbol{\sigma}_1\mathbf{k})\, M\, (\boldsymbol{\sigma}_1\mathbf{k}_0), \qquad M_2 = (\boldsymbol{\sigma}_2\mathbf{k})\, M\, (\boldsymbol{\sigma}_2\mathbf{k}_0),$$
$$M_3 = (\boldsymbol{\sigma}_1\mathbf{k})\,(\boldsymbol{\sigma}_2\mathbf{k})\, M\, (\boldsymbol{\sigma}_1\mathbf{k}_0)\,(\boldsymbol{\sigma}_2\mathbf{k}_0). \tag{2}$$

We expand now the matrix M in terms of the spherical function $Y^{jm}_{ls}(\mathbf{k})$ describing a state of given total angular momentum j, its z-component m, orbital angular momentum l and spin s. Then

$$M(\mathbf{k},\ \mathbf{k}_0;\ \boldsymbol{\sigma}_1,\ \boldsymbol{\sigma}_2) = \sum_{j,\,m} \frac{2\pi}{ik} \sum_{l,\,s;\,l',\,s'} Y^{jm}_{ls}(\mathbf{k})\, Y^{jm+}_{l's'}(\mathbf{k}_0)\, R^{j}_{ls;\,l's'}. \tag{3}$$

The values of s and l are determined from the rule for adding angular momenta, and are as follows: for $s = 0$ (singlet) $l = j$; for $s = 1$ (triplet) $l = j,\ j \pm 1$. For a given value of j, the quantities $R^{j}_{ls,l's'}$ form a symmetric (reversibility of the motion) four-rowed matrix R^j satisfying the condition

$$R^{j+}R^{j} = -R^{j} - R^{j+}, \tag{4}$$

which arises from the unitarity of the S-matrix.

Consider, for example, the first of the transformations (2). Since the operator $(\boldsymbol{\sigma}_1\mathbf{k})$ commutes with the total angular momentum operator,

$$(\boldsymbol{\sigma}_1\mathbf{k})\, Y^{jm}_{ls}(\mathbf{k}) = L^{(1)j}_{l_1s_1,\,ls}\, Y^{jm}_{l_1s_1}(\mathbf{k}). \tag{5}$$

Thus

$$M_1(\mathbf{k},\ \mathbf{k}_0;\ \boldsymbol{\sigma}_1,\ \boldsymbol{\sigma}_2) = \sum_{j,\,m} \frac{2\pi}{ik} \sum_{l,\,s;\,l',\,s'} Y^{jm}_{ls}(\mathbf{k})\, Y^{jm+}_{l's'}(\mathbf{k}_0)\, R^{(1)\,j}_{ls,\,l's'}, \tag{6}$$

where

$$R^{(1)j} = L^{(1)j} R^{j} L^{(1)j+} \tag{7}$$

The matrix $L^{(1)j}$ is a unitary, antisymmetric, Hermitian matrix. Therefore $R^{(1)j}$ satisfies Eq. (4) and has the same symmetry properties as R^j. Therefore the elements of $R^{(1)j}$ may be considered as the elements of a new scattering matrix which leads to the same cross section as M. All this applies as well to the matrices $R^{(2)j} = L^{(2)j} \times R^j L^{(2)j+}$ and $R^{(3)j} = L^{(3)j} R^j L^{(3)j+}$, corresponding to the second and third transformation of (2).

The matrix $L^{(i)j}$ has the form

$$L^{(1,2)j} = \frac{i}{\sqrt{2j+1}} \begin{pmatrix} 0 & 0 & \mp\sqrt{j+1} & \mp\sqrt{j} \\ 0 & 0 & \sqrt{j} & -\sqrt{j+1} \\ \pm\sqrt{j+1} & -\sqrt{j} & 0 & 0 \\ \pm\sqrt{j} & \sqrt{j+1} & 0 & 0 \end{pmatrix},$$
$$L^{(3)j} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2j+1} & -\frac{2\sqrt{j(j+1)}}{2j+1} \\ 0 & 0 & -\frac{2\sqrt{j(j+1)}}{2j+1} & \frac{1}{2j+1} \end{pmatrix}. \tag{8}$$

The first columns (row) correspond to singlet states, the remaining to triplet states in the following ordering of l: $j,\ j+1,\ j-1$. The upper sign corresponds to superscript 1, and the lower sign to superscript 2.

Inasmuch as the operators $(\boldsymbol{\sigma}_{1,2}\mathbf{k})$ in contradistinction to $(\boldsymbol{\sigma}_1\mathbf{k})(\boldsymbol{\sigma}_2\mathbf{k})$ do not commute with the square of the total spin operator $\frac{1}{4}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)^2$, the matrices M_1 and M_2 lead to singlet-triplet transitions. Therefore the first two transformations cannot take place in the case of identical nucleon collisions where singlet-triplet transitions are forbidden by the Pauli principle. This also is true for n — p scattering if isotopic invariance holds.

The operators $(\boldsymbol{\sigma}_i\mathbf{q})$ represent operators which rotate the spin of the i-th nucleon by an angle π about the direction $\mathbf{q}$. This allows to determine the transformation properties of various spin characteristics when R^j is replaced by $R^{(3)j}$. For example, this exchange leads to a change in the sign of the polarization P which takes place in the collision of unpolarized nucleons.

We finally remark that changing the sign of all the phase shifts (taking the complex conjugate of R^j) leaves the cross section unchanged, and changes the sign of $\mathbf{P}_0$. Thus a simultaneous application of this transformation with the transformation R^j into $R^{(3)j}$ leaves unchanged the cross section as well as the polarization. Therefore the two sets of elements of R obtained from one another by means of the indicated transformation, cannot be distinguished through the simplest polarization experiments (double scattering).

[1] S. Minami, Progr. Theor. Phys. **11**, 213 (1954).
[2] Hayakwa, Kawaguchi, and Minami, Progr. Theor. Phys. **12**, 355 (1954).
[3] R. Ryndin and Ia. Smorodinskii, Dokl. Akad. Nauk SSSR **103**, 69 (1955).

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RESONANCE ABSORPTION OF ELECTROMAGNETIC WAVES BY AN INHOMOGENEOUS PLASMA

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THE phenomenological description of the propagation of electromagnetic waves in a plasma is based on the possibility of introducing an index of refraction for the medium. A magneto-active plasma is usually characterized by two indices of refraction. It is well known that at certain values of the electron concentration one of these indices becomes infinite (neglecting the collisions of electrons with heavy particles). This may be called a resonance effect since the singularity in the index of refraction is related to the resonance properties of the plasma.[1,2]

In the resonance region, an electromagnetic wave incident on an inhomogeneous layer is partially or totally absorbed. The first of these effects has been discussed by Ginzburg (cf. Ref. 1, §79, and Ref. 2) for the case of quasi-longitudinal propagation. A calculation of absorption in the region of the singularity in the index of refraction has been carried out by Budden[3] using a simplified model of an inhomogeneous layer. The complete solution of the problem can be obtained in the case in which the plasma is not highly inhomogeneous. The results of an analysis of this kind are given below.

In a weakly inhomogeneous medium, except for one case which is discussed below, the interaction between the ordinary and extraordinary waves can be neglected. For simplicity, we consider transverse propagation although the final results can be generalized quite easily. In transverse propagation the index of refraction for the extraordinary wave has a singularity, the dependence of which on electron concentration is given by the following:

$$n^2(v) = 1 - \frac{v(1-v)}{1-u-v} \qquad \left(v(z) = \frac{4\pi e^2 N(z)}{m\omega^2}; \quad u = \frac{\omega_H^2}{\omega^2}\right) \tag{1}$$

(the wave propagates along the z axis, and the electron concentration N depends on z). The function $n^2(v)$ has two zeros, $v_1(z_1) = 1 - \sqrt{u}$ and $v_2(z_2) = 1 + \sqrt{u}$, and a pole at $v_3(z_3) = 1 - u$. We consider the case $u < 1$, in which the resonance region ($v = v_3$) lies between the zeros of the function $n^2(v)$. The solution for the reflection of waves from such a layer by the "standard-equation" method[4] shows[5] that the reflection coefficient for the region $(v_1 v_2)$ is

$$|R|^2 = 1 - 4e^{-\delta}(1 - e^{-\delta})\sin^2 s, \tag{2}$$

where δ and s are defined by the expressions

$$\delta = 2ik_0 \int_{z_1}^{z_3} \sqrt{n}\, dz; \qquad s = k_0 \int_{z_3}^{z_2} \sqrt{n}\, dz; \qquad \left(k_0 = \frac{\omega}{c}\right). \tag{3}$$

Equation (2) indicates that the maximum value of the absorption coefficient $(1 - |R|^2)$ is approximately 35 per cent.

In calculating absorption in the resonance region it is necessary to take account of the interaction between the different waves only in the case of quasi-longitudinal propagation. In this case, in the region $v \sim 1$ (in the vicinity of which the resonance is found) the index of refraction for the ordinary wave $n_1(\epsilon)$ and for the extraordinary wave $n_2(\epsilon)$ $(\epsilon = 1 - v)$ assume values which are approximately the same and two waves exhibit a strong interaction effect.[2]

If an ordinary wave is incident on the interac-