SOVIET PHYSICS JETP VOLUME 34 (7), NUMBER 3 SEPTEMBER, 1958

INVESTIGATION OF A SHOWER PRODUCED BY A SINGLY-CHARGED PARTICLE OF HIGH ENERGY

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Submitted to JETP editor September 10, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 622-631 (March, 1958)

A (2 + 16p) shower produced by a singly-charged particle of $(5^{+10}_{-3}) \times 10^{12}$ ev energy is analyzed. The energy spectrum of the secondary particles differs from that predicted by Landau's theory, but is consistent with Heisenberg's theory. The inelasticity coefficient is significantly smaller than unity. The fraction of energy consumed in meson production is on the order of 10 — 15%. The ratio of neutral pions to charged shower particles is R = 0.54 ± 0.18, which indicates that a small fraction of the charged shower particles consists of heavy mesons and nucleons.

A (2 + 16p) event was observed in an Ilford G-5 emulsion 600μ thick, exposed in Italy in 1955 at an approximate altitude of 30 km. The energy of the primary particle, estimated by the usual kinematic method, is $(5^{+10}_{-3}) \times 10^{12}$ ev. The shower particles are contained within an angle of 1.7×10^{-1} radians. The central tracks cover a distance up to 5 cm within a single plate. This made it possible to determine the energy of 15 shower particles by direct measurement of the multiple Coulomb scattering.

1. MEASUREMENT OF MOMENTA OF SECONDARY PARTICLES

Figure 1 shows a microdiagram of the analyzed shower. The momenta of the secondary particles were determined by measuring their multiple Coulomb scattering. Measurements were made with a MBI-8M microscope of total magnification 60 × 2.5 × 15. The noise level of the stage with standard glass guide did not exceed 0.06μ per 1000-μ cell.

The geometry of the arrangement of the tracks in the shower did not permit the use of the relative-scattering measuring method. The scattering was therefore measured for each shower track separately. The directly-measured scattering $\overline{D}$ (the second difference of the coordinates of the tracks, determined over a cell length t) consists of true Coulomb scattering, D_c, and of scattering due to all other factors, n. Assuming that these quantities are independent and that they have a Gaussian distribution, we can write

$$\overline{D}_1^2 = \overline{D}_c^2 + n^2, \quad (1)$$

$$\overline{D}_c = kt^{3/2} / p\beta, \quad (2)$$

where k is a constant, p is the momentum, and β the particle velocity in terms of the velocity of light. The quantity n depends on the false scattering, due to microdistortions of the emulsion, nonlinear motion of the microscope stage, asymmetry of the track grains, inaccuracy of reading, fluctuations in microscope temperature (thermal noise), and variation of the microscope focus.

In the case of high-energy particles, n depends essentially on the microdistortions of the emulsion, which vary with a certain power of the cell length. We can therefore put $n \sim t^x$ without introducing an excessive error (Ref. 1).

The value of n can be determined by measuring the scattering of a high-energy particle by three cells along its track. For cells t_1, t_2, and t_3 with a ratio 1 : 2 : 3, Fowler et al.[1] obtained the value of x and derived a formula for n_1 on the basic cell t_1:

$$n_1 = [(27\overline{D}_1^2 - \overline{D}_3^2) / (27 - 3^{2x})]^{1/2}. \quad (3)$$

The value of x is considered constant for a given emulsion. However, the value of x for individual tracks within the same emulsion may differ considerably from the average value. In the measurement of the scattering, when $\overline{D}_c$ is not much greater than n, the elimination of each value of n is of major importance for the determination of the shower-particle energy. The method described below makes it possible to eliminate n for each track. Equation (1) for cells t_1, t_2, and t_4, with a ratio 1 : 2 : 4, can be written

$$\begin{aligned} \overline{D}_1^2 &= \overline{D}_{1c}^2 + n_1^2, \\ \overline{D}_2^2 &= \overline{D}_{2c}^2 + n_2^2 = 8\overline{D}_{1c}^2 + 2^{2x} n_1^2, \\ \overline{D}_4^2 &= \overline{D}_{4c}^2 + n_4^2 = 64\overline{D}_{1c}^2 + (2^{2x})^2 n_1^2. \end{aligned} \quad (4)$$

Such a choice of cells makes it possible to elim-

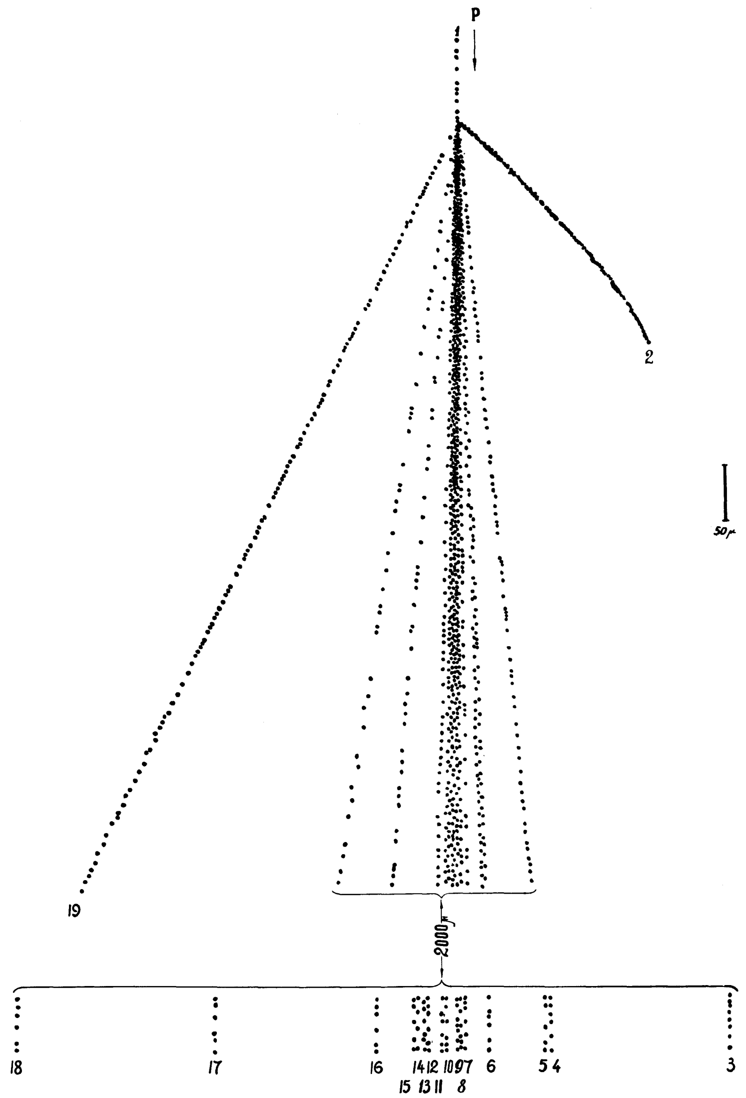

Fig. 1. Microdiagram of the shower, which is formed by a singly-charged particle (probably a proton) of energy $(5^{+10}_{-3}) \times 10^{12}$ ev.

TABLE I

No. tracks	λ	n	10^{-2} t, μ
12	2.2	0.25	40
15	5.6	—	10
13	5	0,13	40
11	1.73	0.38	40
10	3.6	0.07	40
8	1.75	0,46	40
9	4.5	0.18	40
7	1	0.59	40
16	5.5	—	10
6	1.9	0,34	20
5	3.2	0.19	20
14	2.8		10
4	2.7		5
17	2		5
3	1.5		5
18	1.6		5

TABLE II

Number of tracks	$p\beta c$, Bev	θ_i	E_i from Landau's theory	$p_\perp/m_\pi c$	θ_i'	E_i', Bev
12	27±9	3′±2′	2605.1	0.16	5°36′	0.28
15	1.1±0.1	21′±3′	452.9	0.05	179°03′	0.47
13	15±4	22′±5′	434.4	0.66	46°05′	0.19
11	24±8	22′±7′	348.2	1.10	39°02′	0.28
10	14±5	28′±6′	268.3	0.81	56°47′	0.19
8	19±6	37′±1′	249.1	1.48	63°30′	0.27
9	8.3±2.3	40′±6′	217.5	0.68	83°53′	0.16
7	>25	46′±3′	154.9	—	—	—
16	1.5±0.2	1°05′±1′	124.2	0.20	175°14′	0.37
6	8.3±2.2	1°21′±4′	80.5	1.39	117°13′	0.26
5	6.0±1.5	2°03′±2′	61.3	1.84	136°22′	0.34
14	3.6±0.9	2°39′±2′	37.6	1.17	151°27′	0.37
4	1.0±0.3	4°08′±6′	29.8	0.51	173°35′	0.65
17	2.2±0.6	5°04′±2′	19.4	1.37	166°14′	0.80
3	3.3±1.1	7°16′±6′	13.8	2.94	165°35′	1.53
18	1.6±0.9	9°40′±3′	8.3	1.90	169°33′	1.50

inate x from the system of equations (4). The solution yields

$$n_1 = \frac{8\bar{D}_1^2 - D_2^2}{\sqrt{64\bar{D}_1^2 + \bar{D}_4^2 - 16\bar{D}_2^2}} \tag{5}$$

or

$$\bar{D}_{1c} = \left[\frac{\bar{D}_1^2\bar{D}_4^2 - \bar{D}_2^4}{64\bar{D}_1^2 + \bar{D}_4^2 - 16\bar{D}_2^2}\right]^{1/2}. \tag{6}$$

In our measurements, the values of n were calculated from (5). They are listed in Column 3 of Table I for the cells indicated in Column 4. For tracks where formulas (5) cannot be employed because of the small number of measurements, the value of n is assumed equal to the value obtained at the nearest track in which the above method is still applicable. In such cases no value of n is given in Column 3 of Table I. Let us note that the procedure developed here is not applicable in the case when $n \ll D_c$ or when the forward scattering varies with the length of the cell in the same ratio as the Coulomb scattering ($x = 3/2$).

The reliability of the data obtained depends on the ratio $\bar{D}/n = \lambda$ and, naturally, on the number N of independent measurements of the second differences of the coordinates of the track, when the chosen cells do not overlap. The values of λ for the cells indicated in Column 4 of Table I are given in Column 2 of the same table.

When $\lambda > 4$, the effect of the false scattering on the error in the determination of $\bar{D}$ can be neglected. For high-energy particles, owing to the insufficient length of the track, it is frequently necessary to confine oneself to $\lambda < 4$. However, when $\lambda \approx 1$, the measurement of the scattering becomes meaningless. This means physically that the observed scattering is of the same order as the false scattering. Consequently, the error in the determination of the momentum of the particles will not be less than 100% in such cases. For particle No. 7, $\lambda = 1$, and consequently this means that its energy cannot be determined. The value observed for this particle on a cell with t = 4000 μ is $\bar{D} = 0.59 \pm 0.17\mu$. According to formula (2), this scattering corresponds to an energy of 25 Bev. Thus, the energy of this particle can be equal to or greater than 25 Bev.

The values of the energies of the other shower particles are given in Column 2 of Table II. The relative error in the quantity $p\beta$ is

$$\Delta(p\beta)/p\beta = 0.75N^{-1/2}(1 - \lambda^{-2})^{-1/2}. \tag{7}$$

Measurements of the shower tracks were made in cells ranging from 500 to 4,000μ. The basic cells were taken to have t = 500μ and t = 1,000μ. For t = 2,000μ and t = 4,000μ, overlapping of the cells was employed.

2. ANGULAR AND ENERGY DISTRIBUTION OF SHOWER PARTICLES

We used the coordinate method to determine the angles θ_i of the shower particles with respect to the shower axis.

For small angles ($\tan\theta_i \approx \sin\theta_i \approx \theta_i$), the following formula holds

$$\theta_i = \sqrt{\lambda_i^2 + (\varphi_i - \varphi_0)^2}, \tag{8}$$

where φ_0 is the angle of dip of the shower relative to the plane of the emulsion, φ_i is the angle of the dip of the i-th shower particles, and λ_i the angle of the i-th particle relative to the shower axis, measured in the plane of the emulsion. Table II (Column 3) gives the angles θ_i of the shower particles.

Assuming a nucleon-nucleon collision, the energy of a primary particle is estimated from the median and geometric mean angles as well as by the method described in Ref. 2. The values of the energy E_0, estimated in this manner, are in good agreement with each other. Taking into account the possible fluctuation deviation from symmetry (forward and backward) in the distribution of the particles, in the center-of-mass system, we can indicate the following tolerances in the measurement of the energy

$$E_0 = (5^{+10}_{-3}) \cdot 10^{12} \text{ ev}.$$

Comparison with the Landau and Heisenberg theories was made with respect to the angular and energy distributions of the shower particles.

Figure 2 shows a histogram of the angular distributions of the shower particles in the laboratory system. The ordinates represent the relative differential density of the shower particles

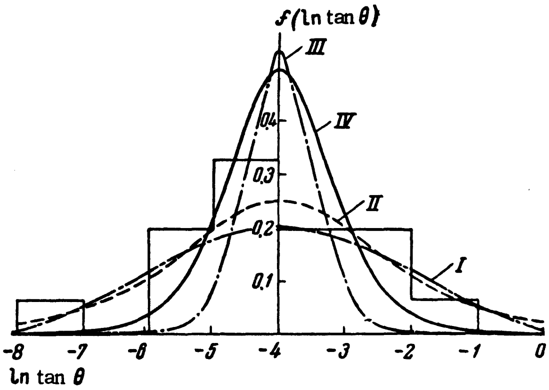

Fig. 2. Histogram of the differential angular distribution of shower particles.

$$f(\ln\tan\theta) = \frac{1}{n_s}\frac{\Delta n_s}{\Delta \ln\tan\theta},$$

as a function of $\ln \tan\theta$, where n_s is the number of charged shower particles and θ is the angle relative to the shower axis. The same diagram shows the corresponding angular-distribution functions of the Landau theory[3] (Curve I)

$$f_{\text{I}}(\ln\tan\theta) = \frac{e^{-L}}{\sqrt{2\pi L}} \exp\{\sqrt{-(2L\ln\tan\theta + \ln^2\tan\theta)}\}, \quad (9)$$

and of the Heisenberg theory,[4] assuming an anisotropic (Curve II) and isotropic angular distribution of the mesons, in the c.m.s. (Curve III):

$$f_{\text{II}}(\ln\tan\theta) = e^{L+\ln\tan\theta} / (1 + e^{L+\ln\tan\theta})^2, \quad (10)$$

$$f_{\text{III}}(\ln\tan\theta) = 3e^{2(L+\ln\tan\theta)} / [1 + e^{2(L+\ln\tan\theta)}]^{5/2}. \quad (11)$$

Formula (10) is valid for large energies of colliding nucleons. The function

$$f_{\text{IV}}(\ln\tan\theta) = 2e^{2(L+\ln\tan\theta)} / [1 + e^{2(L+\ln\tan\theta)}]^2 \quad (12)$$

pertains to the case of a monoenergetic and isotropic distribution of the mesons in the c.m.s. (Curve IV), when the c.m.s. velocity equals the meson velocity in this system. The parameter of the angular distribution is

$$L = -\frac{1}{n_s}\sum_{i=1}^{n_s} \ln\tan\theta_i.$$

With the aid of the method of χ^2 tests,[5] we determined the probability of the symmetry of this shower to be $P(\chi^2) = 75\%$, and compared the theoretical curves I, II, III, and IV with the experimental angular distribution. The probabilities of the histogram of the experimental distribution coinciding with the above theoretical distribution are estimated at 40% (I), 60% (II), 1% (III), and 1% (IV), respectively. The angular distribution of the shower particles is in best agreement with the Heisenberg-theory distribution. However, this

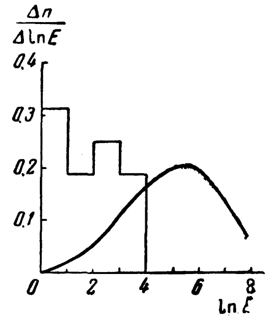

Fig. 3. Histogram of the energy distribution of shower particles in the laboratory system of coordinates. Solid line – curve obtained from the Landau theory.

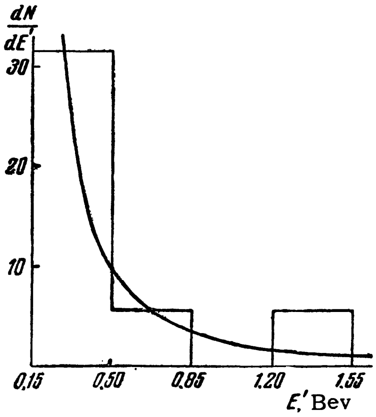

Fig. 4. Histogram of energy distribution of 15 shower particles in the c.m.s. of the colliding nucleons. Curve – energy spectrum according to the Heisenberg theory.

calculation by the χ^2-test method is only tentative, by virtue of the small number of shower particles.

To compare the energy distribution of the shower particles with the Landau theory, we plotted a histogram of the energy distribution of the shower particles in the laboratory system of coordinates, showing the distribution according to Landau (see Fig. 3). The abscissa represents the logarithm of the energy of the shower particles in units of the nucleon rest mass, and the ordinate the relative particle energy-distribution density. The intervals of the histogram ($\Delta \ln E = 1$) are much greater than the errors in the energy distribution, and therefore an account of these errors will affect very little the form of the energy distribution of the shower particles.

It follows from this comparison that the energy distribution of the particles does not correspond to the Landau theory, in that the low-energy shower particles exceed the value expected from the theory. Column 4 of Table II gives the values of the energy, calculated for the corresponding angles in accordance with Landau's theory. The comparison shows that the measured particle energy is one order of magnitude less than the theoretical values.

To compare the energy spectrum with the Heisenberg theory, we recalculated the data to the c.m.s., using a value $\gamma_c = 52$. The values of the angles θ'_i and of the energy E'_i in the c.m.s., under the assumption that all the shower particles are pions, are given in Columns 6 and 7 of Table II. Figure 4 shows the energy distribution in the c.m.s. of 15 shower particles whose energy was measured. The curve corresponds to the energy spectrum of the Heisenberg theory. In this case no discrepancy is observed between the theory and the experimental data.

The inelasticity coefficient estimated from the total energy of the charged shower particles and of the neutral mesons (with the exception of particle No. 7), taking all the possible fluctuations of the primary-particle energy into account, is $0.10^{+0.06}_{-0.02}$ in the c.m.s. Assuming particle No. 7 to be secondary, its energy can be estimated by assuming that the mean transverse momenta in the internal and external cones are equal. The value of the average transverse momentum $\bar{p}_{\perp}$ of shower particles contained within the median angle is $0.9\mu c$ (without particle No. 7); for the external half of the shower particles, this value is $1.4\mu c$ (μ is the pion mass). Such an estimate gives a value of $\sim$ 68 Bev for the energy of particle No. 7. In this case $K \sim 0.15$.

Let us note that if we assume the distribution of the generated mesons to be isotropic in the c.m.s., we can estimate the inelasticity coefficient from the angle of the cone of the shower particles.[6] This value is $K = 0.07$. Consequently, the coefficient of inelasticity ranges from 0.010 to 0.15.

Table III presents, for comparison with the experimental data, the total number of particles N, the average energy of the mesons in the c.m.s., and the inelasticity coefficients as given by the Landau and Heisenberg theories. The average energy of the shower particles in the c.m.s. is determined by using 15 particles. In the Landau theory, the fraction of the energy transferred by the mesons is close to unity, since the nucleons that participate in the collision are not separated in any manner after the collision, and have an energy close to the average value. The calculation was made for an energy of 5,000 Bev. It follows from the table that the experimental values are in better agreement with the Heisenberg theory.

TABLE III

	Total number of particles	Average energy of pions in c.m.s. (Bev)	Coefficients of inelasticity
Experiment	24	0.5	0.10—0.15
From Heisenberg's theory	18	0.6	0.1
From Landau's theory	14—15	6.7	~1

Inasmuch as the energy spectrum in the c.m.s. can be approximated by a power law (see Fig. 4), it is possible to estimate the energy of the primary particle from the relation $\gamma_c = 0.5/\theta_{1/2}$, which yields $E_0 = (1803^{+682}_{-574})$ Bev. This problem is discussed in detail by Takibaev.[7]

If one assumes[8] that the observed shower particles result from a collision between the incident nucleon and a nuclear "tunnel" of varying length, then the number of the nucleons in the tunnel is determined by the values of the median angle $\theta_{1/2}$ and the number of shower particles n_s. An analysis of this shower leads to the conclusion that the number of nucleons in the tunnel should not exceed 2. This is probable also because the excitation energy of the residual nucleus is very small (the total energy of the gray and black tracks is 220 Mev). The investigated shower can therefore be considered as produced in a nucleon-nucleon collision. If particle No. 7 is one of the colliding nucleons, the principal fraction of the energy, in the laboratory system, should be carried away by the neutron with which the collision took place.

3. SOFT COMPONENT ACCOMPANYING THE SHOWER

To study the soft component accompanying the given shower, we scanned the volume of the emulsion within a cone making a half-angle 0.15 radians with the shower axis. The scanning was performed with a total magnification $60 \times 1.5 \times 20$. In the volume scanned we found 10 electron-positron pairs (for nine of these, the distance from the center of the star is indicated in Table IV) and one trident, the data for which are indicated in Table V.

To separate the bremsstrahlung electron-positron pairs from the pairs produced by γ-quanta from the decay of the π^0 meson, we employed the

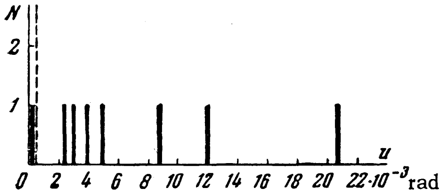

Fig. 5. Number N of electron-positron pairs as a function of the angular distance u to the nearest electron track.

criterion proposed in Ref. 9. We plotted (Fig. 5) along the ordinate the number of pairs as a function of the angular distance u to the nearest electron track of the previously-formed electron-positron pair. The pairs for which $u < 3 \times 10^{-4}$ radians were attributed to bremsstrahlung, and those with $u > 5 \times 10^{-4}$ radians to pairs directly connected with the decay of the π^0 mesons. According to this criterion, pair No. 10, produced $32{,}800\mu$ from the center of the shower, is a bremsstrahlung pair. The remaining nine pairs are produced

TABLE IV

Distance of pair from center of star, r, μ	Pair energy, determined from angle of divergence without accounting for Coulomb scattering, E_1, Bev	Energy of each electron of the pair, determined from the Coulomb scattering, ε_1 and ε_2, Bev	Pair energy, determined from Coulomb scattering, $E_2 = \varepsilon_1 + \varepsilon_2$, Bev	Pair energy, determined from the angle of divergence with allowance for the Coulomb scattering, E_3, Bev
775	$7.7^{+2.5}_{-2.6}$	2.6 ± 0.6 3.8 ± 0.7	6.4 ± 1.3	5^{+11}_{-1}
1155	$22.5^{+20.5}_{-7.3}$	8.8 ± 4.4 6.0 ± 1.5	14.8 ± 5.9	16^{+5}_{-2}
4290	$27.5^{+49.3}_{-10.4}$	5.6 ± 2.1 11.7 ± 5.8	17.3 ± 7.9	35^{+10}_{-9}
8820	$8.7^{+1}_{-2.1}$	1.8 ± 0.4 3.5 ± 0.7	5.3 ± 1.0	8^{+8}_{-1}
9875	14.3^{+4}_{-3}	1.7 ± 0.3 1.0 ± 0.2	2.6 ± 0.5	
15700	$15.6^{+9.4}_{-4.9}$	3.0 ± 0.7 14.8 ± 5.7	17.8 ± 6.4	30^{+50}_{-18}
19550	$0.8^{+0.1}_{-0.3}$	0.20 ± 0.05 0.43 ± 0.10	0.63 ± 0.15	
19650	$1.4^{+0.1}_{-0.2}$	0.27 ± 0.04 0.54 ± 0.10	0.81 ± 0.14	
20800	$8.5^{+1.0}_{-2.0}$	0.78 ± 0.10 1.40 ± 0.20	2.1 ± 0.3	4^{+8}_{-1}

TABLE V

	Primary electron	First electron	Second electron	Third electron
θ_i, rad	0	$(3.5\pm1.3)\cdot10^{-3}$	$(3.8\pm1.3)\cdot10^{-3}$	$(4.3\pm1.3)\cdot10^{-3}$
α_i, rad	—	$2.4\cdot10^{-3}$	$1.2\cdot10^{-3}$	$1.2\cdot10^{-3}$
E_i, Bev	6.4 ± 1.2	0.5 ± 0.1	1.3 ± 0.3	1.4 ± 0.3

by γ-quanta from the π^0-meson decay.

Knowing the number of primary electron-positron pairs accompanying this shower, and using the law of radioactive decay, it is possible to calculate the expected number of π^0 mesons.[9,10] This number is determined from formulas

$$N_{\pi^0} = N_1 f(r)/2, \tag{13}$$

$$f(r) = \frac{1}{(1 - e^{-r/\rho}) - \frac{\lambda}{\lambda - \rho}(e^{-r/\lambda} - e^{-r/\rho})}, \tag{14}$$

where N_1 is the number of γ quanta that decay at a distance r, λ is the average conversion length of the γ quanta in the photoemulsion (for the Ilford G-5 emulsion, $\lambda = 37.5$ mm), ρ is the average range of the π^0 mesons prior to decay in the photoemulsion. The average range of the π^0 mesons depends on the energy, but ρ is much less than λ and consequently the error in the determination of the average energy of the π^0 mesons is almost insignificant in the calculation of the function f(r). Starting with a resultant average energy $E^0_{\pi^0} = 15$ Bev and a lifetime of 2×10^{-15} seconds for the π^0 meson, we find $\rho = 65\mu$. Using formula (13) for three values of r, we found the ratio of the number of the neutral π^0 mesons to the number of charged shower particles, $R = n_{\pi_0}/n_s$. For distances r of 8, 16, and 24 mm from the start of the shower, R is 0.49, 0.45, and 0.59 respectively. The mean value is $R = 0.54 \pm 0.18$; this corresponds to eight π^0 mesons.

Several methods for determining the energy of the electron-positron pairs are described in the literature. We estimated the energy of the pairs by three methods:

1. From the angle between the electron and the positron of the pair,[11] disregarding the effect of the Coulomb scattering. These data are given in Column 2 of Table IV.

2. From a direct measurement of the multiple Coulomb scattering of the electron and the positron. These measurements were made by the procedure described in Sec. 1 of this article. The energy of each electron and positron of the pair, and the energy of the entire pair, are listed in Columns 5 and 6 of Table IV.

3. From the angle between the components of the pair, taking into account the Coulomb scattering.

According to the work by Lorman,[12] the determination of the energy directly from the divergence angle can be used for an estimate of the photon energy not greater than 0.5 Bev. In this case the divergence angle of the electron and the positron can be measured at distances $\leq 100\mu$ from the point of pair production. At such a distance, the Coulomb scattering can be disregarded. For high energies, the divergence angle can be measured reliably at substantially greater distances from the point of pair production, and the contribution of the Coulomb scattering cannot be neglected. Unlike in Ref. 12, in this work we assumed equal distribution of the energy between the electron and the positron, and employed for the rms divergence angle θ a formula which takes into account, along with the Coulomb scattering, also the initial divergence of the components of the pair:

$$\bar{\theta}^2 = 4\frac{m_e^2c^4}{E_\gamma^2}\left(\ln\frac{E_\gamma}{m_ec^2}\right)^2 + 4\pi\frac{k^2t}{E_\gamma^2}. \qquad (15)$$

Here m_e is the mass of the electron, E_γ the energy of the γ-quantum producing the pair, k the constant of the Coulomb scattering, and t the distance from the point of the pair production.

The first term of the right half of Eq. (15) is the r.m.s. divergence angle of the pair, calculated in accordance with Stearns.[13] The second term is the r.m.s. divergence angle of the pair due to the relative Coulomb scattering of the pair components. Using (15), we plotted a family of curves for different energies (Fig. 6). The ordinates represent the r.m.s. divergence angle, as a function of the distance from the point of pair production, in units of 100μ. As can be seen from Fig. 6, most pairs are characterized by an undervalued angle at distances close to the point of production. This is probably due to the fact that at these distances, by virtue of the shrinkage of the emulsion during development, it is impossible to take into account the divergence of the pairs in depth. This method was used by us for pairs with energy ≥ 3 Bev. The measurement results are given in Column 6 of Table IV.

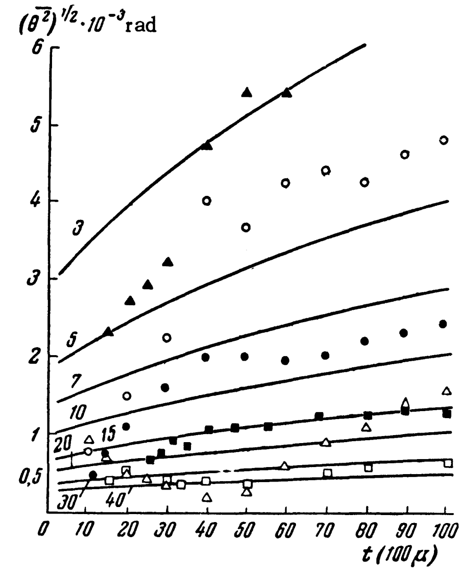

Fig. 6. Rms angle between the electron and positron of a pair as a function of the distance t to the point of pair production. ○ – pair No. 1, ■ – pair No. 2, □ – pair No. 3, ● – pair No. 4, △ – pair No. 6, ▲ – pair No. 9. Numbers on the curves represent Bev.

From the energy distribution of the γ-quanta it is possible to determine the average energy of the π^0 mesons, $\bar{E}_{\pi^0} \approx 2\bar{E}_\gamma$. Calculation of the average energy of the π^0 mesons from Columns 3 and 5 of Table IV yields accordingly

$$\bar{E}_{\pi^0} = 24^{+18}_{-6} \text{ Bev and } \bar{E}_{\pi^0} = 15 \pm 3 \text{ Bev}.$$

The average energy of the charged shower particles is 9 ± 3 Bev. Assuming that all these particles are pions (and this is confirmed by the fact that $R \approx 0.5$), a comparison with the average energy of the π^0 mesons indicates that the estimate of the energy of the γ-quanta by method 1 gives a poorer agreement than that of method 2.

Obviously, the most reliable method for determining the energy of the pairs is the measurement of the mutliple Coulomb scattering of the electron and positron. Unfortunately, this method cannot always be applied. When the energy of the pair is small (< 1 Bev), then, as a consequence of the considerable divergence angle of the electron and the positron, the length of the track is, as a rule, too short for the application of method 2. On the other hand, in the most cases, at electron energies > 30 Bev, measurement of the Coulomb scattering is impossible. Exceptions are pairs with tracks ≥ 4 cm long within the limits of the emulsion.

It is advisable to employ method 1 at energies < 1 Bev and method 3 at energies > 30 Bev.

A trident was formed 19,355μ from the start of pair No. 1. True tridents satisfy the following criterion[10]

$$\theta_i > \bar{\alpha}_i \quad i = 1, 2, 3,$$

where θ_i is the angle that the i-th electron track of the trident makes with the continuation of the primary track; $\bar{\alpha}_i$ is the average angle of multiple scattering (including the Coulomb and all other types of scattering) of the i-th electron. It follows from Table V that this criterion is satisfied here.

On the other hand, according to the plot obtained by Kaplan and Koshiba,[10] the probability that this is a bremsstrahlung pair, less than 0.2μ away from the primary electron track, is nearly 8%. These circumstances make it possible to assert that the trident is a true one. The total length of all electron tracks in the reviewed band is 25 cm. Consequently, the average length of formation of the trident at an energy of 6 Bev is on the order of 25 cm, which is in agreement with the Bhabha theory.[14]

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Translated by J. G. Adashko
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SOVIET PHYSICS JETP VOLUME 34 (7), NUMBER 3 SEPTEMBER, 1958

GENERAL COVARIANT EQUATIONS FOR FIELDS OF ARBITRARY SPIN

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Submitted to JETP editor July 12, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 632-636 (March, 1958)

The Gel'fand-Iaglom field equations are extended to the general theory of relativity.

To obtain a generalized wave equation for a field in general covariant form, one must define covariant differentiation of a generalized wave function describing particles with arbitrary spin. Gel'fand and Iaglom,[1] Dirac,[2] and Fierz and Pauli[3] have studied the generalized wave equation in the special theory of relativity. In the present article, their theory is extended to the general covariant form.

1. SEMIMETRICS AND SEMIMETRIC REPRESENTATION

We introduce the metric g_{ik} in space-time with the aid of the asymmetric matrix $\| \lambda_{i(\alpha)} \|$