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DEPENDENCE OF THE ALPHA-DECAY RATE ON THE ENERGY OF THE ROTATIONAL LEVELS

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It is possible to separate the dependence of the probability of the α-decay on the energy and on the angular momentum carried off by the α-particle by investigating the α-decay of spin $\frac{1}{2}$ nuclei in transitions to the levels of a rotational band with $K = \frac{1}{2}$. It is shown by analyzing the case of Pu^{239} that the dependence of the α-decay on the energy for levels belonging to a particular rotational band is given by the same formula which describes the α-decay of even-even nuclei in ground state transitions.

THE energy dependence of the probability of an α-decay which leaves the daughter nucleus in its ground state is given by the Geiger-Nuttal law. It is usually given in the form

$$\log_{10}\lambda = C - D/\sqrt{E}, \tag{1}$$

where λ is the probability of the α-decay, E is the energy of the emitted α-particles, and C and D are quantities which depend only very weakly on the atomic number Z and are the same for all isotopes of a particular element. Equation (1) was first established empirically. Later it was derived from the theory of the α-decay. The constants C and D are usually determined from experimental data. They have been tabulated by several authors (Refs. 1, 2; see also Ref. 3).

Equation (1) describes particularly well the ground state α-decays of even-even nuclei. Here the spins and parities of parent and daughter nuclei are the same, and the α-decay does not entail a reshuffling of the nucleus. For example, the probabilities of the α-decay of the nuclei Pu^{236}, Pu^{238}, Pu^{240}, and Pu^{242} are 2.4, 76, 7.6×10^3 and 4.0×10^5 years respectively, if calculated with (1); the experimental values are 2.7, 89.6, 6.58×10^3, and 3.76×10^5 years respectively. We used the values $C = 53.35$ and $D = 147.4$ (E in Mev) which according to Ref. 1 should hold for $Z = 94$. Thus formula (1) gives the probabilities of the α-decay of even-even nuclei with an approximate accuracy of 10%.

In odd nuclei, the observed probabilities of α-decay usually turn out to be appreciably smaller than those given by (1). This is evidently due to the change in the nucleon configuration which takes place in general in the α-decay of odd nuclei.

It would be natural to consider the branching ratio of the α-decay to the different levels of a rotational band.

It is well known that Eq. (1) gives too high values of λ for transitions to higher rotational levels. This is to be expected since the corresponding α-particles not only have a smaller energy, but also carry off a certain angular momentum. The probability λ of the α-decay there has to be not only a function of the level energy E, but also of l. For small changes of E and l one can use a series expansion:

$$\log_{10}\lambda(E, l) = \log_{10}\lambda(E_0, 0) + A(E - E_0) + Bl(l+1) + \ldots, \tag{2}$$

It is interesting to ascertain: (i) up to which energies and angular momenta can (2) be used neglecting higher terms, and (ii) whether the energy-dependent part of (2) is equal to that of (1). It turns out that in general this question cannot be easily answered, since the dependence of λ on E cannot usually be separated from that on l. However, the case where the spin of the decaying nucleus equals $\frac{1}{2}$ can be solved at once. Here the levels of the daughter nucleus belong to the rotational band with $K = \frac{1}{2}$ (K is the projection of the angular momentum on the symmetry axis of the nucleus). It follows from kinematical considerations that a level of spin I can be reached if the α-particle carries off an angular momentum of either $I - \frac{1}{2}$ or $I + \frac{1}{2}$. Of these two possibilities, one is of necessity even and the other odd, and thus the wave function of the α-particle has to have even or odd parity. On the other hand, the parity of the wave function of the α-particle is uniquely given by the parity of the ground state of the parent nucleus and by the parity of the levels of the daughter nucleus belonging to the given rotational

α-Decay of Pu^{239} and Levels of U^{235}

Level No.	α-line	Energy of the α-particles (kev)	Energy of the levels (kev)	Intensity of the line (%)	Spin of the daughter nucleus	Angular momentum carried off by α-particle
I	α_0	5147	0	72	1/2	0
II	α_{13}	5134	13.2	16.8	3/2	2
III	α_{52}	5096	51.7	10.7	5/2	2
IV	α_{84}	5064	84	$3.7 \cdot 10^{-2}$	7/2	4
V	α_{151}	4991	151	$1.3 \cdot 10^{-2}$	9/2	4

band. Thus only one of the two possibilities, $I + \frac{1}{2}$ or $I - \frac{1}{2}$, can actually occur, namely that with the required parity. The rotational levels thus form doublets. The transitions to both members of the doublet involve the same angular momentum of the α-particle, and the branching ratio then depends only on the energy.

We consider the decay of Pu^{239}, which recently has been investigated by Novikova et al.[4] This nucleus has spin $\frac{1}{2}$. The experimental results from Ref. 4 are summarized in the table, in which all levels shown belong to the same rotational band with $K = \frac{1}{2}$.

We now shall compare Eq. (1) with the intensities of the α-decay to the levels of the doublet. The experimental ratios of the transition probabilities to the levels II and III and to the levels IV and V are

$$\lambda_{II}/\lambda_{III} = 1.57; \quad \lambda_{IV}/\lambda_V = 2.84.$$

Equation (1) yields for the same transitions

$$\lambda_{II}/\lambda_{III} = 1.75; \quad \lambda_{IV}/\lambda_V = 2.99.$$

Thus Eq. (1) gives the energy dependence of the α-decay in transitions to rotational levels with an accuracy not worse than for ground state transitions of even-even nuclei. The obtained results further show that in the expansion (2) second order terms, in particular, terms containing $[d(\log\lambda)/dE] \times \{d(\log\lambda)/d[\ell(\ell+1)]\}$ are not important for $\ell \leq 4$.

The small discrepancy between the calculated and the experimental ratios λ_i/λ_{i+1}, which is roughly the same in both cases, indicates that one should actually employ in (1) a slightly smaller value for D than given by Bohr et al.[1] Then the calculated ratios would agree with the experimental ratios within the experimental accuracy. This, incidentally, is not astonishing, since the values for C and D as proposed in Ref. 1 have been selected to yield a good fit in a slightly different case — for all isotopes of one element. The obtained accuracy of about 10% actually seems a little unexpected. One would be justified to expect, with a special choice of D, a better fit to a series of rotational levels in a particular nucleus. This conjecture actually turns out to be true.

Note added in proof (January 17, 1958). Our analysis depends essentially on the assumption that the probability of the α-decay depends explicitly only on E and ℓ. However, in principle, λ could also depend explicitly on the spin I of the daughter nucleus. In this case the above considerations are not satisfactory and there does not seem to exist any manner in which the dependence of λ on E, ℓ, and I could be determined without recourse to some kind of a model.

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