tions of Eyges and Fernbach. The functions differ by less than 10% for $x_r \sim 1-5$. The photon LDF calculated by us using the first three moments differs from the more accurate function by less than 10%.

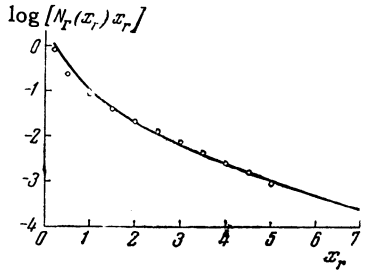

FIG. 2

The electron LDF[1] is also shown in Fig. 1 (dot-dash curve). It should be noted that, even for $x_r = 10^{-4}$ which corresponds to distances of 2×10^{-5} radiation units for particles with energy $\geq 10^8$ ev (i.e. distances < 0.5 cm in air at sea-level), the values of the photon LDF are only three times larger than those of the electron LDF, in spite of the fact that for $x_r \to 0$ the function $x_r N_\Gamma(x_r)$ diverges as $\ln x_r$, while $x_r N_p(x_r)$ remains finite.

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MOTION OF ELECTRON ALONG SELF-INTERSECTING TRAJECTORIES

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In Ref. 1 we derived an equation for the motion of an electron with arbitrary law of dispersion $E(k)$ in a magnetic field ($H = H_z$) and obtained (following I. M. Lifshitz and Kosevich[2]) with the aid of the quasi-classical approximation the following equation

$$S(E, k_3) = 2\pi\alpha_0^{-2}(n + 1/2), \quad \alpha_0^2 = hc/eH,$$

for the energy levels of an electron moving in closed trajectories (the intersections of the surface $E(k) = \text{const}$, and the plane $k_3 = \text{const}$). In the present work we shall consider a case when the trajectory has a form of a closed self-intersecting curve ("figure eight"). However, the consideration given below pertains also to the case when the "figure eight" has a narrow neck and when it breaks up into two closed regions.

Near the point of self-intersection it is impossible to employ the quasi-classical approximation. For the region near such a point (where the trajectory can be represented by two hyperbolas, which degenerate into straight lines upon exact self-intersection), it is necessary to write the exact solution. A similar problem was solved in Ref. 3. It turns out that near the point of self-intersection the exact solution is expressed in terms of degenerate hypergeometric functions, similar to the manner in which the solution near the point of the classical turn is expressed in terms of the Airy functions. The "joining" of the quasi-classical solution (away from the point of self-intersection) and the exact solution (in the vicinity of this point) gives a quantization condition in the form

$$S = 4\pi\alpha_0^{-2}(n + 1/2 + \gamma_{1,2}(\lambda)); \tag{1}$$

here S is the total area of the curve, $\gamma_{1,2}(\lambda)$ are functions, to be determined below, of the quantity

$$\lambda = (\varkappa_0/\varepsilon)^2 \sqrt{R/\varkappa_0}, \tag{2}$$

where $\kappa_0 = \kappa_{10} a_1$, κ_{10} is the value of κ_1 at the boundary (for exact self-intersection $\kappa_{10} = 0$), $\epsilon = a/\alpha_0$, R is the radius of curvature of the trajectory at the point of self-intersection ($\sqrt{\kappa_0/R}$ is the slope of the tangent at the point of self-intersection for $\kappa_{10} = 0$, or the slope of the asymptote of the hyperbola in the case of inexact self-intersection). The quantities γ_1 and γ_2 are determined in the following manner

$$\gamma_1 = -\frac{1}{8} - \varphi_1 - \frac{\lambda}{4}\ln\left|\frac{\lambda}{4e}\right|, \quad \gamma_2 = -\frac{1}{8} - \varphi_2 - \frac{\lambda}{4}\ln\left|\frac{\lambda}{4e}\right|,$$
$$R_1 e^{i\varphi_1} = 1/\Gamma\left(\frac{1}{4} + i\,\frac{\lambda}{4}\right), \quad R_2 e^{i\varphi_2} = e^{-i\pi/4}/\Gamma\left(\frac{3}{4} + i\,\frac{\lambda}{4}\right) \tag{3}$$

In order to gain an idea of the splitting of the energy levels upon gradual deformation of the "figure eight," it is enough to consider the following cases.

(1) When $\lambda < 0$ and $|\lambda| \gg 1$, corresponding to two individual regions (κ_{10} is imaginary), $\gamma_1 = \gamma_2 = 0$. Then the area of each region is determined by the usual equation $S = 2\pi\alpha_0^{-2}(n + 1/2)$, and the total area is

$$S = 4\pi\alpha_0^{-2}(n + 1/2) \tag{4}$$

(2) When $\lambda = 0$ (exact self-intersection, $\kappa_{10} = 0$), $\gamma_1 = -\gamma_3 = 1/8$. In this case the area of the entire curve is expressed by the formula

$$S = 4\pi\alpha_0^{-2}(n + 1/2 \pm 1/8). \quad (5)$$

Both levels, corresponding to a given value of n, are separated by a distance that is one quarter as small as that of the levels corresponding to neighboring n and equal γ. The levels are thus equidistant in pairs.

(3) When $\lambda \gg 1$ (wide neck, κ_{10} large), $\gamma_1 = -\gamma_2 = 1/4$. Formula (1) for the total area now becomes $S = 4\pi\alpha_0^{-2}(n + 1/2 \pm 1/4)$, which can also be written in the usual form

$$S = 2\pi\alpha_0^{-2}(n + 1/2). \quad (6)$$

The levels are again equidistant, but at distances half as small than in the case of two individual regions [formula (4)]. The levels are no longer equidistant when the curve is nearly self-intersecting and the conditions for the applicability of the quasiclassical approximation are no longer satisfied.

Let us consider the de Haas — van Alphen effect for trajectories with self intersection. The first two (most significant) terms of the oscillating portion of the number of electron states (with energy from 0 to E) will be of the form

$$\sin\left(\frac{1}{2}\alpha_0^2 S_m - \frac{\pi}{4}\right)\cos 2\pi\gamma_m - \frac{1}{2\sqrt{2}}\sin\left(\alpha_0^2 S_m - \frac{\pi}{4}\right)\cos 4\pi\gamma_m; \quad (7)$$

S_m is the extremal value of the area, and γ_m is the corresponding value of γ.

When $\gamma = 0$ (two individual regions), the oscillation takes place with a frequency corresponding to the cross-section area of the individual region. Then, as γ increases (merging of the regions and formation of one common region), the first term diminishes and the term with double the frequency starts assuming an ever increasing role. When $\gamma = 1/4$ (merging regions), the first term vanishes, i.e., the frequency of oscillations already corresponds to the total area of the curve.

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ANGULAR DISTRIBUTION IN THE REACTIONS $K^+ \to 2\pi^+ + \pi^-$ *and* $K^+ \to 2\pi^0 + \pi^+$

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BY angular distribution we shall understand the dependence of the disintegration probability on the angle ϑ between the relative momentum of the identically charged π mesons $\mathbf{k}_{12} = \mathbf{p}_1 - \mathbf{p}_2$ and the momentum of the third meson $\mathbf{p}_3$.

As is known,[1,2] neglecting the interaction of the π mesons in the final state, the matrix elements for both decays do not depend on ϑ with an accuracy up to terms $\sim k_{12}^2 p_3^2$, since the angular momenta ℓ, L (Refs. 1 and 2) can assume the values $\ell = L = 0, 2, 4, \ldots$, and the contribution of the corresponding states to the matrix elements are $\sim k_{12} p_3$. The latter is due to the fact that the particles in states with $\ell, L \neq 0$, in order to leave the region of their creation, have to overcome the centrifugal barrier, whose penetration coefficient is proportional to $k_{12}^{\ell} p_3^{\ell}$.

However, in the presence of interaction the particles can go into a state with $\ell, L \neq 0$ and give a contribution to the angular distribution without passing the centrifugal barrier. This case arises when the particles, created in a state with $\ell = L = 0$, leave the region of their creation, whereupon one of the pairs of particles 1 and 3 or 2 and 3 gets close and interacts. In such an interaction the angular momenta ℓ and L are not conserved, but the total angular momentum is conserved. This makes possible the transition from a state with $\ell, L = 0$ to a state $\ell, L \neq 0$ with the same total angular momentum. It can be shown[3] that the am-