(2) When $\lambda = 0$ (exact self-intersection, $\kappa_{10} = 0$), $\gamma_1 = -\gamma_3 = 1/8$. In this case the area of the entire curve is expressed by the formula

$$S = 4\pi\alpha_0^{-2}(n + 1/2 \pm 1/8). \quad (5)$$

Both levels, corresponding to a given value of n, are separated by a distance that is one quarter as small as that of the levels corresponding to neighboring n and equal $\gamma$. The levels are thus equidistant in pairs.

(3) When $\lambda \gg 1$ (wide neck, $\kappa_{10}$ large), $\gamma_1 = -\gamma_2 = 1/4$. Formula (1) for the total area now becomes $S = 4\pi\alpha_0^{-2}(n + 1/2 \pm 1/4)$, which can also be written in the usual form

$$S = 2\pi\alpha_0^{-2}(n + 1/2). \quad (6)$$

The levels are again equidistant, but at distances half as small than in the case of two individual regions [formula (4)]. The levels are no longer equidistant when the curve is nearly self-intersecting and the conditions for the applicability of the quasiclassical approximation are no longer satisfied.

Let us consider the de Haas — van Alphen effect for trajectories with self intersection. The first two (most significant) terms of the oscillating portion of the number of electron states (with energy from 0 to E) will be of the form

$$\sin\left(\frac{1}{2}\alpha_0^2 S_m - \frac{\pi}{4}\right)\cos 2\pi\gamma_m - \frac{1}{2\sqrt{2}}\sin\left(\alpha_0^2 S_m - \frac{\pi}{4}\right)\cos 4\pi\gamma_m; \quad (7)$$

$S_m$ is the extremal value of the area, and $\gamma_m$ is the corresponding value of $\gamma$.

When $\gamma = 0$ (two individual regions), the oscillation takes place with a frequency corresponding to the cross-section area of the individual region. Then, as $\gamma$ increases (merging of the regions and formation of one common region), the first term diminishes and the term with double the frequency starts assuming an ever increasing role. When $\gamma = 1/4$ (merging regions), the first term vanishes, i.e., the frequency of oscillations already corresponds to the total area of the curve.

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## *ANGULAR DISTRIBUTION IN THE REACTIONS $K^+ \to 2\pi^+ + \pi^-$ and $K^+ \to 2\pi^0 + \pi^+$*

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By angular distribution we shall understand the dependence of the disintegration probability on the angle $\vartheta$ between the relative momentum of the identically charged $\pi$ mesons $\mathbf{k}_{12} = \mathbf{p}_1 - \mathbf{p}_2$ and the momentum of the third meson $\mathbf{p}_3$.

As is known,[1,2] neglecting the interaction of the $\pi$ mesons in the final state, the matrix elements for both decays do not depend on $\vartheta$ with an accuracy up to terms $\sim k_{12}^2 p_3^2$, since the angular momenta $\ell, L$ (Refs. 1 and 2) can assume the values $\ell = L = 0, 2, 4, \ldots$, and the contribution of the corresponding states to the matrix elements are $\sim k_{12}p_3$. The latter is due to the fact that the particles in states with $\ell, L \neq 0$, in order to leave the region of their creation, have to overcome the centrifugal barrier, whose penetration coefficient is proportional to $k_{12}^{\ell} p_3^{\ell}$.

However, in the presence of interaction the particles can go into a state with $\ell, L \neq 0$ and give a contribution to the angular distribution without passing the centrifugal barrier. This case arises when the particles, created in a state with $\ell = L = 0$, leave the region of their creation, whereupon one of the pairs of particles 1 and 3 or 2 and 3 gets close and interacts. In such an interaction the angular momenta $\ell$ and L are not conserved, but the total angular momentum is conserved. This makes possible the transition from a state with $\ell, L = 0$ to a state $\ell, L \neq 0$ with the same total angular momentum. It can be shown[3] that the amplitude of such a transition is determined by the amplitudes of the scattering of the pairs of particles from each other and is proportional to $\sqrt{E}$ ($E$ is the relative kinetic energy of three $\pi$ mesons) for transitions into states with arbitrary, possible $\ell$ and L. For sufficiently small energy, the contribution of these processes to the angular distribution is thus more important than the direct passage through the centrifugal barrier. In this case the angular distribution is determined not by the specific character of the disintegration interaction, but by the amplitudes for the scattering of one $\pi$ meson from another.

In order to find the angular distribution in this case, it is sufficient to know the wave functions of the system of three $\pi$ mesons in the small region of radius $\mathbf{r}_0$ where the particles are created. It is shown in Ref. 3 that $\psi(2\pi^+, \pi^-)$ can in this region be written, for example, in the form

$$\psi(2\pi^+, \pi^-) = \Big\{1 - ik_{12}a_2 - \frac{i}{3}(k_{12}+k_{23})(a_2+2a_0) + J\frac{\varkappa^2}{9}(5a_2^2 + 11a_2a_0 + 2a_0^2)\Big\} f^{(-)} + \Big\{-\frac{i}{3}(k_{13}+k_{23})(a_2-a_0) + J\frac{\varkappa^2}{9}(13a_2^2 - 11a_2a_0 - 2a_0^2)\Big\} f^{(+)} + O(\varkappa^2) + O(\varkappa^3); \quad (1)$$

$f^{(-)}, f^{(+)}$ are the wave functions of the systems $(2\pi^+, \pi^-)$ and $(2\pi^0, \pi^+)$, respectively, at zero energy; $a_0, a_2$ are the amplitudes for the scattering of a $\pi$ meson from a $\pi$ meson at zero energy in states with isotopic spin 0 and 2; $J$ is a known function of $k_{12}/\kappa$ and $\vartheta$; $\kappa = \sqrt{m_\pi E/\hbar}$. An analogous formula holds for $\psi(2\pi^0, \pi^-)$. With the help of these formulae, the matrix elements for both disintegrations can be expressed through the matrix elements at zero energy $\langle f^{(\mp)}|\hat{W}|\psi_{K^+}\rangle$ and the amplitudes $a_2$ and $a_0$.

The result of raising the respective expressions to the second power depends essentially on whether or not "time-parity" is conserved in these disintegrations. If "time-parity" is conserved then the $\langle f^{(\mp)}|\hat{W}|\psi_{K^+}\rangle$ are real. In this case the angular distribution differs from a spherically symmetric one only by terms of order $\kappa^2$, inasmuch as the terms of first order in 1 are purely imaginary. Using an approximation to the expression for $J$, limiting oneself to lowest powers in $\cos\vartheta$, and integrating over the energy of the third particle, one obtains for the disintegration probabilities the expressions

$$dW^{(-)}(\vartheta) = W^{(-)}\{1 + \cos^2\vartheta\,(mE/\hbar^2)\,[0.07a_2^2 + 0.1a_2a_0 - 0.07a_0^2 + \rho\,(0.25a_2^2 - 0.32a_2a_0 + 0.07a_0^2) + 0.03\,(a_2-a_0)^2\rho^2]\}\, d\cos\vartheta,$$

$$dW^{(+)}(\vartheta) = W^+\{1 + \cos^2\vartheta\,(mE/\hbar^2)\,[0.1a_2^2 + 0.03a_2a_0 + 0.03a_0^2 + \rho^{-1}(0.12a_2^2 - 0.17a_2a_0 + 0.05a_0^2)]\}\, d\cos\vartheta;$$

$$\rho = W^-/W^+.$$

If "time-parity" is not conserved the $\langle f^{(\pm)}|\hat{W}|\psi_{K^+}\rangle$ are complex. In this case the angular and energy distribution changes already in terms of first order of $\kappa$. In this case, taking into account only terms of first order, we obtain for the absolute squares of the matrix elements the expressions

$$|\langle\psi(2\pi^+, \pi^-)|\hat{W}|\psi_{K^+}\rangle|^2 = W^-\{1 - (2\rho/3)(k_{13}+k_{23})(a_2-a_0)\sin\varphi\}, \quad |\langle\psi(2\pi^0, \pi^+)|\hat{W}|\psi_{K^+}\rangle|^2 = W^+\{1 - (4/3\rho)\,k_{12}(a_2-a_0)\sin\varphi\};$$

$\varphi$ is the relative phase of $\langle f^{(+)}|\hat{W}|\psi_{K^+}\rangle$ and $\langle f^{(-)}|\hat{W}|\psi_{K^+}\rangle$.

In all the preceding formulae, the $\pi$ mesons are taken to be nonrelativistic. In order to take into account the relativistic corrections it is sufficient to change in the final formulae, the angle $\vartheta$ to the angle $\vartheta'$ between the momentum $\mathbf{p}_3$ and the relative momentum of the identically-charged $\pi$ mesons in the system of their center of mass.


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## *USE OF MOVING HIGH-FREQUENCY POTENTIAL WELLS FOR THE ACCELERATION OF CHARGED PARTICLES*


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THE motion of charged particles (charge $e$, mass $m$, $e/m = \eta$) in high-frequency electromagnetic fields $\mathbf{E}(\mathbf{r})e^{i\omega t}$, $\mathbf{H}(\mathbf{r})e^{i\omega t}$ may be approximately represented as small oscillations $\mathbf{r}_1 = -(\eta/\omega^2)\mathbf{E}(\mathbf{r}_0)e^{i\omega t}$ relative to a comparatively slowly-varying mean position $\mathbf{r}_0(t)$. In the non-