plitude of such a transition is determined by the amplitudes of the scattering of the pairs of particles from each other and is proportional to $\sqrt{E}$ (E is the relative kinetic energy of three π mesons) for transitions into states with arbitrary, possible ℓ and L. For sufficiently small energy, the contribution of these processes to the angular distribution is thus more important than the direct passage through the centrifugal barrier. In this case the angular distribution is determined not by the specific character of the disintegration interaction, but by the amplitudes for the scattering of one π meson from another.

In order to find the angular distribution in this case, it is sufficient to know the wave functions of the system of three π mesons in the small region of radius $\mathbf{r}_0$ where the particles are created. It is shown in Ref. 3 that $\psi(2\pi^+, \pi^-)$ can in this region be written, for example, in the form

$$\psi(2\pi^+, \pi^-) = \Big\{1 - ik_{12}a_2 - \frac{i}{3}(k_{12}+k_{23})(a_2+2a_0) + J\frac{\varkappa^2}{9}(5a_2^2+11a_2a_0+2a_0^2)\Big\} f^{(-)} + \Big\{-\frac{i}{3}(k_{13}+k_{23})(a_2-a_0) + J\frac{\varkappa^2}{9}(13a_2^2-11a_2a_0-2a_0^2)\Big\} f^{(+)} + O(\varkappa^2) + O(\varkappa^3); \quad (1)$$

$f^{(-)}, f^{(+)}$ are the wave functions of the systems $(2\pi^+, \pi^-)$ and $(2\pi^0, \pi^+)$, respectively, at zero energy; a_0, a_2 are the amplitudes for the scattering of a π meson from a π meson at zero energy in states with isotopic spin 0 and 2; J is a known function of k_{12}/κ and ϑ; $\kappa = \sqrt{m_\pi E/\hbar}$. An analogous formula holds for $\psi(2\pi^0, \pi^-)$. With the help of these formulae, the matrix elements for both disintegrations can be expressed through the matrix elements at zero energy $\langle f^{(\mp)}|\hat{W}|\psi_{K^+}\rangle$ and the amplitudes a_2 and a_0.

The result of raising the respective expressions to the second power depends essentially on whether or not "time-parity" is conserved in these disintegrations. If "time-parity" is conserved then the $\langle f^{(\mp)}|\hat{W}|\psi_{K^+}\rangle$ are real. In this case the angular distribution differs from a spherically symmetric one only by terms of order κ^2, inasmuch as the terms of first order in 1 are purely imaginary. Using an approximation to the expression for J, limiting oneself to lowest powers in $\cos\vartheta$, and integrating over the energy of the third particle, one obtains for the disintegration probabilities the expressions

$$dW^{(-)}(\vartheta) = W^{(-)}\{1+\cos^2\vartheta\,(mE/\hbar^2)\,[0.07a_2^2+0.1a_2a_0 - 0.07a_0^2+\rho\,(0.25a_2^2-0.32a_2a_0+0.07a_0^2) + 0.03\,(a_2-a_0)^2\rho^2]\}\,d\cos\vartheta,$$

$$dW^{(+)}(\vartheta) = W^{+}\{1+\cos^2\vartheta\,(mE/\hbar^2)\,[0.1a_2^2+0.03a_2a_0 + 0.03a_0^2+\rho^{-1}(0.12a_2^2-0.17a_2a_0+0.05a_0^2)]\}\,d\cos\vartheta;$$

$$\rho = W^-/W^+.$$

If "time-parity" is not conserved the $\langle f^{(\pm)}|\hat{W}|\psi_{K^+}\rangle$ are complex. In this case the angular and energy distribution changes already in terms of first order of κ. In this case, taking into account only terms of first order, we obtain for the absolute squares of the matrix elements the expressions

$$|\langle\psi(2\pi^+, \pi^-)|\hat{W}|\psi_{K^+}\rangle|^2 = W^-\{1-(2\rho/3)(k_{13}+k_{23})(a_2-a_0)\sin\varphi\}, \quad |\langle\psi(2\pi^0, \pi^+)|\hat{W}|\psi_{K^+}\rangle|^2 = W^+\{1-(4/3\rho)\,k_{12}(a_2-a_0)\sin\varphi\};$$

φ is the relative phase of $\langle f^{(+)}|\hat{W}|\psi_{K^+}\rangle$ and $\langle f^{(-)}|\hat{W}|\psi_{K^+}\rangle$.

In all the preceding formulae, the π mesons are taken to be nonrelativistic. In order to take into account the relativistic corrections it is sufficient to change in the final formulae, the angle ϑ to the angle ϑ' between the momentum $\mathbf{p}_3$ and the relative momentum of the identically-charged π mesons in the system of their center of mass.

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USE OF MOVING HIGH-FREQUENCY POTENTIAL WELLS FOR THE ACCELERATION OF CHARGED PARTICLES

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THE motion of charged particles (charge e, mass m, $e/m = \eta$) in high-frequency electromagnetic fields $\mathbf{E}(\mathbf{r})e^{i\omega t}$, $\mathbf{H}(\mathbf{r})e^{i\omega t}$ may be approximately represented as small oscillations $\mathbf{r}_1 = -(\eta/\omega^2)\mathbf{E}(\mathbf{r}_0)e^{i\omega t}$ relative to a comparatively slowly-varying mean position $\mathbf{r}_0(t)$. In the nonrelativistic case, the coordinate $\mathbf{r}_0(t)$ satisfies the equation[1]

$$\ddot{\mathbf{r}}_0 = -\nabla\Phi(\mathbf{r}_0); \quad \Phi(\mathbf{r}_0) = (\eta/2\omega)^2 |\mathbf{E}|^2. \tag{1}$$

As previously shown,[1] absolute minima of the potential $\Phi(\mathbf{r}_0)$ exist in heterogeneous fields of definite configuration. Localization of particles with charge of any sign is possible in the potential wells that correspond to these minima. Upon using oscillations with various frequencies we obtain, generally speaking, a time-varying potential contour $\Phi(\mathbf{r}_0, t)$. In such a manner we may, in particular, bring about accelerated motion of the potential wells and, as a result, produce acceleration of the charged particles localized therein.[2]

We consider two cylindrical waves, progressing in opposite $(\pm z)$ directions. If their frequencies and amplitudes are identical, they form a standing wave $\mathbf{E}_0(x,y,z)e^{i\omega t}$, where $\mathbf{E}_0(x,y,z)$ is a real function. Let the structure of this field be such that the potential corresponding to it, $\Phi_0 = (\eta/2\omega)^2|\mathbf{E}_0(x,y,z)|^2$, has absolute minima (for example, a wave of type TM_{01} in a circular waveguide, of type TM_{11} in a rectangular waveguide, etc.). For a displacement of the potential wells along z it is necessary to change the phase of one wave from its opposite; in particular, their displacement with constant velocity v_0 is brought about when two waves with different frequencies $\omega_{1,2} = \omega_0 \pm \Delta\omega$ are combined. Limiting ourselves to nonrelativistic motion $(|\Delta\omega| \ll \omega_0)$ and neglecting the difference in field structure of the opposing waves, we obtain for the total field

$$\mathbf{E} = \mathbf{E}_0(x, y, z - v_0 t)\, e^{i(\omega t - z\Delta h)}, \tag{2}$$

where $2\Delta h = h(\omega_1) - h(\omega_2)$, and $h(\omega)$ is the propagation constant. The potential Φ corresponding to this field has the form $\Phi = \Phi_0(x, y, z - v_0 t)$. The displacement velocity of the potential wells

$$v_0 = 2\Delta\omega / [h(\omega_1) + h(\omega_2)] \tag{3}$$

turns out to be proportional to the difference in frequencies of the opposing waves, so that the capture and consequent acceleration of a particle is brought about by varying the frequency of the generator which excites one of the waves.

In the case of a relativistic velocity v_0, the potential wells (in the accompanying frame of reference) are somewhat deformed. However, as before, their displacement velocity is determined by the relationship (3).

Inasmuch as in the accompanying frame of reference an accelerated particle is at all times oscillating with the frequency of the external field, the effectiveness of such an accelerator is less than that of the usual linear one. In fact, the ratio of the "actual field" $(\nabla_z\Phi)/\eta$ to the corresponding field in a linear accelerator with identical value of permissible intensity E_{max} is equal to

$$\eta^{-1}|\nabla_z\Phi|/E_{\max} \approx h\eta E_{\max}/2\omega^2 = {}^1/_2\, h\,|\mathbf{r}_1|_{\max} \ll 1.$$

However, an accelerator with high-frequency potential wells does have definite advantages. First of all, the application of transverse magnetic waves (TM), which give rise to three dimensional potential wells, obviates the necessity for supplementary focusing of the particles in the transverse plane. Furthermore, since the effect of the capture and acceleration of particles does not depend on the sign of their charge, it is possible to apply this principle for the acceleration of quasi-neutral plasma bunches. Finally, the utilization of waves with any phase velocity, both greater and smaller than the velocity of light, is permissible. Consequently, instead of periodic structures, application of the customary smooth waveguides is allowed. It certainly follows by implication that in such a waveguide, as immediately apparent from (3), the equality $v_0 = c$ is unattainable.

In conclusion, we note that in the presence of a supplementary focusing magnetic field $(H_z = \text{const})$ in the accelerator, waves of the transverse electric type (TE and TEM) may be applied. In this case the expression for the potential Φ will have the form

$$\Phi = |\mathbf{E}|^2\eta^2/4(\omega^2 - \omega_H^2); \quad \omega_H = |\eta|H_z/c.$$

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