CONTRIBUTION TO THE THEORY OF SURFACE IMPEDANCE OF METALS IN ANOMALOUS SKIN EFFECT

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REFERENCES 1 and 1a contain the development of a theory of the surface impedance under conditions of anomalous skin effect, when the least characteristic quantity of the problem is the skin depth δ (small compared with the mean free path ℓ of the electron, compared with the distance vT traversed by the electrons during one period T of the high frequency field, and compared with the Larmor radius r in the presence of a constant magnetic field). The electrons in the metal were considered there as an ideal gas of Fermi quasi particles with a dispersion law $\epsilon = \epsilon(\mathbf{p})$ (ϵ is the energy and $\mathbf{p}$ the quasi momentum of the quasi particle).

Actually, the interaction between electrons is far from being small, and this leads to the need of describing the conduction electrons as a Fermi liquid.

The formulas for the current density and the kinetic equation were written for this case by Silin,[2] starting with the theory of a Fermi liquid as proposed by Landau[3]

$$\mathbf{j} = \frac{2e}{h^3}\int \frac{\partial \varepsilon_0}{\partial \mathbf{p}}\left(f' - \frac{\partial f_0}{\partial \varepsilon_0}\hat{L}f'\right)d\mathbf{p};$$

$$i\omega f' + \left(v_z\frac{\partial}{\partial z} + \frac{e}{c}[\mathbf{v}\times\mathbf{H}]\frac{\partial}{\partial \mathbf{p}}\right)\left(f' - \frac{\partial f_0}{\partial \varepsilon_0}\hat{L}f'\right) - I\left(f' - \frac{\partial f_0}{\partial \varepsilon_0}\hat{L}f'\right)$$

$$= -\frac{\partial f_0}{\partial \varepsilon_0}e\mathbf{v}\mathbf{E}; \quad \hat{L}f' = \int \Phi(\mathbf{p}, \mathbf{p}')f'(\mathbf{p}')d\mathbf{p}'.$$

Here f' is an addition to the equilibrium Fermi function $f_0(\epsilon_0)$, $\mathbf{v} = \partial\epsilon_0/\partial\mathbf{p}$ is the velocity of the electrons in the equilibrium state, $\Phi(\mathbf{p}, \mathbf{p}')$ is the function that characterizes the interaction, I is the collision integral, and z the direction of the inward normal to the metal surface.

It was shown in Refs. 1 and 1a that when $\hat{L} \equiv 0$, only electrons with $v_z \ll v$ are of significance in the determination of the surface impedance, for it is exactly at these values of v_z that the function f' has a sharp maximum near the surface at the distances of interest to us. But for such values of v_z, obviously, $|(\partial f_0/\partial\epsilon_0)\hat{L}f'| \ll |f'|$, so that the singularity in v_z disappears when integrating over the momenta (compare with the proof of the possibility of introducing the free-path time in anomalous skin effect for all temperatures, in Ref. 1a). This means that in the zeroth approximation with respect to the anomaly, the only one of real importance, the Fermi-liquid theory gives exactly the same results as the Fermi-gas theory.

The correctness of the above statement can also be readily verified directly, by replacing

$$\hat{L}f' = \int \Phi(\mathbf{p}, \mathbf{p}')f'(\mathbf{p}')d\mathbf{p}' \approx \int \Phi(v_x, v_y, 0; \mathbf{v}')f'(\mathbf{p}')d\mathbf{p}'$$

(the validity of such a replacement in the zeroth approximation with respect to the anomaly, i.e., with respect to $\max(\delta/\ell, \delta/r, \delta\omega/v)$, can be verified by substitution), and by carrying out calculations analogous to those of Ref. 1a.

Note added in proof. After this work was completed, an article by Silin [V. P. Silin, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1282, (1957), Soviet Phys. **6**, 985 (1958)] appeared, in which another method was used to obtained the very same result for the particular case of no constant magnetic field.

[1] G. E. H. Reuter and E. H. Sondheimer, Proc. Roy. Soc. **195**, 336 (1948).

[1a] M. Ia. Azbel' and E. A. Kaner, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 811 (1956), **32**, 896 (1957); Soviet Phys. JETP **3**, 772 (1956), **5**, 730 (1957).

[2] V. P. Silin, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 707 (1958), Soviet Phys. JETP **7**, 486 (1958) (this issue).

[3] L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 1058 (1956), Soviet Phys. JETP **3**, 920 (1956).

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