*In the presence of a common resonant level, the situation becomes considerably more complicated, owing to the resonant contribution of the transitions through the common level, which acts as an intermediate one with the difference frequency.[1,2] In the usual electron-nuclear level scheme in the Overhauser effect, when the resonant fields for the electrons and nuclei have a common level, failure to take these transitions into account causes the constant hyperfine structure to tend to zero in the final result, which, as is known, gives a finite effect.[1] There is no common level if both resonant fields are due to electron transitions at different nuclear orientations.

†The prime indicates that one of the levels should be replaced by the normalization condition. Formally, the system of equations for T_a corresponds to a certain equivalent dc circuit.

‡We disregard level shifts (for example, the influence of polarization on nuclei on the position of the electron resonance[3]). We notice also that in experiments in which the Overhauser effect is measured with two fields,[4] there is no interaction between the fields, in view of the smallness of one of the resonant fields.

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ON THE THEORY OF PLASMA WAVES IN A DEGENERATE ELECTRON LIQUID

V. P. SILIN

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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PLASMA waves in a degenerate electron gas were, apparently, first considered by Gol' dman.[1] However, the electrons in metals can hardly be considered as a gas. It is thus of interest to study the plasma oscillations of a degenerate electron liquid. According to Landau's theory of a Fermi liquid[2] the transport equation for the non-equilibrium correction δn to the distribution function of the quasi-particles (electrons) of a degenerate electron liquid has the form,[3]

$$\frac{\partial \delta n}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}}\left\{\delta n - \delta\varepsilon\,\frac{\partial n_0}{\partial \varepsilon_0}\right\} + e\mathbf{E}\mathbf{v}\,\frac{\partial n_0}{\partial \varepsilon_0} = 0. \qquad (1)$$

Here n_0 is the equilibrium distribution function, ϵ_0 the electron energy in the equilibrium state, and

$$\delta\varepsilon = \int \Phi\,(\mathbf{p},\mathbf{p}')\,\delta n\,(\mathbf{p}',\mathbf{r})\,d\mathbf{p}', \qquad (2)$$

where Φ is typical for the theory of a Fermi liquid, reflecting the short-range correlation of the particles. Finally $\mathbf{E}$ is the electric field which is determined from the equation

$$\operatorname{div}\mathbf{E} = 4\pi e\int \delta n d\mathbf{p}. \qquad (3)$$

In Eq. (1) collisions are neglected since it is assumed that the frequency of the plasma oscillations is much larger than the collision frequencies.

Considering solutions of Eq. (1) of the form $\delta n_{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{r})-i\omega t}$, and restricting ourselves to the case of long wavelengths, which allows us to expand in powers of k, we obtain from Eqs. (1) to (3), assuming that the Fermi surface is spherical, the following dispersion relation for the dependence of the frequency ω of the plasma waves on the wave vector at long waves

$$\omega^2 = \omega_0^2 + v_0 V_0\left(\frac{3}{5} + A_0 + \frac{4}{25}A_2\right)k^2, \qquad (4)$$

where v_0 and p_0 are the velocity and momentum of an electron on the Fermi surface, and A_0 and A_2 coefficients in the expansion in Legendre polynomials

$$\frac{8\pi p_0^2\Phi}{3\,(2\pi\hbar)^2 v_0} = \sum_n A_n P_n(\cos\chi)$$

(χ is the angle between the vectors $\mathbf{p}$ and $\mathbf{p}'$). Finally

$$\omega_0^2 = 4\pi e^2\cdot 8\pi p_0^2 V_0/3\,(2\pi\hbar)^3,\; V_0 = v_0\,(1 + A_1).$$

For a perfect Fermi gas of electrons, $A_n = 0$ and (4) goes over into the corresponding formula of Gol' dman's paper. The author gave in Ref. 4 an estimate of the coefficient A_1 for a number of real metals. It was then shown that it was not at all allowed to neglect this quantity compared to unity.

It is useful to make an estimate for the coefficients A_n for the case when the function Φ is determined by the forward-scattering amplitude calculated in Born approximation for a screened Coulomb potential. In that case

$$A_n = \frac{4}{3\pi}\,\frac{e^2}{\hbar v_0}\,\frac{2n+1}{2}\int_{-1}^{1} dx\; P_n(x)\left\{\xi - \frac{1}{4}\,\frac{1}{1 - x + {}^1/_2\xi}\right\}$$

and correspondingly

$$A_0 = \frac{4}{3\pi}\frac{e^2}{\hbar v_0}\left\{\xi - \frac{1}{8}\ln(1+4\xi)\right\};$$

$$A_1 = \frac{1}{\pi}\frac{e^2}{\hbar v_0}\left\{1 - \frac{1}{2}\left(1+\frac{1}{2\xi}\right)\ln(1+4\xi)\right\};$$

$$A_2 = \frac{5}{2\pi}\frac{e^2}{\hbar v_0}\left\{1+\frac{1}{2\xi} - \left(\frac{1}{3}+\frac{1}{2\xi}+\frac{1}{8\xi^2}\right)\ln(1+4\xi)\right\},$$

where $\xi = (p_0/\hbar k_D)^2 = \pi h v_0/4e^2$, and $k_D = \sqrt{(4/\pi)}\, e^2/\hbar v_0 (p_0/\hbar)$ is the quantity which is the inverse of the screening radius. For most metals $v_0 \sim 10^8\,\text{cm-sec}^{-1}$ and hence $e^2/\hbar v_0 = 2.2$. Here $A_0 = 0.25$, $A_1 = 0.05$, and $A_2 = -0.05$.

If we consider anisotropic metals in the approximation $\omega_0^2/v_0^2 \gg k^2 \gg \omega_0^2/c^2$ (where c is the velocity of light), we can also speak of longitudinal waves,

$$\omega^2 = \frac{4\pi e^2}{k^2}\frac{2}{(2\pi\hbar)^3}\int\frac{dS}{v}(\mathbf{kv})(\mathbf{kV}), \qquad (5)$$

where the integral is taken over the Fermi surface where dS is an element of that surface, and where

$$\mathbf{V} = \mathbf{v} + \frac{2}{(2\pi\hbar)^3}\int\frac{dS'}{v'}\Phi(\mathbf{p},\mathbf{p}')\,\mathbf{v}'.$$

The frequency of the plasma waves determined by equation (5) can also be found from the equation $k_\alpha k_\beta \epsilon_{\alpha\beta} = 0$, where $\epsilon_{\alpha\beta}$ is the dielectric constant of a degenerate electron liquid.[4]

Everything stated here can bear a relation to real metals only, apparently, if the energy of the plasma oscillation ($\sim\hbar\omega_0$) is small compared to the distance from the conduction band to the nearest filled band. One must then also take into account the contribution ϵ_0 to the dielectric constant, which comes not only from the conduction electrons.

We note finally that the dependence of the frequency of the plasma waves on the direction of the vector **k** can lead to a broadening of the line of discrete energy losses of electrons passing through non-cubic metals.

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