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ON THE RELATION BETWEEN THE ANGULAR AND ENERGY DISTRIBUTIONS OF PARTICLES IN HIGH-ENERGY NUCLEAR INTERACTIONS

G. B. ZHDANOV

P. N. Lebedev Physical Institute, Academy of Sciences, U.S.S.R.

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In continuation of Ref. 1, various experimental data on the magnitude of the transverse momentum component of particles produced in high-energy nuclear interactions are considered. The data are compared with the one-dimensional hydrodynamical theory of multiple particle production.

In study of high-energy nuclear interactions by means of photographic emulsions, it is generally possible to determine the angular distribution of secondary particles with sufficient accuracy, whereas the energy of the particles can be measured only to a limited degree. The knowledge of the energy characteristics of an event is, however, of considerable help not only for the understanding of the interaction process, but also for the correct interpretation of data on the angular distribution, especially in the not very rare cases where there is no frame of reference in which the distribution possesses mirror symmetry with respect to the plane with $\theta = 90°$.

Rozental' and Milekhin[1] were the first to draw attention to the experimental data and theoretical considerations in favor of existence of a connection between the angle of emission θ of a particle and its energy E. They indicate that the projection of momentum on the plane perpendicular to the direction of motion of the primary particle $p_\perp$ vary only slightly over a relatively large region of the angle θ and corresponds roughly to p of particles in thermal motion at the temperature μc^2 (where μ denotes the π-meson mass).* The purpose of the present paper is to consider a much wider sphere of experimental evidence supporting the above view.

2. EXPERIMENTAL DATA ON THE TRANSVERSE MOMENTUM OF SECONDARY PARTICLES OF $10^9 - 10^{13}$ ev.

It is more convenient to consider the distribution of transverse momentum $p_\perp$ in a logarithmic scale, particularly in units of $X = \log_{10}(p_\perp/\mu c)$, since, as a rule the relative error of momentum (or more accurately, energy) measurements (and consequently that of $p_\perp$) does not decrease with its value and even increases in certain cases. We introduce, therefore, a smaller error into the transverse momentum distribution ascribing the same statistical weight to various measurements by using the logarithmic scale. It should be also mentioned that, as it will be seen later, the theoretical and experimental distributions of the variable X can be approximated satisfactorily by the Gaussian curve

$$N(X) = N_0 \exp\{(X - \overline{X})/\sigma\sqrt{2}\}^2, \qquad (1)$$

Any such distribution can be sufficiently well characterized by two parameters: the mean value of X and the dispersion σ.

A. Cloud Chamber Method

Angles of emission of secondary particles with energies up to 10^{11} ev were measured using a cloud chamber placed in a magnetic field. The distributions shown in Fig. 1 (taken from Ref. 2) refer to the angles of emission of secondary particles $\theta_0 =$ 30, 50, and 90° in the center-of-mass system. The primary particles were protons of $E_0 = 2.2$ Bev. Data obtained from cosmic rays[3,4] referring to primary energies of 5 and 30 Bev are shown in Figs. 2 and 3, respectively. The dashed parts of the histograms (which were taken into account in constructing the resulting dashed Gaussian curves) were obtained by an appropriate extrapolation of the events in which only the lower limit of the measured momentum was known.

B. Emulsion Method: Measurements of the Multiple Scattering of Secondary Particles

Besides the 34 measurements of the single event reported by Debenedetti et al.[5] and which was al-

*In the following we call $p_\perp$ simply the transverse momentum, and measure it in units of μc.

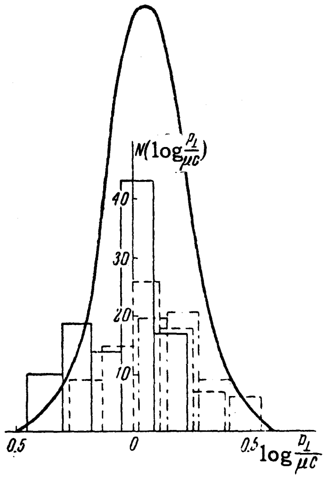

FIG. 1. Distribution of secondary particles with respect to the transverse momentum $p_\perp$ (in units of μc), obtained by means of an accelerator for $\overline{E}_0 = 2.2$ Bev.[2] Total number of particles N = 244. Mean (geometrical) value $(\overline{p}_\perp)_{geom} = 1.1\ \mu c$. Dispersion of the distribution $\sqrt{D(p_\perp)}/\overline{p}_\perp \approx 0.4$. The solid, dot –, and dash-dot histograms of $p_\perp$ distributions correspond to angles $\theta_0 = 30°$, $\theta_1 = 50°$, and $\theta_0 = 90°$ respectively.

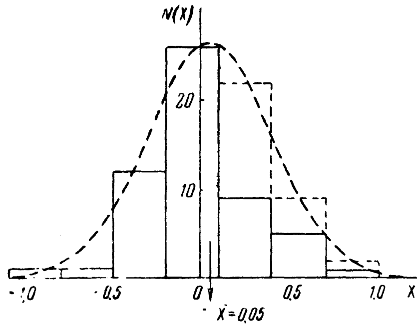

FIG. 2. Distribution with respect to $p_\perp$ of secondary particles for $\overline{E}_0 = 5$ Bev[3] (cloud chamber method). N = 72, $(\overline{p}_\perp)_{geom} = 1.1\ \mu c$, $\sqrt{D(p_\perp)}/\overline{p}_\perp = 0.8$.

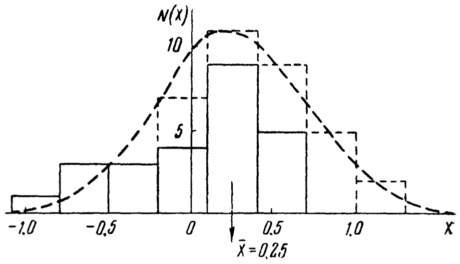

FIG. 3. Distribution of secondary particles with respect to $p_\perp$ for $E_0 = 30$ Bev[4] (cloud chamber method). N = 41, $(\overline{p}_\perp)_{geom} = 1.8\ \mu c$, $\sqrt{D(p_\perp)}/\overline{p}_\perp = 1.0$.

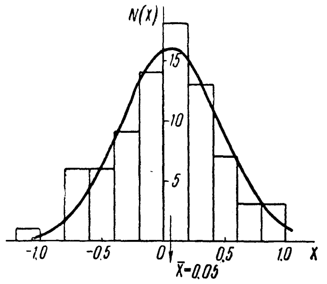

FIG. 4. Distribution of secondary particles with respect to $p_\perp$ for $E_0 \sim 5 \cdot 10^{12}$ ev. Data of Ref. 5–7 and of the author (multiple scattering method, emulsion). N = 80, $(\overline{p}_\perp)_{geom} = 1.1\ \mu c$, $\sqrt{D(p_\perp)}/\overline{p}_\perp = 0.9$.

ready mentioned in Ref. 1, we had at our disposal additional 46 measurements of 4 events of 10^{12} – 10^{13} ev which were partially published in Refs. 6 and 7. The distribution shown in Fig. 4 is based upon all data available. This time we have not included the 8 cases (out of 88) when only the lower limit of the value of $p_\perp$ had been known; the resulting mean value of the transverse momentum is therefore slightly lower.

C. Emulsion Method: Analysis of Secondary Interactions

In the cases when secondary interactions have been observed, it is possible to determine the corresponding value of γ_c from the angular distribution, making use of the fact that the distribution is usually symmetrical in the center-of-mass system. Passing from γ_c to the energy of particles, it is possible to account for the tunnel effect by using the results of Ref. 8. Data on all 15 cases known to us, in which it has been possible to measure the angles of emission and the energies of secondary particles, are given in the table.

It can be seen from the table that both the mean transverse momentum and the spread about the mean are considerably larger than in all previous (and also following) methods of determination of $p_\perp$. There is a reason to think that the large spread in values of $p_\perp$ is in that case connected with large (compared with Poissonian) fluctuations of the angular distribution and that the large value of $p_\perp$ obtained is due to possible deviations from asymmetry in the angular distribution (in the mean) in interactions between particles with different mass (essentially π-mesons with nuclei).

D. Emulsion Method: Analysis of the Angular and Energy Distributions of Particles of the Soft Component

The study of the electron-photon component of the products of nuclear interactions reveals two possibilities for determination of the transverse momentum of π^0-mesons. Firstly, it is possible to measure the position of electron pairs of non-bremsstrahlung origin* with respect to the axis of the primary star, and to find the energy of those pairs from the opening angle of their components.†

*Bremsstrahlung photons, incidentally, posses a transverse momentum of the order of $m_e c$ only (where m_e is the electron mass).

†The energy determination followed the method of Ref. 13.

Primary interaction	Secondary interactions: Type of star	γ_c	Angle with shower axis, θ	Energy in lab. system: a n—n interaction, Bev	Energy in lab. system: b tunnel effect interaction, Bev	$p_\perp/\mu c$: a	$p_\perp/\mu c$: b	Remarks
$1+37_\alpha$, $\gamma_c=200$	$3+10_p$	13 ± 4	$6\cdot10^{-4}$	350^{+250}_{-200}	700^{+500}_{-400}	$1.5^{+1.1}_{-0.9}$	$3.0^{+2.2}_{-1.8}$	
	$9+13_p$	5.5 ± 1.5	$3\cdot10^{-3}$	60^{+40}_{-30}	180^{+120}_{-90}	$1.3^{+0.9}_{-0.6}$	$3.9^{+2.7}_{-1.8}$	
$8+30_n$, $\gamma_c=55$	$4+11_p$	8 ± 2	$5\cdot10^{-3}$	130^{+70}_{-60}	130^{+70}_{-60}	$4.6^{+2.5}_{2.1}$	$4.6^{+2.5}_{-2.1}$	
$1+51_n$, $\gamma_c=22$	$13+10_p$	3.7 ± 1.1	~0.01	30^{+20}_{-12}	60^{+40}_{-25}	~7	~14	
$1+29_Z$, $\gamma_c=20$	$4+5_n$	7 ± 3	~0.05	100^{+100}_{-70}	100^{+100}_{-70}	~35	~35	
	$14+17_p$	3 ± 1	0.025	20^{+15}_{-11}	100^{+75}_{-55}	$3.5^{+2.5}_{-2.0}$	~15	
$21+78_Z$, $\gamma_c=15$	$0+6_p$	~100 (?)	$3\cdot10^{-3}$	$\sim20{,}000$ (?)	$\sim20{,}000$ (?)	~500 (?)	~500 (?)	
	$17+8_p$	20 ± 6	$7\cdot10^{-3}$	760^{+500}_{-420}	760^{+500}_{-420}	35^{+27}_{-18}	35^{+27}_{-18}	
	$4+12_p$	5 ± 1.5	$3.5\cdot10^{-3}$	50^{+35}_{-25}	100^{+70}_{-50}	$1.25^{+0.9}_{-0.6}$	$2.5^{+1.8}_{-1.2}$	
$5+30_p$, $\gamma_c=26$	$3+55_n$	10 ± 1.5	$1.5\cdot10^{-3}$	220 ± 60	1000	2.5 ± 0.75	10	Ref. 9
$20+56_p$, $\gamma_c=70$	$22+22_p$	4	$4\cdot10^{-3}$	30	150	0.85	4	Ref. 10
	$3+9$	13	$4\cdot10^{-3}$	320	500	9	15	
	$23+37_n$	9	$5\cdot10^{-3}$	160	800	6	30	
$0+14_\alpha$, $\gamma_c=420$	$0+16_p$	~30	$1.1\cdot10^{-4}$	~2000	~4000	1.6	3.2	Ref. 11
	$3+18_p$	~20	$5.4\cdot10^{-4}$	~1000	~3000	4	12	
Geometrical mean (of 14 cases) for $p_\perp \mu c$.						~4	~10	

Interactions with $n_s < 5$ are not given. The angles θ are measured in projection on the emulsion plane.

Advantage is taken of the fact that the additional transverse momentum imparted to the photons in the π^0-decay is found to be several times smaller than the initial momentum of the π^0-meson. The second method consists in determination of the photon energy from the development of electromagnetic cascades initiated by them. By a special combination of additional absorbers, it is possible to create such conditions (as it has been done in emulsion chamber experiments of a group of Japanese physicists[12]) that cascades due to different photons are well separated from each other.

In the determination of $p_\perp$ according to the first method we made use of the data of Refs. 13, 14, and 11. The calculated distribution is shown in Fig. 5. The distribution obtained using the second method, and the data of Ref. 2, as well as the theoretical curve for the temperature of evaporation equal to $\mu c^2/k$ calculated according to Ref. 1 are shown in Fig. 6. Comparing Fig. 6 with Figs. 2—5, we can see that the last method yields the narrowest distribution which, at the same time, is not in disagreement with the theoretical curve. This distribution of the transverse momentum is based on all available cosmic ray data, and although one cannot exclude the possibility that the result is due to experimental errors, we shall assume the theoretical distribution given in Fig. 6 as the basis for the following discussion.

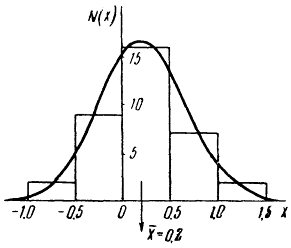

FIG. 5. $p_\perp$ distribution of π^0-mesons obtained from the energy and angle of emission of electron pairs (data of Refs. 12, 14, and 11). N=36, $(\bar{p}_\perp)_{geom} = 1.6\ \mu c$, $\sqrt{D(p_\perp)}/\bar{p}_\perp$ 1.0.

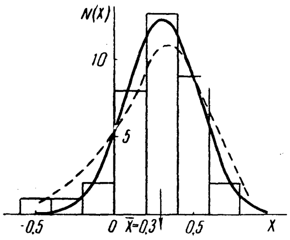

FIG. 6. $p_\perp$ distribution of π^0-mesons obtained from data of Ref. 12, for electromagnetic cascades in emulsion chamber. N = 36, $(\bar{p}_\perp)_{geom} = 2\ \mu c$, $\sqrt{D(p_\perp)}/\bar{p}_\perp = 0.5$. The dotted curve represents distribution according to one-dimensional hydrodynamical theory[1] for disintegration temperature $T_k = \mu c^2/k$.

E. Data from Extensive Air Showers

The multiplicity of processes contributing to the development of extensive air showers makes it impossible to obtain sufficiently accurate data on the transverse momentum of shower particles at the place of production. In view of the considerably higher energies involved, it is nevertheless interesting to compare even rough estimates of $p_\perp$ with previous results.

The possibility of estimating $p_\perp$ is basically connected with the analysis of the angular distribution of the penetrating component of showers at various depths underground (cf., e.g., Refs. 15, 16). Defining the mean angle of emission of mesons as the ratio of the mean radius of lateral distribution to the effective altitude of shower initiation (about 10 km) and determining the mean energy from the range in ground we found that, for energies up to 10^{12} ev, the mean value of the transverse momentum of π-mesons should be $1-5$ (in units of μc). The same principle of determination of transverse momentum can be used for the soft component of showers, giving results consistent with the above. It can be maintained that, for a primary energy of $10^{12}-10^{14}$ ev, the distribution of secondary particles with respect to transverse momentum is in good agreement with the one-dimensional hydrodynamical theory of multiple particle production if we regard transverse deviation of produced particles solely as a result of their thermal motion at the temperature of evaporation, close to $\mu c^2/k$.

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