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POLARIZATION AND ANGULAR DISTRIBUTION OF X-RAYS EMITTED AFTER NUCLEAR CAPTURE OF ELECTRONS AND AFTER CONVERSION TRANSITIONS

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Results obtained previously[1] are extended to K and L capture of any order of forbiddenness, taking parity nonconservation into account. It is shown that observation of the correlation of the polarization of x-rays emitted after K or L capture with nuclear spin directions or with the polarization of subsequent gamma rays may provide important information concerning β interactions. The angular distribution of x-rays is anisotropic when capture occurs from the L_{III} subshell.

Formulas are presented which permit determination of the spins and parities of nuclear levels from the observation of x-ray polarization following conversion transitions.

1. POLARIZATION AND ANGULAR DISTRIBUTION OF X-RAYS EMITTED FOLLOWING ELECTRON CAPTURE

In an earlier paper[1] the author proposed observation of the correlation between the angular polarization of x-rays emitted in the filling of atomic shell vacancies following allowed K capture and the orientation of nuclear spins or the polarization of subsequent gamma radiation. It was shown that information might thus be obtained regarding the relative signs of the scalar and tensor β-interaction constants. A number of recent experiments[2] have confirmed Lee and Yang's[3] hypothesis of parity nonconservation in β decay. We have extended the results of Ref. 1 to take the nonconservation of parity into account.

Allowed K and L captures are of greatest interest in connection with the unambiguous interpretation of experimental results. Correlation calculations for these allowed transitions as well as for forbidden transitions are given in Appendix I. At this point we shll present and discuss the final formulas.

We shall first consider the case in which a nucleus before an allowed K capture is located in an external field and we do not study the direction and polarization of the gamma ray which may be emitted after K capture. When an x-ray is emitted in an electronic transition to a vacated K level from a level with a momentum $I = 1/2$ (such as L_{II}) the correlation between the circular polarization of this x-ray and the nuclear spin orientation is given by

$$W_{jX}(\theta) = a+b+\sigma_X \rho\,\{{}^1/_2\,[j_0(j_0+1) - j_1(j_1+1)+2]\,b+\varkappa\sqrt{j_0(j_0+1)}\}\cos\theta; \tag{1}$$

$$a = (|c_S|^2+|c_S'|^2)\,|K_S|^2+(|c_V|^2+|c_V'|^2)\,|K_V|^2 - 2\mathrm{Re}\,(c_S c_V^* - c_S' c_V'^*)\,K_S K_V^*;$$

$$b = (|c_T|^2+|c_T'|^2)\,|K_T|^2+(|c_A|^2+|c_A'|^2)\,|K_A|^2 - 2\mathrm{Re}\,(c_T c_A^* - c_T' c_A'^*)\,K_T K_A^*; \tag{2}$$

$$\varkappa = 2\mathrm{Re}\,[(c_S c_T^* + c_S' c_T'^*)\,K_S K_T^* - (c_V c_T^* - c_V' c_T'^*)\,K_V K_T^* - (c_S c_A^* - c_S' c_A'^*)\,K_S K_A^* + (c_V c_A^* + c_V' c_A'^*)\,K_V K_A^*];$$

$$\rho = \sum_{\mu_0}\mu_0 w(\mu_0)/j_0(j_0+1). \tag{3}$$

θ denotes the angle between the direction of the x-ray and the principal nuclear spin orientation; j_0 is the nuclear spin in the initial state, μ_0 is its projection on the z axis, and $w(\mu_0)$ is the probability of a given value of μ_0; ρ gives the degree of polarization of the nuclear spins; $\sigma_X = 1$ or -1, respectively, for right or left circular polarization of the x-rays; c_S, c_V, c_T and c_A are the scalar, vector, tensor and axial-vector (pseudovector) β-interaction constants; c_S', c_V', c_T' and c_A' are analogous constants in terms which are allowed only when parity is not conserved (containing in addition $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$, where $\gamma_4 = -\beta$; γ_i, γ_5 and β are Dirac matrices).

In calculating (1) we have used the general form of the β interaction which was proposed by Lee and Yang.[3] K_S and K_T are nuclear matrix elements for the scalar and tensor β interactions, with K_V and K_A for the vector and pseudovector β interactions. In the nonrelativistic approximation for nucleons $K_S = -K_V$ and $K_T = -K_A$. The expression for the correlation does not depend on the magnitudes of K_S and K_T but only on their ratio K_S/K_T:

$$K_S = \int \psi^*_{j_1\mu_1}\beta\psi_{j_0\mu_0}\,d\mathbf{r},$$

$$C^{j_0\mu_0}_{1\Lambda j_1\mu_1}K_T = (-1)^{1+\Lambda}\int \psi^*_{j_1\mu_1}\beta\sigma_{-\Lambda}\psi_{j_0\mu_0}\,d\mathbf{r}; \tag{4}$$

$\psi_{j_0\mu_0}$ and $\psi_{j_1\mu_1}$ are the nuclear wave functions before and after electron capture; $\sigma_0 = \sigma_z$, $\sigma_{\pm 1} = \pm\sqrt{1/2}\,(\sigma_x \pm i\sigma_y)$; σ_x, σ_y, σ_z are Pauli matrices; $C^{j_0\mu_0}_{1\Lambda j_1\mu_1}$ is a Clebsch-Gordan coefficient; j_1 and μ_1 are the nuclear spin and its projection after K capture. The matrix elements K_S and K_T are independent of the magnetic quantum numbers μ_0, μ_1 and Λ.

If a gamma ray is emitted after K capture, for the determination of a, b and κ it is possible to study the correlation between the circular polarizations of the x-ray and gamma ray. It is then not necessary to use oriented nuclei. If the x-ray is emitted in an electronic transition to the K shell from a level of mechanical moment $I = 1/2$ and the gamma ray is of multipole order 2^L the correlation is given by

$$W_{\gamma X}(\theta) = a+b-\sigma_X\sigma_\gamma\,[j_1(j_1+1)-j_2(j_2+1)+L(L+1)] \times [4j_1(j_1+1)\,L(L+1)]^{-1}\{[j_1(j_1+1)-j_0(j_0+1)+2]\,b + 2\varkappa\sqrt{j_1(j_1+1)}\}\cos\theta. \tag{5}$$

Here θ is the angle between the directions of the x-ray and gamma ray, and $\sigma_\gamma = 1$ or -1 for right or left circular polarization of the gamma ray.

For mixed 2^L magnetic and 2^{L+1} electric multipole transitions $[j_1(j_1+1)-j_2(j_2+1)+L(L+1)]$ must be replaced by

$$\{j_1(j_1+1)-j_2(j_2+1)+L(L+1) + f^2(2L+3)^{-1/2}L\,[j_1(j_1+1)-j_2(j_2+1) + (L+1)(L+2)] - 2fL(L+2)\,[L(j_2+j_1+L+2)(j_2+j_1-L) \times (j_1-j_2+L+1) \times (j_2-j_1+L+1)/(2L+1)]^{1/2}(2L+3)^{-1}\};$$

f^2 is the ratio of the electric and magnetic radiation intensities, with $f = \pm|f|$.

If the emission of the observed gamma ray does not directly follow K capture, but is separated from the latter by cascade emission of one or more photons which are not observed the curly bracket in (5) must be preceded by the factor[4]

$$\prod_{k=1}^{N-1}\left\{\frac{j_k(j_k+1)+j_{k+1}(j_{k+1}+1)-L_k(L_k+1)}{2[j_k(j_k+1)j_{k+1}(j_{k+1}+1)]^{1/2}}\right\}, \quad (6)$$

where L_k is the multipole order of the intermediate photons:

$$j_0(K)\,j_1(L_1)\,j_2(L_2)\,j_3\ldots\ldots j_N(L)\,j_{N+1},$$

while L is the multipole order of the observed photon. If one of the intermediate gamma transitions is mixed, the expressions for magnetic and electric transitions must be added with weights proportional to their intensities. Interference terms drop out upon integration over the angles of gamma emission.

When an electron fills a vacancy in a K shell by a transition from a level with $I = 3/2$ (such as L_{III}), then in (1) and (5) σ_x must be preceded by the factor $-1/2$. Since the transition probability from an $I = 3/2$ level is almost twice as large as from $I = 1/2$ the total correlation is very small. It is therefore necessary to determine the polarization of x-rays coming from $I = 3/2$ and $I = 1/2$, separating them according to their energies. It is most convenient to study the K_{α_1} and K_{α_2} lines. It must be kept in mind that an external magnetic field can change the separations and order of atomic levels. The character of this change for different specific cases is easily established either experimentally or theoretically.

We note that for atoms where the $P_{3/2}$ shell has not been completely built up the polarization of photons coming from the $P_{1/2}$ level is not entirely compensated by the opposite polarization of photons coming from the $P_{3/2}$ level, and it is not necessary to distinguish them energetically. In this case it is best to use the lightest nuclei, where the L shell is not filled. Light nuclei in which K capture occurs possess short lifetimes as a rule and are not easily used in experiments requiring the orientation of nuclear spins. It is then easier to investigate the polarization correlation of x-rays and gamma rays.

For allowed L_I or L_{II} capture, the correlation is of the same character as for K capture. However, in this case there is incomplete compensation of right and left circular polarization for the combined L_β, L_γ, etc. radiation. A theoretical calculation of the remaining degree of polarization is difficult and would require good knowledge of the electronic wave functions.

For K, L_I, and L_{II} capture the correlations (1) and (5) exist only if the nuclei possess dipolar polarization; these correlations do not exist for aligned nuclei, which can be used only to study L_{III} capture. In this case there will be an anisotropic angular distribution of x-rays relative to the orientation of the aligned nuclei. For unoriented nuclei with L_{III} capture followed by gamma-ray emission there exists an angular correlation between x-rays and gamma rays. Only the tensor and pseudovector interactions contribute to allowed L_{III} capture. The x-ray angular distribution is given by

$$W(\theta) = 1 + \sqrt{3(2j_0+1)/10}\,A_I W(j_1 1 j_0 2;\ j_0 1)$$
$$\times (3\cos^2\theta - 1)\,P^j_{20};$$
$$A_{1/2} = 1/2,\quad A_{3/2} = -2/5,\quad A_{5/2} = 1/10. \quad (7)$$

The quantities A_I depend only on the total angular momentum I of the electron which fills the vacancy in the L_{III} subshell. W(abcd; ef) is the Racah function. For aligned nuclei without investigation of the possible gamma rays we have

$$P^j_{20} = 5\sum_{\mu_0}[3\mu_0^2 - j_0(j_0+1)]$$
$$\times w(\mu_0)\,[(2j_0-1)\,j_0(j_0+1)(2j_0+3)]^{-1/2}. \quad (8)$$

When the angular correlation of x-rays and gamma rays is investigated for unoriented nuclei we have

$$P^j_{20} = 5\sqrt{(2j_1+1)(2L+1)}\left[1 - \frac{3}{L(L+1)}\right]$$
$$\times W(j_2 L j_1 2;\ j_1 L)\,C^{L0}_{L020}. \quad (9)$$

Equation (7) does not contain nuclear matrix elements of L capture nor the constants c and c′, which are included within a general factor that goes not influence the angular correlation (7) and is therefore omitted here.

A study of the correlation expressed by (7) does not provide information concerning β interactions, but it can serve for determination of the angular momenta of nuclear levels.

The existing experimental information concerning the constants c and c′ is incomplete and contradictory according to the present theory. There is therefore considerable interest in the determination of combinations of c and c′ which have not yet been investigated. a, b, and κ in (1) and (5) are among such combinations, and a study of these equations may throw light on the nature of β interactions. We note that the expression for the polarization of β particles emitted by polarized nuclei[5] also contains c and c′ in combinations similar to a, b and κ; but the same expression contains c and c′ in other combinations. Therefore (1) and (5) can be interpreted more simply and unambiguously.

For forbidden K and L capture, the polarization and angular correlations of x-rays can be determined from (25). For first forbiddenness $N = 1$; L and $L' = 0, 1, 2$; $i = 1/2$ and $3/2$. For $j_1 = j_0 \pm 2$ in capture from the L_{III} subshell the correlation formula agrees with (7) if $\sqrt{3}\, W(j_1 1 j_0 2; j_0 1)$ is replaced by $\sqrt{7}\, W(j_1 2 j_0 2; j_0 2)$. For forbidden K, L_I and L_{II} captures the correlation formulas are more complicated than (1) and (5) as they contain some unknown ratios of nuclear matrix elements which must be determined experimentally.

2. POLARIZATION AND ANGULAR DISTRIBUTION OF X-RAYS FOLLOWING CONVERSION TRANSITIONS

In Ref. 1 it was shown that observation of the circular polarization of x-rays emitted when vacancies in atomic shells are filled following conversion transitions of oriented nuclei can provide the same information regarding nuclear levels that comes from experiments on the angular correlation of conversion electrons and subsequent (or preceding) gamma rays. When gamma rays are emitted following conversion transitions, instead of using oriented nuclei we can study the correlation between the circular polarizations of x-rays and gamma rays.

Appendix II contains the derivation of the general formula for the conversion transition probability of an oriented nucleus with subsequent emission of x-rays and gamma rays of given polarization. We shall here give the final formulas for the most important cases. We first consider a conversion transition by an oriented nucleus when subsequent gamma rays do not interest us. For K-shell conversion, when the vacancy is filled by an electron with $I = 1/2$ (let us say $p_{1/2}$), the circular polarization of the x-rays, according to (25), (41) and (42) is given by

$$W_{j_0 X}(\theta) = 1 + \sigma_X \rho\, [j_0 (j_0 + 1) - j_1 (j_1 + 1) + L (L + 1)]\, Q_L \cos\theta; \quad (10)$$

$$Q_L = \frac{1}{2L(L+1)} \frac{(L+1)\,|B_L^-|^2 - L\,|B_L^+|^2}{(L+1)\,|B_L^+|^2 + L\,|B_L^-|^2}. \quad (11)$$

For an EL electric transition

$$B_L^+ = \sqrt{1/L(L+1)}\,[R_3 + R_4 + 2R_6]_{J=L+1/2} M_L^{el}; \quad (12)$$

$$B_L^- = \sqrt{1/L(L+1)}\,[L(R_3 + R_4) - (2L+1) R_5 - R_6]_{J=L-1/2} M_L^{el}. \quad (13)$$

For an ML magnetic transition

$$B_L^+ = -(L/L+1)\,[R_1 + R_2]_{J=L+1/2} M_L^{mg}; \quad (14)$$

$$B_L^- = [R_1 + R_2]_{J=L-1/2} M_L^{mg}. \quad (15)$$

The quantities $R_1, R_2, \dots R_6$ are radial integrals which depend on the multipole order of the L conversion transition, on the total and orbital angular momenta of the converted electron in the shell J_0 and $l_0 = J_0 + \lambda_0$; $\lambda_0 = \pm 1/2$, and on the corresponding angular momenta of the electron after conversion J and $l = J + \lambda$. The explicit form of R_i is given in the book by M. E. Rose.[6] In the deduction it was assumed that a part is played by a conversion transition of a single definite multipole order. There are possible instances in which contributions from E2 and M1 transitions are of comparable magnitude. When the nuclear spin is not changed in conversion ($j_1 = j_0$), comparable contributions can come from E0, M1, and E2 transitions. Formulas (10) and (16) cannot be applied to these cases.

The quantities M_L^{el} and M_L^{mg} in (12) – (15) are reduced matrix elements of the nuclear transition and depend on the structure of the nucleus. In the cases represented by (10) and (16) they drop out and do not affect the correlation.

The correlation between the circular polarizations of an x-ray K_{α_2} quantum (electron transition from $I = 1/2$) and a gamma ray after K conversion of an unoriented nucleus is given by

$$W_{\gamma X}(\theta) = 1 - [j_1 (j_1 + 1) - j_2 (j_2 + 1) + L_\gamma (L_\gamma + 1)]\, [j_1 (j_1 + 1) - j_0 (j_0 + 1) + L (L + 1)]\, [2L_\gamma (L_\gamma + 1)\, j_1 (j_1 + 1)]^{-1} Q_L \sigma_X \sigma_\gamma \cos\theta; \quad (16)$$

θ is the angle between the photon directions $\sigma_X = 1$ or -1 and $\sigma_\gamma = 1$ or -1, depending upon whether the respective photons possess right or left circular polarization.

When an x-ray quantum is emitted through the filling of a K-vacancy by an electron with $I = 3/2$, in (10) and (16) σ_X must be preceded by the factor $-1/2$. Therefore all statements made in section 1 concerning the need for distinguishing the energies of the K_{α_1} and K_{α_2} lines remain valid.

When the nuclear spin is not changed through conversion a study of $W_{j_0 X}(\theta)$ and $W_{\gamma X}(\theta)$ enables us to determine the contribution and relative phase of the matrix elements of E0 and M1 transitions. According to (25), (41) and (42) the polarization of K_{α_2} photons emitted in the K-conversion transition of a polarized nucleus ($j_1 = j_0$) is given by

$$W_{j_0 X}(\theta) = 1 - \sqrt{j_0(j_0+1)}\sigma_X \rho Q_{10}\cos\theta; \tag{17}$$

$$Q_{10} = \frac{B_0^+ B_1^{-*} + B_0^{+*} B_1^-}{\sqrt{3}\sum_L (2L+1)^{-1}\left[L\,|B_L^-|^2 + (L+1)\,|B_L^+|^2\right]}. \tag{18}$$

In (18) the summation can be taken over $0 \le L \le 2j_0$, but it is usually sufficient to limit this to $L = 0, 1$, and 2, i.e., to E0, M1, and E2 transitions. B_0^+ and B_1^- in (18) must be taken for E0 and M1 transitions.

The $K\alpha_2 - \gamma$ polarization correlation is given by

$$W_{\gamma X}(\theta) = 1 + \left[j_1(j_1+1) - j_2(j_2+1) + L_\gamma(L_\gamma+1)\right]$$
$$\times\left[2L_\gamma(L_\gamma+1)\sqrt{j_0(j_0+1)}\right]^{-1} Q_{10}\sigma_X\sigma_\gamma\cos\theta. \tag{19}$$

The determination of M_0^{el}/M_1^{mg}, which is possible with the aid of (17) and (19), furnishes information concerning the structure of the nucleus.

When conversion occurs in shells with $J_0 > 1/2$, such as L_{III}, the x-ray angular distribution is anisotropic with respect to the nuclear spin orientation or direction of subsequent gamma rays. For L_{III}-conversion the angular distribution is given by

$$W_{jX}(\theta) = 1 + A_l R_{j_1 j_0}(3\cos^2\theta - 1)\,P^j{}_{20},$$

$$R_{j_1 j_0} = \Big[\sum_{LL'J}(2J+1)\sqrt{(2j_0+1)/5}\;W\left(\tfrac{3}{2}J2L', L\tfrac{3}{2}\right)$$
$$\times W(j_1 L j_0 2;\; j_0 L')\;\; B_L^J B_{L'}^{J*}\Big]$$
$$\times\Big[\sum_{LJ}(2J+1)(2L+1)^{-1}|B_L^J|^2\Big]^{-1}. \tag{20}$$

Here B_L^J means $B_{J\lambda L}^{J_0\lambda_0}$ (see Appendix II) for L_{III}-conversion ($J_0 = 3/2$, $\lambda_0 = -1/2$). The sum $R_{j_1 j_0}$ is taken over the possible values: $J = L \pm 1/2$, $L \pm 3/2$. When for the conversion transition it is sufficient to consider only a single multipole order L_0, so that $L = L' = L_0$, the summation over L and L′ in $R_{j_1 j_0}$ vanishes and M_L drops out of the formula.

When an E0 transition is possible, the angular distribution is determined by the interference of E0 and M1. $R_{j_1 j_0}$ contains terms for $L = L' = 0, 1, 2$; $L = 0$ and $L' = 1$; $L = 1$ and $L' = 0$. This case furnishes the same information that can be obtained by investigating Eq. (17) or (19).

APPENDIX I

We shall derive a general formula for the polarization and angular distribution of x-rays emitted after electron capture, for both allowed and forbidden transitions.

Let us consider a nucleus that is oriented in an external field. As heretofore, j_0 and j_1 will denote the spin of the initial and final nucleus, respectively; J_0 and $l = J_0 + \lambda_0$ are the moments of the orbital electron that is absorbed by the nucleus and I and $l_I = I + \lambda_I$ are the moments of the electron which jumps into the vacancy. The wave function of the captured electron is

$$\psi^{(n)}_{J_0 M_0 \lambda_0}(\mathrm{r}) = \begin{Bmatrix} -i\,F^{(n)}_{J_0\lambda_0}(r)\;\hat{Y}^{-\lambda_0}_{J_0 M_0}(\vartheta,\varphi) \\ 2\lambda_0\,G^{(n)}_{J_0\lambda_0}(r)\;\hat{Y}^{\lambda_0}_{J_0 M_0}(\vartheta,\varphi)\end{Bmatrix}, \tag{21}$$

where $F^{(n)}_{J_0\lambda_0}(r)$ is the small and $G^{(n)}_{J_0\lambda_0}(r)$ is the large radial component, and $\hat{Y}^\lambda_{JM}(\vartheta,\varphi)$ is a spherical spinor[7] with components equal to the components of the $(h-J)$-vector $Y^{h\tau}_{JM}$ for $h = 1/2$, as follows:

$$\left[Y^{h\tau}_{JM}(\vartheta,\varphi)\right]_\gamma = (-1)^{h-\gamma}\,C^{JM}_{J+\tau, M+\gamma, h-\gamma}\;Y_{J+\tau, M+\gamma}(\vartheta,\varphi). \tag{22}$$

The orientation of the initial nucleus is given by the polarization tensor

$$P^{j_0}_{g_0\eta_0} = \frac{2g_0+1}{2j_0+1}\sum_{\mu_0\mu'_0} C^{j_0\mu_0}_{j_0\mu'_0 g_0\eta_0}\,\rho(\mu_0\mu'_0), \tag{23}$$

where $\rho(\mu_0\mu'_0)$ is the density matrix of the initial state. If the axis of quantization is chosen to be the physically distinguished axis of principal nuclear spin orientation, then

$$\rho(\mu_0\mu'_0) = w(\mu_0)\,\delta_{\mu_0\mu'_0},$$

where $w(\mu_0)$ is the probability of finding the nucleus in a state with the projection μ_0 on the z axis, and $\sum_{\mu_0} w(\mu_0) = 1$. After electron capture the nucleus may remain in an excited state and emit several gamma rays successively. The polarization tensor of this nucleus can be expressed in terms of observed quantities which characterize the subsequent radiative transition:

$$P^{j_1}_{g_1\eta_1} = \frac{2g_1+1}{2j_1+1}\sum_{\mu_1\mu'_1} C^{j_1\mu_1}_{j_1\mu'_1 g_1\eta_1} H_{j_1\mu_1} H^*_{j_1\mu'_1}, \tag{24}$$

where $H_{j_1\mu_1}$ is the matrix element of the nuclear gamma transition.

The spin state of the atomic shell after capture is given by the tensor $P^{J_0}_{g\eta}$, which we shall express in terms of quantities that characterize the x-ray that accompanies the filling of the vacated level. Equation (24) remains valid for $P^{J_0}_{g\eta}$ with replacement of j_1, μ_1, g_1, η_1 by J_0, M_0, g, η and replacement of $H_{j_1\mu_1}$ by the x-ray transition matrix element $H_{J_0\mu_0}$.

Thus the state of the atom before the conversion transition is determined by the character of the polarization of the nucleus (the tensor $P^{j_0}_{g_0\eta_0}$) and of the atomic shell. The polarization of the atomic shells which do not participate in the transition is of no interest here. The cases of practical interest are K and L conversion. Since the total angular momentum of a filled K or L shell is zero these shells are not oriented in an external field and the electronic polarization tensor for the initial state can be taken as unity. In the final state the orientation of the nucleus will be given by $P^{j_1}_{g_1\eta_1}$ and the orientation of the shell by $P^{J_0}_{g\eta}$.

Using the method of calculation described in Refs. 1 and 5, we obtain the following expression* for the probability of a conversion transition in an atom:

$$W_{j_0\gamma X} = \sum (-1)^{g_0}(2j_0+1)$$

$$\times (2i+1)\sqrt{(2j_0+1)(2j_1+1)(2J_0+1)/(2g+1)}$$

$$\times W(J_0 i g L', L J_0)\, X(j_1 j_1 g_1, j_0 j_0 g_0, L L' g)$$

$$\times C^{g\eta}_{g_1\eta_1 g_0\eta_0} P^{j_1 *}_{g_1\eta_1} P^{j_0 *}_{g_0\eta_0} P^{J_0}_{g\eta} B^{J_0\lambda_0}_{i\nu L} B^{J_0\lambda_0 *}_{i\nu L'}. \tag{25}$$

Here $W(abcd;ef)$ are Racah functions, which are tabulated in Ref. 8, $X(abc,def,ghi)$ are Fano functions, which are tabulated in Refs. 9 and 10, and $C^{c\gamma}_{a\alpha b\beta}$ are Clebsch-Gordan coefficients, which are tabulated in Ref. 11. The total angular momentum carried away by the neutrino is denoted by i and $l_\nu = i + \nu$, $\nu = \pm 1/2$ denotes its orbital angular momentum. The summation in (25) is taken over the permissible values of all indices. The values of j_0, j_1, J_0, λ_0 and the order of forbiddenness of the transition, denoted by N, are given. For a transition of definite order of forbiddenness (25) retains only a few terms of the sum, because the quantities in (25) differ from zero only for definite values of the indices:

$$C^{j_0\mu_0}_{L\Lambda j_1\mu_1} B^{j_0\lambda_0}_{i\nu L} = \sum_{h=0}^{1}\sum_{\tau=-h}^{h}\int \psi^*_{j_1\mu_1} S^L_{h\tau} Y^{h\tau *}_{L\Lambda}\psi_{j_0\mu_0}\,d\mathbf{r};$$

$$S^L_{h\tau} = \Big\{ [\beta\,(c_S R_1^- + ic'_S R_2^+) + (c_V R_1^+ + ic'_V R_2^-)]\delta_{h0}$$

$$+ [\beta\sigma(c_T R_1^- + ic'_T R_2^+) + \sigma(c_A R_1^+ + ic'_A R_2^-)]\,\delta_{h1}\Big\}\,\delta_{L+\tau,\,N}$$

$$-\{[\gamma_5(ic_A R_2^- + c'_A R_1^+) - \beta\gamma_5(ic_P R_2^+ + c'_P R_1^-)]\,\delta_{h0}$$

$$+ [\alpha\,(ic_V R_2^- + c'_V R_2^+)$$

$$-\beta\alpha(ic_T R_2^+ + c'_T R_1^-)]\,\delta_{h1}\}\,\delta_{L+\tau,\,N-1}. \tag{26}$$

Here $\psi_{j_0\mu_0}$ and $\psi_{j_0\mu_1}$ are the nuclear wave functions before and after capture. The entire dependence of the right-hand side of (26) on the magnetic quantum numbers μ_0, μ_1 and Λ is contained in the Clebsch-Gordan coefficient while $B^{J_0\lambda_0}_{i\nu L}$ is independent of them.

$$R_1^{\pm} = \sum_{\omega\chi}[\delta_{\omega-\lambda_0}\delta_{\chi-\nu}F^{(n)}_{J_0\lambda_0}(r)$$

$$\pm 4\lambda_0\nu\delta_{\omega\lambda_0}\delta_{\chi\nu}G^{(n)}_{J_0\lambda_0}(r)]\,N^{h1}_{i\chi J_0\omega}; \tag{27}$$

$$R_2^{\pm} = \sum_{\omega\chi}[2\lambda_0\,\delta_{\omega\lambda_0}\delta_{\chi-\lambda}G^{(n)}_{J_0\lambda_0}(r)$$

$$\pm 2\nu\,\delta_{\omega-\lambda_0}\delta_{\chi\nu}F^{(n)}_{J_0\lambda_0}(r)]\,N^{hL}_{i\chi J_0\omega}; \tag{28}$$

$$N^{hL}_{i\chi J_0\omega} = [(qr)^{l_1}/(2l_1+1)!!]$$

$$\times\sqrt{2(2h+1)(2L+1)(2f+1)(2l_1+1)}$$

$$\times\sqrt{2J_0+1}\;C^{l_1 0}_{l0f0}\,X(l_1 l f,\, i J_0 L,\, \tfrac{1}{2}\tfrac{1}{2}h), \tag{29}$$

$$l = J_0+\omega,\ l_1 = i+\chi,\ f = L+\tau,$$

$$-h \leqslant \tau \leqslant h,\ \omega = \pm\tfrac{1}{2},$$

where $\chi = \pm 1, 2$; the parity of $(L+\tau)$ is equal to the parity of $(J_0 + i + \omega + \chi)$; q is the energy of the neutrino.

The angular distribution and polarization of x-rays are given in (25) by the tensor $P^{J_0}_{g\eta}$. Since in this case it is sufficient to consider only electric dipole radiation, following Ref. 1 $P^{J_0}_{g\eta}$ in (24) becomes

$$P^{J_0}_{g\eta} = (2g+1)(-1)^{g+\eta}\sqrt{3(2J_0+1)}\;W(J_0 I g 1;\, 1 J_0)$$

$$\times\sum_{\sigma\sigma'} i^{\sigma-\sigma'}\exp[i(\sigma-\sigma')\alpha]\;C^{1\sigma}_{1\sigma' g\varphi}\,D^g_{-\eta\varphi}(\phi,\theta,0). \tag{30}$$

*Without making specific mention of the fact, we shall always omit general factors which do not affect the correlation.

The angles θ and ϕ give the direction of the x-ray momentum $\mathbf{k}(k, \theta, \phi)$. Its linear polarization is given by the vector ϵ, whose components in a right-handed coordinate system with $\mathbf{k}$ along the z axis are

$$\varepsilon_0 = \varepsilon_z = 0, \quad \varepsilon_{\pm 1} \equiv \pm(\varepsilon_x \pm i\varepsilon_y)/\sqrt{2} = \pm e^{\pm i\alpha}/\sqrt{2}. \tag{31}$$

The angle α is computed from the x axis in the plane perpendicular to $\mathbf{k}$. $D^{g}_{\eta\varphi}(\phi, \theta, 0)$ is the familiar matrix of the irreducible representation of the rotation group.[12] It is defined here so that

$$\psi_{j\mu}(\theta\phi) = \sum_{\mu'} D^{j}_{\mu\mu'}(\phi, \theta, 0)\, \psi_{j\mu'}(0,0) \tag{32}$$

and, in particular, $Y_{lm}(\theta, \phi) = \sqrt{(2l+1)/4\pi}\, D^{l}_{m0}(\phi, \theta, 0)$. The spherical harmonic Y_{lm} agrees with Ref. 13 and differs by the factor $(-1)^m$ from Ref. 11. The possible values of g in (30) are 0, 1, and 2.

According to (30), $P^{J_0}_{00} = 1$ and

$$P^{J_0}_{1\eta} = -\frac{\sqrt{3\pi}}{2}\, \frac{J_0(J_0+1) - I(I+1) + 2}{\sqrt{J_0(J_0+1)}}\, \sigma_X Y^{*}_{1\eta}(\theta, \phi), \tag{33}$$

where $\sigma_X = 1$ or -1 for right or left circularly polarized light quanta, respectively. If the detector does not distinguish quanta of different circular polarizations, $P^{J_0}_{1\eta}$ is equal to the sum of the right-hand side of (33) for $\sigma_X = 1$ and -1, so that $P^{J_0}_{2\eta} = 0$. $P^{J_0}_{2\eta}$ gives the linear polarization of the x-rays:

$$P^{J_0}_{2\eta} = 2\sqrt{6\pi(2J_0+1)}\, W(J_0 J\, 21;\ 1 J_0)\{Y^{*}_{2\eta}(\theta, \phi) - \sqrt{15/8\pi}\,(-1)^{\eta}\,[e^{2i\alpha} D^{2}_{-\eta 2}(\phi, \theta, 0) + e^{-2i\alpha} D^{2}_{-\eta -2}(\phi, \theta, 0)]\}. \tag{34}$$

If triple correlation is not observed between the x-rays, gamma rays and nuclear spin orientation, and if the Z axis is a physically distinguished direction, then (25) contains no terms for which η, η_1, and η_0 are not zero:

$$P^{J_0}_{20} = 2\sqrt{6\pi(2J_0+1)}\, W(J_0 I\, 21;\ 1 J_0)\,\{Y_{20}(\theta, \phi) - \sqrt{6}\cos 2\alpha\, Y_{22}(\theta, 0)\}. \tag{35}$$

$P^{j_1}_{g_1\eta_1}$, which characterizes the radiative transition of the nucleus, can be represented in a form similar to (30). For a 2^L-multipole transition

$$P^{j_1}_{g_1\eta_1} = (2g_1+1) \times \sqrt{(2j_1+1)(2L+1)} \sum_{\sigma_\gamma \sigma'_\gamma} \xi^{(\sigma_\gamma - \sigma'_\gamma)/2} \exp\{i(\sigma_\gamma - \sigma'_\gamma)\alpha_\gamma\} \times C^{L_\gamma \sigma_\gamma}_{L_\gamma \sigma_\gamma g_1 \varphi}\, W(L_\gamma j_2 g_1 j_1;\ j_1 L_\gamma)\, D^{g_1}_{\eta_1 \varphi}(\phi_\gamma, \theta_\gamma, 0). \tag{36}$$

θ_γ and ϕ_γ are the directional angles of the gamma ray. The angle α_γ for gamma rays is analogous to α for x-rays and is defined in accordance with (31). The term with $\sigma_\gamma = \sigma'_\gamma = 1$ gives $P^{j_1}_{g_1\eta_1}$ for light quanta with right circular polarization; the term with $\sigma_\gamma = \sigma'_\gamma = -1$ does the same for light quanta with left circular polarization; and the sum of the terms with $\sigma_\gamma = 1$, $\sigma'_\gamma = -1$ and $\sigma_\gamma = -1$, $\sigma'_\gamma = 1$ gives $P^{j_1}_{g_1\eta_1}$ for linearly polarized light quanta. $\xi = 1$ or -1, respectively, for magnetic or electric transitions.

If only the gamma-ray angular distribution is observed, by summing (36) with respect to the polarizations we obtain

$$P^{j_1}_{g_1\eta_1} = \sqrt{(2g_1+1)(2j_1+1)(2L_\gamma+1)} \times \left[1 - \frac{g_1(g_1+1)}{2L_\gamma(L_\gamma+1)}\right] W(j_2 L_\gamma j_1 g_1;\ j_1 L_\gamma) \times C^{L_\gamma 0}_{L_\gamma 0 g_1 0}\, Y_{g_1\eta_1}(\theta_\gamma, \phi_\gamma). \tag{37}$$

The above formulas can be used to study electron capture of any order of forbiddenness. However, an unambiguous interpretation of experimental findings is obtained most simply and conveniently for allowed transitions, which we shall now consider in detail. For allowed transitions $N = 0$ and the second curly bracket in $S^{L}_{h\tau}$ gives no contribution since $L + \tau \geq 0$. In K capture we have $J_0 = 1/2$, $\lambda_0 = -1/2$ and thus $g = 0$ or 1. For an allowed transition $i = 1/2$, $\chi = -1/2$; therefore L and $L' = 0$ or 1, and, correspondingly, $\tau = 0$ or -1. In $B^{J_0\lambda_0}_{i\nu L}$ we may leave only two indices and write simply $B_{L\nu}$. For $B_{L\nu}$ we obtain

$$B_{0\nu} = (c_S \delta_{\nu,-1/2} + i c'_S \delta_{\nu 1/2}) K_S - (c_V \delta_{\nu,-1/2} - i c'_V \delta_{\nu 1/2}) K_V; \tag{38}$$

$$B_{1\nu} = \sqrt{3}\,[(c_T \delta_{\nu,-1/2} + i c'_T \delta_{\nu 1/2}) K_T - (c_A \delta_{\nu,-1/2} - i c'_A \delta_{\nu 1/2}) K_A]; \tag{39}$$

$$K_S = \int \psi^{*}_{j_1\mu_1} \beta G^{(1)}_{1/2,-1/2}(r)\, \psi_{j_0\mu_0} d\mathbf{r}, \qquad C^{j_0\mu_0}_{1\Lambda j_1\mu_1} K_T = (-1)^{1+\Lambda} \int \psi^{*}_{j_1\mu_1} G^{(1)}_{1/2,-1/2}(r)\, \beta\sigma_{-\Lambda}\, \psi_{j_0\mu_0} d\mathbf{r}. \tag{40}$$

K_V and K_A differ from K_S and K_T, respectively, by the absence of the matrix β. In nonrelativistic approximation for nucleons $K_S = -K_V$, $K_T = -K_A$. In (40) $\sigma_0 = \sigma_z$, $\sigma_{\pm 1} = (\sigma_x \pm i\sigma_y)/\sqrt{2}$. The expression for correlation does not depend directly on K_S and K_T but only on their ratio K_S/K_T. Since the electronic wavelength is larger than nuclear dimensions, the function $G^{(n)}_{1/2,-1/2}(r)$ can be taken outside of the integral sign and combined in the expression for K_S/K_T. This corresponds to neglecting quantities smaller than $(\alpha Z)^2$ compared with unity. For L capture or forbidden K capture, the correlation expression contains integrals such as (40) with $G^{(n)}_{J_0\lambda_0}(r)$ and $F^{(n)}_{J_0\lambda_0}(r)$. These quantities can be taken outside of the integral sign at a point r on the nuclear boundary. As previously, the error is $\lesssim (\alpha Z)^2$. Since for an allowed K capture $g = 0$ or 1, then, according to (30), the x-rays are not linearly polarized and the angular distribution is isotropic, but circular polarization exists. When the initial nucleus is oriented in an external field and the direction and polarization of gamma rays is not being considered, then we must set in (25) $g_1 = 0$ and thus $g_0 = g = 0.1$. In this case correlation between the circular polarization of the x-rays and the nuclear spin orientations can be expected only when the nucleus possesses dipolar polarization (by the Gorter-Rose method,[14] for example). When the nuclei are aligned (as by the method of Bleaney[15] or Pound[16]), there is no correlation. When triple correlation is observed $g_1 \neq 0$, $g_0 \neq g$ and aligned nuclei can be used.

APPENDIX II

We shall give a general formula for the conversion-transition probability of an oriented nucleus with subsequent emission of x-rays and gamma rays of definite polarization. The only difference from the problem in Appendix I is the fact that an electron jumps from a discrete level to the continuous spectrum and is not captured by the nucleus. As in Appendix I, we are not interested in the direction of emission of the particle (in this case the electron, previously the neutrino). The correlation formula is (25) with the following changes: (1) The neutrino total and orbital angular momenta i and $l\nu = i + \nu$ are replaced by the total and orbital angular momenta J and $l = J + \lambda$ of the ejected electron; (2) L and L′ are understood to mean the multipole order of the conversion transition; (3) Eq. (41) or Eq. (42) is used for $B^{J_0\lambda_0}_{J\lambda L}$. The remaining notation is the same, with the same meaning, as in Eq. (25). For electric conversion transitions we have

$$B^{J_0\lambda_0}_{J\lambda L} = \sqrt{(2J_0+1)(2L+1)/L(L+1)}\ \{\sqrt{2J_0-2\lambda_0+1}$$
$$\times W(LJJ_0-\lambda_0\tfrac{1}{2};\ J_0J-\lambda)$$
$$\times C^{J-\lambda 0}_{J_0-\lambda_0 0 L 0}[LR_3 + (L + (2J_0+1)\lambda_0 - (2J+1)\lambda)R_6]$$
$$+ \sqrt{2J_0+2\lambda_0+1}\, W(LJJ_0+\lambda_0\tfrac{1}{2};\, J_0J+\lambda)$$
$$\times C^{J+\lambda 0}_{J_0+\lambda_0 0 L 0}[LR_4 - (L - (2J_0+1)\lambda_0$$
$$+ (2J+1)\lambda)R_5]\}\, M^{\mathrm{el}}_L. \quad (41)$$

For magnetic conversion transitions we have

$$B^{J_0\lambda_0}_{J\lambda L} = -\sqrt{(2J_0+1)(2L+1)/L(L+1)}$$
$$\times [\lambda(2J+1) + \lambda_0(2J_0+1)]$$
$$\times \{\sqrt{2J_0+2\lambda_0+1}\, W(LJJ_0+\lambda_0\tfrac{1}{2};\, J_0J-\lambda)$$
$$\times C^{J-\lambda 0}_{J_0+\lambda_0 0 L 0} R_1 + \sqrt{2J_0-2\lambda_0+1}$$
$$\times W(LJJ_0-\lambda_0\tfrac{1}{2};\, J_0J+\lambda)\, C^{J+\lambda 0}_{J_0-\lambda_0 0 L 0} R_2\}\, M^{\mathrm{mg}}_L. \quad (42)$$

The radial integrals $R_1, R_2, \ldots, R_6$, used in Rose's book,[6] are taken over all space. A considerable correction is required if it is taken into account that the electronic wave function differs from a Coulomb function inside the nucleus. For K conversion, Sliv and Band[17] calculated the integrals subject to this correction and used them to obtain internal conversion coefficients.

For a conversion transition of a single definite multipole order, say L_0, we must put $L = L' = L_0$ in (25). For a mixture of ML_0 and $E(L_0+1)$, in addition to the terms for which $L = L' = L_0$ and $L = L' = L_0 + 1$ Eq. (25) contains terms with $L = L_0$, $L' = L_0 + 1$ and $L = L_0 + 1$, $L' = L_0$.

M^{el}_L and M^{mg}_L in (41) and (42), which are the matrix elements of the nuclear transition, depend on the structure of the nucleus. The correlation formulas do not contain the M_L but only their ratio $M_L/M_{L'}$. When only one value $L = L'$ plays a part the M_L drop out of the formulas. The corrections[17,18] associated with the interior of the nucleus must depend on its structure, but with the exception of E0 transitions and, in part, M1 transitions this dependence has very little effect on the results.[17]

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THE ANOMALOUS SKIN EFFECT IN THE INFRA-RED REGION

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The surface impedance of metals in the infra-red region at low temperatures has been calculated, taking into account the effect of interelectron collisions. The electrons are considered as a Fermi liquid with an arbitrary dispersion law for the quasi-particles.

THE purpose of this paper is to calculate the effect of interelectron collisions on the optical properties of metals in the infra-red region of the spectrum. To this end we shall assume that the frequency of the incident light satisfies the conditions

$$\omega l / v \gg 1 \quad (1)$$

(v is the speed of the electrons and l is the free path length) and

$$\omega\delta / v \gg 1, \quad (2)$$

where δ is the depth to which the field penetrates in the metal. The physical significance of the first condition is obvious, and the second implies that the field in which the electrons are moving may be considered to be uniform. It is also assumed that the metal has a large negative dielectric constant and a small absorption, which can be treated as a perturbation. This condition limits the validity of the formulas to the high-frequency region. The calculations will be carried out for the case of very low temperatures, where $kT \ll \hbar\omega$.

We have made no assumptions at all about the law of dispersion for the electrons, the type of scattering probability which enters into the theory, nor the magnitude of the interactions between electrons. In particular it must be emphasized that we are not assuming either that the electron dispersion is approximately quadratic, or that the electron interaction is weak.

First of all we must consider the current due to a single moving electron (or, strictly speaking, a single Fermi particle) assuming that the electrons form a Fermi liquid. In the same way that the energy $\epsilon(\mathbf{p})$ of each individual particle is given by the formula

$$\partial E = \int \varepsilon(\mathbf{p})\, \partial n(\mathbf{p})\, d\tau$$

[E is the total energy, n the distribution function

ERRATA TO VOLUME 7

Page	Reads	Should Read
533, title	Nuclear magnetic moments of Sr^{87} and Mg^{95}	Nuclear magnetic moments of Sr^{87}
645 Eq. (1)	$\ldots + \varkappa \sqrt{j_0 (j_0 + 1)}$	$\ldots - \varkappa \sqrt{j_0 (j_0 + 1)}$
647 Eq. (11)	$(L + 1) \lvert B_L^- \rvert^2 - L \lvert B_L^+ \rvert^2$	$L (L + 1) \left[\lvert B_L^- \rvert^2 - \lvert B_L^+ \rvert^2 \right]$
894 Eq. (12)	$\varepsilon_{11} = 1 - \sum \frac{\cdots}{\sqrt{\pi/\mu}}$	$\varepsilon_{11} = 1 - \sum \frac{\cdots}{\sqrt{\pi}\ \mu}$
897 Eq. (45)	$\sqrt{\pi/2}$	$\sqrt{\pi/8}$
979 Table II, heading	$\lvert E_\gamma > 50$ Mev $\lvert E_\gamma > 50$ Mev	$\lvert E_\gamma < 50$ Mev $\lvert E_\gamma > 50$ Mev
1023 Figure caption		a) $\omega < \omega_H$, b) $\omega > \omega_H$
1123 Eq. (2)	$\Gamma = \mu_2/\mu_1$	$\Gamma = \mu_2/\mu_1,\ \mu_\perp = (\mu_1^2 - \mu_2^2)/\mu_1$

ERRATA TO VOLUME 8

Page	Reads	Should Read
375 Figure caption	a) positrons of energy up to 0.4 ϵ, b) positrons of energy up to 0.3 ϵ.	a) positrons of energy up to 0.3 ϵ, b) positrons of energy up to 0.4 ϵ.
816 Beginning of Eq. (8)	$I_2^5 = (4\pi)^2 \ldots$	$I_2^2 = (4\pi)^5 \ldots$