ON THE LATERAL DISTRIBUTION OF PARTICLES IN EXTENSIVE AIR SHOWERS

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Submitted to JETP editor November 11, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 956-961 (April, 1958)

The relative role of various factors affecting the lateral divergence of charged particles in extensive air showers is considered. It is shown that the lateral distribution of electrons is completely determined by Coulomb scattering. The lateral distribution of μ mesons is to a large degree determined by Coulomb scattering and deflection in the magnetic field of the earth. The transverse momentum transferred to π and K mesons in an elementary nuclear cascade act is respectively $\leq 1.5 \times 10^8$ ev/c and $\leq 5 \times 10^8$ ev/c.

RECENTLY a number of experiments[1-5] were carried out to study in detail the lateral density distributions of electrons, μ mesons, and nuclear-active particles in extensive atmospheric cosmic ray showers in the lower layers of the atmosphere. In addition, the lateral distribution of electrons and μ mesons was investigaged for distances up to several hundred meters from the shower axis.

In connection with these experiments, it is expedient to consider the question of the relative effect of various factors determining the lateral divergence of shower particles.

Shower particles may deviate from the flight direction of the primary particle which produced the entire shower because of the following reasons:

I. The angular divergence of secondary particles: (a) in the elementary nuclear cascade act and (b) in spontaneous decay acts.

II. Coulomb scattering (of charged particles) by air nuclei.

III. Lorentz forces determined by the earth's magnetic field (acting on the charged particles).

After the discovery of the so-called "anomalous" shower width[6] it was hypothesized[7] that the basic cause of the wide lateral divergence of shower particles is reason I(a). However, a more detailed analysis of the effects of the second and third reasons shows that this notion must be reconsidered.

Let us first consider the lateral distribution of electrons. Electrons in an extensive atmospheric shower originate in electron-photon cascade showers generated by γ-quanta from the decay of π^0 mesons. The latter are formed in the elementary nuclear-cascade acts. As was shown in Ref. 7, from the point of view of the laws of conservation of energy and momentum, the maximum possible π^0-meson emission angles in these acts are sufficiently large, so that the cores of the individual cascade showers could deviate from the axis of the entire shower by a distance on the order of several hundred meters, creating at these distances electron densities which agree with experimental data.

According to the calculations of Moliere[8] and Eyges,[9] available until recently, the electrons in electron-photon cascade showers could not occur in considerable quantities at such large distances. However, during the past few years Nishimura and Kamata,[10] and also Greisen,[10] gave a more accurate solution of the problem on the lateral distribution of electrons in electron-photon cascade showers, which leads to a different qualitative conclusion.

Let us compare the experimental lateral electron distribution[1-5] with theoretical distributions according to Nishimura and Kamata, for the various degrees of s-development of an electron-photon cascade shower. In spite of the fact that the development of the electron-photon component in a real shower is different on the whole than in an electron-photon cascade shower, such a comparison of lateral distributions is justifiable, since the latter are determined primarily by the energy spectrum of electrons and depends quite weakly on the "previous history" of the electron-photon cascade shower.

The lateral distribution of electrons from Refs. 1 — 5 is shown in Fig. 1. The unit of length is $r_1 = E_S X_0/\beta$ (E_S = 19 Mev, β = 72 Mev, and X_0 = 270 m above sea level). The theoretical curves are shown for the parameter s = 1, 1.25, and 1.4. All the curves are normalized for areas under them. The experimental points fit well the theoretical curve for s = 1.25 up to distances r = 3 (Ref. 5) and do not depart appreciably from the curve with

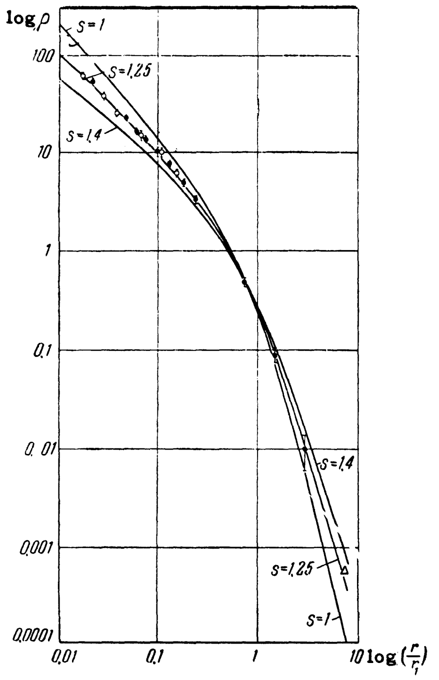

FIG. 1. Comparison of experimental lateral electron distribution in extensive atmospheric showers[1-5] with the theoretical electron distribution in a photon-electron shower according to Nishimura and Kamata.[10] Δ – experimental data of Ref. 1; o, • – data of Ref. 5.

s = 1.25 for larger distances up to r = 7 (Ref. 4).

The agreement of the experimental and theoretical curves for one definite value of s in such a wide range of distances is, of course, not accidental and it is natural to suppose that this takes place because the energy spectrum of shower electrons coincides with the energy spectrum of the electron-photon cascade shower of age s, whereas the lateral divergence of electrons is determined entirely by Coulomb scattering. Unfortunately, the electron energy spectrum has not been measured to date, but it is worth noting that the electron absorption coefficient, obtained by investigating the barometric effect and altitude behavior of the shower, agrees with the value s = 1.2.[5,11] Thus, if the energy spectrum of the electrons in the shower coincides with the energy spectrum of electrons in an electron-photon cascade shower of age s = 1.2, then the experimentally observed extensive lateral divergence of the electrons can be explained only by the Coulomb scattering of the electrons in the individual electron-photon cascade showers, without assuming that the cores of these showers leave the basic core of the extensive atmospheric shower.*

*As was shown by Cocconi,[12] the earth's magnetic field does not exert any substantial effect on the lateral distribution of electrons for showers observed in the lower layers of the atmosphere.

Let us consider now the lateral distribution of μ mesons. μ mesons originate from the decay of $\pi^{\pm}$ and $K^{\pm}$ mesons in the upper layers of the atmosphere. According to experimental data[13] μ mesons originate in extensive atmospheric showers at altitudes of 5 to 20 km above sea level. μ mesons cover large distances from the place of origin to the observation level, experiencing practically no interactions involving considerable energy losses, and losing energy slowly by ionizing air atoms. The lateral distribution of low-energy μ mesons was investigated by a number of workers.[3-5] According to the experimental data, the meson density falls off considerably slower than the electron current density even over large distances, on the order of hundreds of meters from the shower axis.

The presence of a large free path brings us to the important role played in the lateral divergence of μ mesons both by the Coulomb scattering from air nuclei and by deflections in the earth's magnetic field, even if the muons have a relatively high energy. Inasmuch as $\rho_{\mu}(r)$, even for the maximum distances from the axis investigated in the above references, does not drop off faster than $1/r^2$, we shall not consider quantitatively the Coulomb scattering effect and deflections in the earth's magnetic field. Let us limit ourselves to the evaluation of the mean square deflection $\sqrt{\overline{r_s^2}}$ of μ mesons with energy E by multiple scattering, and to their deflection in a magnetic field r_M. It is obvious that:

$$\overline{r_s^2} = \int_0^H \frac{h^2 E_s^2}{[E+\beta t_0(1-e^{-\alpha h})]^2}\, t_0 \alpha e^{-\alpha h}\, dh. \qquad (1)$$

Where, H is the altitude above the observation level at which the μ mesons are produced, t_0 the depth of observation level in shower units, E_s the parameter of multiple-scattering theory, β the ionization losses calculated for one shower unit in air (E_s = 21 Mev, β = 72 Mev), α the coefficient in the barometric formula: $\alpha = (1/7000)\ \mathrm{m}^{-1}$. For r_m we have the following expression:

$$r_m = \int_0^H h\,dh/\rho; \quad \rho = E + \beta t_0 (1 - e^{-\alpha h})/300\,\mathcal{H}, \qquad (2)$$

where $\mathcal{H}$ is the component of the earth's magnetic field perpendicular to the axis of the extensive atmospheric shower. Assuming the altitude of μ meson generation to be H = 10 km above sea level, and considering that the mean energy of the μ mesons at large distances from the shower axis is close to the minimum energy recorded by the detector, equal in the above experiments to 3×10^8 ev, we obtain the values of $\sqrt{\overline{r_s^2}}$ and r_m shown below.* (In doing this we can estimate only the min-

*The data are for sea level.

imum the experimental value of $\sqrt{\overline{r_s^2}}$).

$\sqrt{r^2_{exp.}}$	$\sqrt{\overline{r_s^2}}$	r_m
$\geqslant 400$ m	300 m	200 m.

Thus, the lateral distribution of low-energy μ mesons, ordinarily investigated in experiments, is determined essentially by the Coulomb scattering of the μ mesons and by their deflection in the earth's magnetic field. The angular divergence of the μ-meson "ancestors" in the elementary nuclear cascade acts is also evidently an essential factor in the lateral divergence of the μ mesons. However, the relative role of all these factors cannot be definitely determined at the present time without additional experimental data on the lateral distribution of μ mesons, their energy spectrum, and also the altitude of their production in the atmosphere.*

It is especially interesting to analyze data on the lateral distribution of high-energy μ mesons, since such mesons originate in elementary nuclear-cascade acts of higher energies. Such data were obtained by George[14] and Cocconi.[15] However, these are qualitative in nature, since the lateral distribution of the μ mesons was investigated by the correlation-curve method. Recently Andronikashvili and Bibilashvili,[16] using the method of correlated hodoscopes, obtained the lateral distribution of μ mesons of energy $E \geq 1.5 \times 10^{10}$ ev in extensive showers at sea level. The experimental data[16] are shown in Fig. 2. As can be seen, the lateral distribution is investigated up to distances where the drop-off in the function $\rho_\mu(r)$ becomes $\sim 1/r^3$. Consequently, the experimental data enable one to evaluate the integral $\int \rho_\mu(r) \times 2\pi r dr$, corresponding to the full number of μ mesons in the shower, since the integral does not diverge at the upper limit. Thus, it is meaningful to make a more detailed examination of the effect of Coulomb scattering and of the magnetic field on the lateral divergence of μ mesons.†

Let us find the lateral distribution function $\rho_\mu(r)$ for the case of μ-meson divergence due to Coulomb scattering and the deflection in the earth's magnetic field, neglecting factor I. The lateral distribution of multiply-scattered μ mesons with

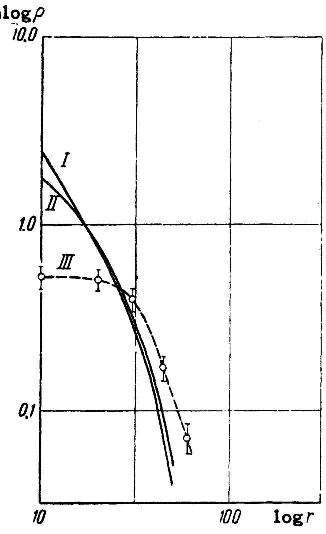

FIG. 2. Lateral distribution of μ mesons in extensive atmospheric showers. I – theoretical curve $\rho_\mu(r_1 > E_{min})$; $\gamma = 0.7$, H = 10 km. II – the same for $\gamma = 1.3$, H = 10 km; III – experimental data of Ref. 16.

energies $E \gg \beta t_0(1 - e^{-\alpha H})$, has the form of a Gaussian distribution

$$\rho_\mu(r, E)\, dE = C(E) \exp\{-r^2/\overline{r^2}(E)\}\, dE,$$

where $\overline{r^2}(E)$ is determined by the above-mentioned formula. C is determined from the condition

$$\int \rho_\mu(r, E)\, dE\, 2\pi r dr = F(E)\, dE,$$

if

$$F(E)\, dE = AE^{-(\gamma+1)} dE,$$

then

$$C = AE^{-(\gamma+1)}/\pi\overline{r^2}(E) = AE^{-(\gamma-1)}/\pi B(H, t_0),$$

since

$$\overline{r^2} = E^{-2} B(H, t_0).$$

The action of the earth's magnetic field results in a lateral sorting out of the μ mesons according to their energy and charge. If the shower axis is vertical, the positive μ mesons are displaced to the east, and the negative ones in the opposite direction. At the same time, all μ mesons of energy E have the same deflection, $r_m(E) = (H^2/2E) \times 300\mathcal{H}$. Thus, the simultaneous action of the Coulomb scattering and the deflection in the earth's magnetic field can be taken into account in the following expression for $\rho_\mu(r)$, which gives the lateral distribution in the west-east direction.*

$$\rho_\mu(r_1 > E_{min}) = \int_{E_{min}}^{\infty} \frac{A}{\pi B(H, t_0)} E^{-(\gamma-1)} \exp\left\{\frac{-(r - r_m)^2 E^2}{B(H, t_0)}\right\} dE$$

*It is necessary to consider the sensitivity of the values $\sqrt{\overline{r_s^2}}$ and r_m to the altitude H of μ meson production. The value of r_m is quite sensitive to H. According to (2), without taking into account ionization losses, $r_m \sim H^2$. The value of $\overline{r_s^2}$ is considerably less sensitive to H when $\alpha H > 1$: $\overline{r_s^2} \sim |(2/\alpha^2) - H^2 e^{-\alpha H}|$. However, $\overline{r_s^2} \sim (\alpha H)^3$ when $\alpha H < 1$.

†It is impossible to calculate $\overline{r^2}$ from experimental data for this would require us to investigate $\rho_\mu(r)$ up to distances corresponding to a $1/r^5$ drop off.

*The experimental setup was oriented in this direction.

$$+\int_{E_{\min}}^{\infty}\frac{A}{B(H,t_0)}E^{-(\gamma-1)}\exp\left\{\frac{-(r+r_m)^2E^2}{B(H,t_0)}\right\}dE.$$

In Fig. 2 are shown the theoretical curves calculated for various assumptions concerning γ (Ref. 17), for $E_{min} = 1.5 \times 10^{10}$ ev, H = 10 km, and the latitude of Tbilisi. The comparison of the theoretical and experimental curves shows that the lateral divergence of high-energy μ mesons is essentially dependent on factors II and III.*

Using the cited data we can estimate of the value of the transverse momentum acquired by the secondary particles, the $\pi^{\pm}$ and $K^{\pm}$ mesons, in elementary nuclear-cascade acts. The transverse momentum of s secondary particle of energy E is $p = (E/c)\vartheta$, where ϑ is the emission angle of the secondary particle in the laboratory system. If the μ-meson energy spectrum has the form $F(E)dE = AE^{-(\gamma+1)}dE$ for $E > E_{min}$, then, neglecting ionization losses and the μ-meson decay probability (this is true for $E \gg 10^{10}$ ev), one can obtain the differential spectrum of the decayed $\pi^{\pm}$ mesons and $K^{\pm}$ mesons:

$$f_{\pi(K)}=\frac{A}{\left[1-\left(\frac{m}{M}\right)^2\right]\gamma E}\left(E_{\min}^{-\gamma}-E^{\gamma}\right)$$

$$\text{for } E_{\min}\leqslant E\leqslant\left(\frac{M}{m}\right)^2E_{\min};$$

$$f_{\pi(K)}=\frac{A}{\left[1-\left(\frac{m}{M}\right)^2\right]\gamma}\left[\left(\frac{m}{M}\right)^{-2\gamma}-1\right]E^{-(\gamma+1)}$$

$$\text{for } E\geqslant\left(\frac{M}{m}\right)^2E_{\min},$$

Where M is the mass of $\pi^{\pm}$ or $K^{\pm}$ meson, m the mass of the μ meson, and E_{min} the minimum energy of the recorded μ mesons (1.5×10^{10} ev in the experiments of Ref. 16).

Let us determine now the value of energy E* which is possessed by more than half of decayed $\pi^{\pm}$ (or $K^{\pm}$) mesons. For the case of $\pi^{\pm}$ mesons, $E^* = 2E_{min} = 3 \times 10^{10}$ ev. For the case of $K^{\pm}$ mesons, $E^* = 7E_{min} = 10^{11}$ ev, $p^* = E^*\vartheta/c$ and since $\vartheta < 50\ m/10^4\ m = 5 \times 10^{-3}$, then $p^*_{\pi} < 1.5 \times 10^8$ ev; and $p^*_K < 5 \times 10^8$ ev.†

*The observed discrepancy between the experimental and theoretical curves can be understood if one takes into account the specific geometry of the setup used in Ref. 16. The detector of high-energy μ mesons was placed at a depth of 27 meters underground. The apparatus to locate the shower axis was placed on the surface of the earth. The inclination of the shower axis from the vertical could cause a systematic error in the determination of the distance from the shower axis to the detector, on the order of half of the indicated depth.

†The transverse momentum acquired by the μ meson on account of factor Ib, has an order of magnitude of $\mu_{\pi}c/2$ for the $\pi \to \mu + \nu$ decay; for the $K \to \mu + \nu$ decay, $\mu_K c/2 = 2\mu_{\pi}c$.

In conclusion it is necessary to note that the Coulomb scattering must also be taken into account in analyzing data for the lateral distribution of charged nuclear-active particles in extensive atmospheric showers.

The author wishes to express his thanks to S. B. Vernov and G. T. Zatsepin for discussion of the above questions.

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Translated by H. Kruglak
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