ANGULAR ANISOTROPY IN $\pi^+ - \mu^+ - e^+$ DECAY OBSERVED IN A PROPANE BUBBLE CHAMBER

A. I. ALIKHANIAN, V. G. KIRILLOV-URGIUMOV, L. P. KOTENKO, E. P. KUZNETSOV, and Iu. S. POPOV

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor December 12, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 1101-1109 (May, 1958)

The angular anisotropy in $\mu^+ - e^+$ and $\pi^+ - \mu^+$ decays was measured. The dependence of the $\mu^+ - e^+$ decay anisotropy on the electron energy is not inconsistent with the two-component-neutrino theory. No angular anisotropy is displayed by $\pi^+ - \mu^+$ decays.

1. INTRODUCTION

THE experimental data accumulated on angular correlation in $\mu - e$ and $\pi - \mu$ decays is currently of interest. In the former case, it can be used to verify the two-component neutrino theory and to understand the nature of the depolarization of the μ mesons remaining in media. In the latter case it can essentially establish the presence or absence of the correlation itself.

As was shown by Lee and Yang,[1] Landau,[2] and others[3,4] who have developed the theory of the two-component neutrino, the degree and character of the angular anisotropy in $\mu - e$ decay depend substantially on the energy of the decay electrons. Let ϵ be the decay-electron energy in units of maximum energy in the $\mu - e$ decay. The angular distribution is then, according to the two-component neutrino theory,

$$dN \sim 2\varepsilon^2 [(3 - 2\varepsilon) + \xi \cos\vartheta (2\varepsilon - 1)]\, d\varepsilon\, d\Omega, \qquad (1)$$

where ϑ is the angle between the momentum of an electron emitted inside a solid angle $d\Omega$ and the spin of the μ meson (in the $\pi^+ - \mu^+ - e^+$ decay with stoppage of the π^+ meson the spin is parallel to the momentum of the μ^+ meson), and ξ is a parameter that takes the character of the interaction into account. Integration of (1) over all electron energies yields

$$dN \sim (1 + {}^1/_3\, \xi \cos\vartheta)\, d\Omega = (1 + a \cos\vartheta)\, d\Omega. \qquad (2)$$

The theoretical value of ξ ranges from -1 to $+1$ and a lies accordingly between $-{}^1/_3$ and $+{}^1/_3$. The coefficient a of formula (2) is a parameter of the theory, while the experimentally-determined coefficient depends on the degree of depolarization of the μ mesons. We shall therefore denote the experimental value of this coefficient by A, and the correction for depolarization by a.

We have studied the angular anisotropy in the $\pi^+ - \mu^+ - e^+$ decay with discrimination by decay-electron energy.

2. EXPERIMENTAL CONDITIONS

The $\pi^+ - \mu^+ - e^+$ decays were registered in a 750-cc propane bubble chamber.[5] The chamber was exposed in a π^+-meson beam using the phasotron of the Joint Institute for Nuclear Research. The π^+ mesons were formed on an external polyethylene target by protons with energies of ~ 660 Mev. The target was 70 cm long, making it possible to increase the π^+-meson yield through the use of the two reactions of π^+-meson creation, $p + p \rightarrow \pi^+ + p + n$ and $p + p \rightarrow \pi^+ + d$.

The π^+ mesons, with an approximate energy

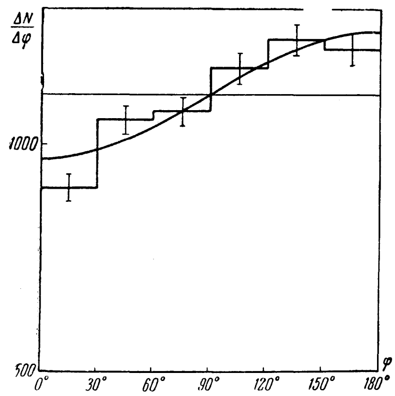

FIG. 1. Angular distribution of electron momenta relative to the μ-meson momentum in $\pi^+ - \mu^+ - e^+$ decay. The abscissas indicate the projection of the spatial angle φ between the μ-meson and the electron momenta, and the ordinate indicates the number of electrons over the interval of the angle φ. The smooth curve corresponds to $dN \sim [(1 + A(\pi^2/16) \cos\varphi)\, d\varphi]$ at $A = -0.22 \pm 0.03$. Horizontal line – isotropic distribution. $N = 6670$.

of 180 Mev, were aimed with a deflecting magnet into a collimator, and were then slowed down by a copper absorber so that they were stopped inside the chamber. The total distance from target to chamber was 7 m.

To prevent distortion of the angular distribution by precession of the stopped π^+ mesons in the stray field of the accelerator, the chamber was shielded and the field inside the chamber did not exceed one gauss. The accelerator beam was switched off during the dead time of the chamber. This made possible, in particular, to regular visual monitoring of the quality of the tracks in the chamber. Approximately 8,000 photographs were made during the time of chamber operation, and 6,670 $\pi^+ - \mu^+ - e^+$ decay events were registered.

3. ANGULAR ANISOTROPY IN $\mu^+ - e^+$ DECAY. DEPENDENCE OF ANGULAR ANISOTROPY ON THE DECAY-ELECTRON ENERGY

We have measured the projections of the spatial angles between the momenta of the π^+ meson and the electron on the plane of the camera film. Compared with the measurements of spatial angles, the measurement of angle projections, in addition to being relatively simple, has also the advantage that it becomes unnecessary to introduce corrections for the angle distortion in the propane (the orthogonal projections of spatial angles do not change in a refracting medium). The error in the measurement of the angle was 1 to 3°, depending on the length of the track. To eliminate possible edge effects, which can distort the angular distribution, the only events analyzed were those in which the μ^+ meson was not closer than 3 mm from the wall of the chamber. Events identified as decay of stopped μ^+ mesons may include μ^+ mesons scattered prior to stopping or a π^+ meson decaying in flight. An estimate shows that these extraneous events comprise not more than 1% of the total number of $\pi^+ - \mu^+ - e^+$ decays.

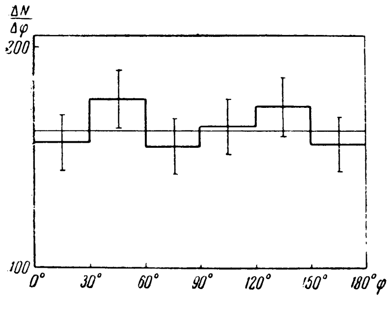

FIG. 2. The same as in Fig. 1 for the group of $\pi^+ - \mu^+ - e^+$ decays with soft electrons; N = 980.

In the case when the projections of the spatial angles on a plane is measured, the distribution (2), as can be readily shown (see Appendix), is transformed into

$$dN \sim [1 + a(\pi^2/16)\cos\varphi]\, d\varphi, \qquad (3)$$

provided the directions of the μ-meson momenta are distributed in space isotropically (as will be shown below, this condition is satisfied for $\pi^+ - \mu^+ - e^+$ decays). Here φ is the projection of the spatial angle ϑ. Figure 1 shows the distribution of the projections of the angles between the initial momenta of the μ^+ meson and the electron for 6,670 events of $\pi^+ - \mu^+ - e^+$ decay. It is seen that the experimental distribution is well approximated by formula (3).

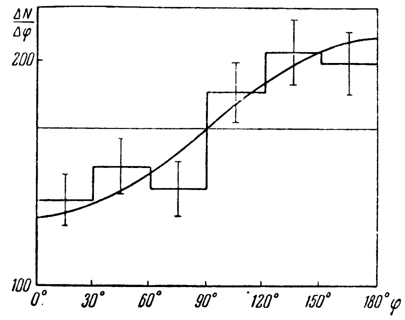

FIG. 3. The same as in Fig. 1 for the group of $\pi^+ - \mu^+ - e^+$ decays with hard electrons. The smooth curve corresponds to $dN \sim (1 + 0.715\, A \cos\varphi)\, d\varphi$ at $A = -0.33 \pm 0.07$; N = 1023.

The coefficient A, obtained from the "forward-backward" ratio (the ratio of the number of electrons emitted in the 90 — 180° interval to the number of electrons emitted between 0 and 90°) is $A = -0.22 \pm 0.03$ (Ref. 6).

The error in A is the statistical rms error. It must be noted that had we measured the spatial angles, we would have decreased the error in A by merely 22%.

To estimate the correctness of the two-component neutrino theory, it would be interesting to separate the various groups with decay electrons of different energies from the total number of $\pi^+ - \mu^+ - e^+$ decay events, and to compare the anisotropy for these groups with that predicted by the two-component neutrino theory. In the group of events with low-energy electrons, we have selected such decay events, in which, over a propane path 2 cm long, the electrons experienced multiple Coulomb scattering at an angle not less than 2 to 3°, corresponding roughly to a limiting energy of 30 to 40 Mev for these events. A total of 980 such events was found. The histogram for the projections of the angles between the momenta of the μ^+ meson and the electron, plotted for these events, is shown in Fig. 2. It is seen that we have here complete isotropy. If we calculate formally the coefficient A, using the above method, we find

$$A_s = -0.014 \pm 0.08.$$

Figure 3 shows the histogram for $\pi^+ - \mu^+ - e^+$

decays with hard electrons. This group includes events in which the electrons were scattered not more than 2 mm over 3 to 4 cm of their path projections. A total of 1023 such events was found. Such a selection does not include events with decay electrons that have a small projection, the numerical coefficient in formula (3) is not $\pi^2/16 = 0.616$, but greater. Calculations show that it amounts to 0.715. The coefficient A for events with hard electrons, found from the "forward-backward" ratio, is

$$A_h = -0.33 \pm 0.07.$$

The isotropy for cases with soft electrons, and the increased anisotropy compared with the integral histogram for the cases with hard electrons, are in qualitative agreement with the concepts of the theory of the two-component neutrino, according to which the anisotropy should lead to an increase in the decay electron energy.

For a quantitative comparison it is essential, first of all, to take into account the depolarization of the stopped μ^+ mesons in the propane. We shall not carry out a quantitative comparison for cases with hard electrons, owing to the difficulties in estimating their energy, and also owing to the small difference (taking the errors into account) between the coefficient A_s and the integral coefficient A, which equals -0.22 ± 0.03. We took account of the depolarization in propane in the manner used by Chadwick, Wilkinson et al.,[7,8] who introduced a depolarization correction for emulsion. It is known from experiments in which the angular correlation in μ^+-e^+ decay is measured with the aid of counters, that maximum anisotropy, which incidentally is of constant value, is observed when a μ^+ meson decays in graphite and in certain pure metals, for example aluminum and magnesium.[9,10] Furthermore, this anisotropy is the same for all these materials. It can be assumed that there is no depolarization in these substances. However, the coefficient A obtained in these experiments cannot be identified with the parameter a of Eq. (1), for the degree of polarization of the μ^+-meson beam is not known exactly, nor do we know the cutoff limit of the decay-electron spectrum on the low-energy side. Depolarization in propane can be accounted for by measuring the coefficients A for carbon and propane, in the same setup, by the counter method. Such measurements are available in Ref. 10, which gives for carbon $A = -0.27 \pm 0.02$ and for propane $A = -0.18 \pm 0.015$.

The ratio of these two quantities will characterize the change in the coefficient A due to depolarization in propane. To introduce a correction for the depolarization in our measurements, it is necessary to multiply our measured values of the coefficient A by this ratio. We then obtain for the entire electron spectrum

$$a = A(0.27/0.18) = -0.33 \pm 0.06.$$

According to the theory of the two-component neutrino, a can range from -0.33 to $+0.33$, i.e., there is agreement with experiment. Introducing the depolarization correction for the group of decays with soft electrons (980 events), we get

$$a_s = A_s(0.27/0.18) = -0.02 \pm 0.11.$$

To calculate the theoretical values of the parameter a_s, it is necessary to integrate the distribution (1) to the limiting energy of the soft electrons. For a more accurate determination of the limiting energy, we selected approximately 200 events of $\pi^+-\mu^+-e^+$ decays with an electron track length not less than 2 cm. We then calculated the ratio of the number of events with soft electrons to the number of remaining events, and obtained a value of 0.375. Then, integrating (1) over the angles, we obtained the distribution over the electron energies only: $dN(\epsilon) \sim \epsilon^2(3-2\epsilon)d\epsilon$ (which coincides with the well-known Michel formula with the parameter $\rho = 0.75$). The limiting energy was found from the relation

$$\int_0^{\varepsilon_{lim}} dN(\varepsilon) \Big/ \int_1^{\varepsilon_{lim}} dN(\varepsilon) = 0.375.$$

We obtained $\epsilon_{lim} = 0.56$ or $E_{lim} = 30$ Mev (at $E_{max} = 53.6$ Mev).

Next, integrating (1) to ϵ_{lim}, we get

$$dN \sim (1 + a_s \cos\vartheta)\, d\Omega$$

for $a = -0.33$, and $a_s = +0.07$. Our value, $a_s = -0.02 \pm 0.11$, agrees with the theoretical within experimental error. Thus, the quantitative reduction in angular anisotropy, observed with diminishing electron energies in the μ^+-e^+ decay, is in agreement with the predictions of the two-component neutrino theory.

The table lists data of five measurements of the angular $\mu-e$ anisotropy in $\pi^+-\mu^+-e^+$ decay in propane bubble chambers. We took into account the correction for depolarization (fourth and fifth column), and averaged the parameter a from the five sets of data. In the next to the last line we give the average value of a obtained by Wilkinson[8] from eight investigations in emulsion. There is good agreement between the values of the parameter a obtained in emulsion and in a propane chamber. This indicates, in particular,

Method	Reference	A	a	ξ	Number of events
Propane chamber	[11]	-0.18 ± 0.05	$-0.27\ \pm0.08$	-0.81 ± 0.24	1188
"	[12]	-0.19 ± 0.04	-0.285 ± 0.07	-0.85 ± 0.20	3500
"	[13]	-0.16 ± 0.03	$-0.24\ \pm0.05$	-0.72 ± 0.17	3734
"	[14]	-0.19 ± 0.03	-0.285 ± 0.05	-0.85 ± 0.16	6760
"	Our data	-0.22 ± 0.03	$-0.33\ \pm0.06$	-0.99 ± 0.17	6670
	Average	-0.19 ± 0.02	$-0.28\ \pm0.03$	-0.84 ± 0.09	21802
Ilford G-5 emulsion	Average[8]		-0.287 ± 0.039	-0.87 ± 0.12	16000
Average values obtained with emulsions and propane chambers			-0.283 ± 0.023	-0.85 ± 0.07	

that correct allowance has been made for the depolarization in media. The last line gives the average values of the parameters a and ξ obtained with propane chambers and with emulsions. The parameter a does not exceed the maximum value (with a minus sign) called for by the theory of the two-component neutrino, i.e., -0.33, and accordingly ξ does not exceed -1.

4. ANGULAR DISTRIBUTION IN $\pi^+ - \mu^+$ DECAY

Figure 4 shows a distribution histogram of the projections of the angles between the π^+- and μ^+-meson momenta for the same 6,670 $\pi^+ - \mu^+ - e^+$ decay events. It is seen that the distribution is isotropic between 0 and 150°. The smaller number of particles in the last angle interval is due to oversights. We have noted that the greater the number of extraneous particles in the chamber, the more oversights occur for decays in which the projection of the angle between the π^+ and μ^+-meson momenta is close to 180°. If a series of measurements is made with small chamber loading, isotropy is observed over the entire interval of angles, i.e., from 0 to 180°. The distribution of the angles between the π^+ and μ^+-meson momenta can thus be assumed isotropic.

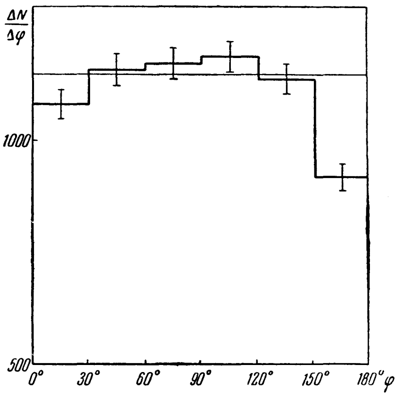

FIG. 4. Angular distribution of μ-meson momenta relative to the π-meson momentum in $\pi^+ - \mu^+ - e^+$ decay. The abscissas indicate the projection of the spatial angle between the π- and μ-meson momenta at the decay point, and the ordinates the number of mesons within the angle interval. The horizontal line represents isotropic distribution. N = 6670.

Since in our case of the momentum of the π^+ meson did not deviate more than 18° from the beam axis at the end of the range of the π^+ mesons, it follows from the isotropy in the angular distribution between the π^+ and μ^+-meson momenta that the distribution of the angles between the momentum of the μ^+ meson and the axis of the beam is also isotropic in $\pi^+ - \mu^+$ decays.

CONCLUSIONS

1. An increase in angular anisotropy with electron energy is observed in $\mu^+ - e^+$ decay; this increase does not contradict the two-component neutrino theory.

2. The value of the coefficient A in the distribution of the angles between the μ^+-meson and the electron momenta, obtained in the registration of $\pi^+ - \mu^+ - e^+$ decays in a propane chamber, is

$$A = -0.22 \pm 0.03.$$

The value of this parameter, averaged over five investigations in propane chambers, with correction for depolarization, is

$$a = -0.28 \pm 0.03,$$

which agrees with the average values of the parameter, $a = -0.287 \pm 0.039$, obtained in nine emulsion investigations.[8] The average a obtained in both propane and emulsion is

$$a = -0.283 \pm 0.023$$

and correspondingly, the value of ξ is

$$\xi = -0.85 \pm 0.07.$$

3. The angles between the meson momenta are

isotropically distributed in $\pi^+ - \mu^+$ decay.

We take this occasion to thank Professor V. P. Dzhelepov for the opportunity to carry out the experiment with the phasotron of the Joint Institute for Nuclear Research, to B. A. Dolgoshein for useful discussions, to L. A. Kuzin, A. V. Samoilov, and F. M. Sergeev for participating in the processing of the material, and to A. A. Bedniakov for helping in the work with the phasotron.

APPENDIX

CONNECTION BETWEEN SPATIAL ANGLE DISTRIBUTION AND THE DISTRIBUTION OF THE PLANE PROJECTIONS OF THE ANGLES IN $\mu-e$ AND $\pi-\mu-e$ DECAYS

1. Isotropic Distribution in Projection ($\pi-\mu$ Decays)

Let a beam of particles move in the direction Ox and decay at the point 0, the angular distribution of the decaying particles being isotropic. We then have (Fig. 5)

$$dN = \mathrm{const}\cdot d\Omega = \mathrm{const}\cdot \sin\theta\, d\theta\, d\varphi.$$

In observing the decay along the z axis, i.e., in projecting the spatial distribution on the XY plane, we fix the projection of the spatial angle ϑ between the initial direction Ox and the momentum of the particle OA. Here the projection of the angle ϑ coincides with the angle φ. The observer, so to speak, integrates over the angle

$$dN(\varphi) = \mathrm{const}\cdot d\varphi \int_0^\pi \sin\theta\, d\theta = 2\,\mathrm{const}\cdot d\varphi = \mathrm{const}\cdot d\varphi.$$

Consequently, we also have an isotropic angular distribution in the projection.

2. Distribution $dN \sim (1 + a\cos\vartheta)\,d\Omega$ In Projection on a Plane ($\mu-e$ Decays).

The distribution of the momenta of the decaying particles about Ox is now expressed by the formula

$$dN \sim (1 + a\cos\vartheta)\,d\Omega.$$

The problem consists of expressing this distribution in terms of the projection of the angle ϑ on the plane XY, i.e., in terms of φ (see Fig. 5).

It is easy to show that the connection between the angles, θ, φ, and ϑ is given by

$$\sin\theta\cos\varphi = \cos\vartheta.$$

Inserting this expression into the formula for the distribution over ϑ and integrating again over θ, we get

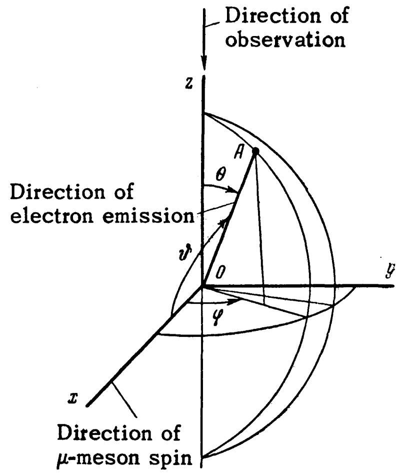

FIG. 5

$$dN(\varphi) \sim \int_0^\pi (1 + a\sin\theta\cos\varphi)\sin\theta\, d\theta\, d\varphi$$
$$= 2\,[1 + (a\pi/4)\cos\varphi]\, d\varphi,$$
$$dN(\varphi) \sim [1 + (a\pi/4)\cos\varphi]\, d\varphi.$$

3. Distribution $dN \sim (1 + a\cos\vartheta)\,d\Omega$ in Plane Projection, Subject to the Condition that the Direction from which ϑ is Measured is Isotropically Distributed in Space ($\pi-\mu-e$ Decays)

The problem is as follows: The π meson moves along Ox, and the decay μ mesons are emitted isotropically. The momenta of the decay electrons have a spatial distribution $dN \sim (1 + a\cos\theta')\,d\Omega$ relative to the direction of the μ meson (see Fig. 6). The observer sees the pic-

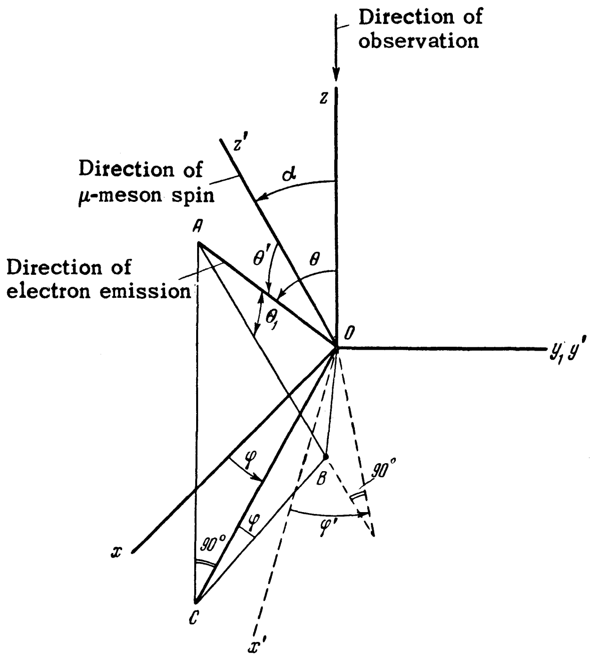

FIG. 6

ture in the XY plane. It is required to find the distribution of the projections of the spatial angles θ' relative to the projection of the μ-meson momentum on the XY plane.

Since we always measure the projection of the angle between the meson and the electron, we can consider the projection of the momentum of the meson to be directed at all times along Ox (the μ meson moves in the ZX plane), and we have the picture shown in Fig. 6.

In x′y′z′ coordinates the electron distribution is

$$dN = \mathrm{const}\cdot(1 + a\cos\theta')\,d\Omega.$$

But now the momentum of the μ meson makes an angle $(90-\alpha)$ with the plane of observation.

It is necessary to find the connection between the angles α, φ, θ, and θ'. Then, integrating over the corresponding angles (taking into account the isotropic distribution of the μ-meson momenta in space), we find the dependence of the electron emission on the angle φ. This dependence is of the form

$$\cos\theta' = \sin\theta\sin\alpha\cos\varphi + \cos\theta\cos\alpha.$$

Putting AO = a, AB = b, CB = c, OC = d, and AC = f, and noting that $\angle$ OAD = θ' and $\angle$ OCB = φ, we write for OB

$$a^2 + b^2 - 2ba\cos\theta' = c^2 + d^2 - 2cd\cos\varphi,$$

where

$$b = f/\cos\alpha = a\cos\theta/\cos\alpha;$$

$$c = f\tan\alpha = a\cos\theta\tan\alpha;\quad d = a\sin\theta.$$

Solving this equation with respect to $\cos\theta'$, we obtain the relation given above between the angles α, φ, θ, and θ'. It must be noted that the angle α assumes values from 0 to π and, in addition, we assume an isotropic distribution of the μ-meson momenta in space. To take this factor into account we must multiply the entire distribution by $\sin\alpha$:

$$dN = \mathrm{const}\cdot[1 + a(\cos\theta\cos\alpha + \sin\theta\sin\alpha\cos\varphi)]\sin\alpha\,d\alpha\,d\Omega,$$

where $d\Omega = \sin\theta\,d\theta\,d\varphi$. Here the angles α and θ vary from 0 to π. Integrating over θ and α we get

$$\int_0^\pi\int_0^\pi \sin\alpha\sin\theta\,d\theta\,d\varphi = 4;$$

$$a\int_0^\pi\int_0^\pi \cos\theta\cos\alpha\sin\theta\sin\alpha\,d\theta\,d\alpha = 0;$$

$$a\cos\varphi\int_0^\pi\int_0^\pi \sin\theta\sin\alpha\sin\theta\sin\alpha\,d\theta\,d\alpha = (a\pi^2/4)\cos\varphi,$$

i.e.,

$$dN \sim [1 + (\alpha\pi^2/16)\cos\varphi]\,d\varphi.$$

[1] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).

[2] L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 407 (1957), Soviet Phys. JETP **5**, 336 (1957).

[3] A. Salam, Nuovo cimento **5**, 299 (1957).

[4] B. F. Touchek, Nuovo cimento **5**, 754 (1957).

[5] Kotenko, Popov, and Kuznetsov **Приборы и техника эксперимента** (Instr. and Meas. Engg) No. 1, 1957, p. 36.

[6] Alikhanian, Kirillov-Ugriumov, Kotenko, Kuznetsov, and Popov, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 253 (1958), Soviet Phys. JETP **7**, 176 (1958).

[7] Chadwick, Durrani, Eisberg, Tones, Wignall, and Wilkinson, Phil. Mag. **2**, 684 (1957).

[8] D. H. Wilkinson, Nuovo cimento **6**, 517 (1957).

[9] Barley, Coffin, Garwin, Lederman, and Weinrich, Bull. Am. Phys. Soc. Ser. II **2**, 205 (1957).

[10] Swanson, Campbell, Garwin, Sens, Telegdi, Wright, and Yovanovich, Bull. Am. Phys. Soc. Ser. II **2**, 205 (1957).

[11] Pless, Brenner, Williams, Barrari, Hildebrand, Milburn, Ramsey, Shapiro, Strauch, Street, and Young, Report of the 1957 Rochester Conference.

[12] S. C. Wright, Report of the 1957 Rochester Conference.

[13] Aliston, Evans, Morgen, Newport, Williams, and Kirk, Phil. Mag. **2**, 1143 (1957).

[14] Barmin, Kanavets, Morozov, Pershin, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 830 (1958), Soviet Phys. JETP **7**, 573 (1958).

Translated by J. G. Adashko
230