$$[E_i(\mathbf{r})E_k(\mathbf{r}')]_\omega$$
$$= 2\hbar\coth\frac{\hbar\omega}{2T}\left\{A\frac{\omega^2}{c^2(\delta^2 l)^{1/3}}\delta_{ik} - \frac{\omega^2}{c^2}\frac{\delta^2 l^2}{6}\frac{\partial^2}{\partial x_i\partial x_k}\delta(\rho)\right\}, \quad (18)$$

where
$$A = 1729/2^{1/3}\pi^{2/3}3^{1/6} \approx 570.$$

Here the spectral density of the energy per unit volume of the electric field in the metal, W_ω, is equal to
$$W_\omega = \frac{1}{8\pi}\{E^2\}_\omega = \frac{3A\hbar}{4\pi}\coth\frac{\hbar\omega}{2T}\cdot\frac{\omega^2}{c^2(\delta^2 l)^{1/3}}. \quad (19)$$

In averaging over an infinitely small volume, the second component in Eq. (18) drops out.

2. $\delta \ll l \ll \rho$. For calculation of the integrals entering into Eq. (15), it is appropriate in this case to extend the path of integration to infinity[5] (we consider k a complex variable, Im k → ∞), taking into account all the singularities of the integral. The singularities of the integrand are the zeros of the denominator and k = i (branch point). After transformation, the integrals reduce to a sum of residues and an integral from i to i∞.

It is easy to see that for $\delta \ll l$ the sum of the residues is appreciably less than the integral, the asymptotic value of which leads to a correlation function of the following sort:

$$[E_i(\mathbf{r})E_k(\mathbf{r}')]_\omega = \frac{\hbar\omega}{6\pi\sigma_0}\coth\frac{\hbar\omega}{2T}\left[\frac{\delta_{ik}}{\rho} - \frac{\nu_i\nu_k}{l\ln^2(\rho/l)}\right]\frac{e^{-\rho/l}}{\rho^2}. \quad (20)$$

The authors take this opportunity to express their gratitude to L. D. Landau and E. M. Lifshitz for acquainting us with the book, Electrodynamics of Continuous Media, before publication.

[1] S. M. Rytov, Теория электрических флуктуаций и теплового излучения (Theory of Electrical Fluctuations and Thermal Radiation) Acad. of Sci. Press, 1953.

[2] M. L. Levin, Dokl. Akad. Nauk SSSR **102**, 53 (1955).

[3] F. V. Bunkin, Dissertation, Phys. Inst. Acad. Sci. U.S.S.R., 1955.

[4] L. D. Landau and E. M. Lifshitz, Электродинамика сплошных сред (Electrodynamics of Continuous Media), Gostekhizdat, 1957

[5] G. E. H. Reuter and E. H. Sondheimer, Proc. Roy. Soc. (London) **195**, 336 (1948).

Translated by R. T. Beyer
237

SOVIET PHYSICS JETP VOLUME 34(7), NUMBER 5 NOVEMBER, 1958

QUANTUM OSCILLATIONS OF THE HIGH-FREQUENCY SURFACE IMPEDANCE

M. Ia. AZBEL'

Physico-Technical Institute, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor November 26, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 1158-1168 (May, 1958)

On the basis of general formulas obtained earlier by the author, a quantum-mechanical formula is found for the total surface impedance of metals at high frequencies, where the skin depth is small in comparison with the Larmor radius and with the electronic mean free path. The analysis is carried out for an arbitrary law of dispersion for the conduction electrons. Cases involving constant magnetic fields both parallel to the surface of the metal and inclined with respect to it are studied. The influence of the thickness of the metallic film on the quantum oscillations is ascertained. It is shown that an experimental study of the surface impedance in a strong magnetic field makes it possible in principle to reconstruct the shape of the Fermi surface and to determine the velocities of the electrons on it.

INTRODUCTION

In Ref. 1 we found the first non-vanishing quantum correction to the current density Δ_j^{qu} at high frequencies in a film whose width D satisfies the inequality

$$D > d = \left|\frac{c}{eH}\int_{t_0'}^{t_0''} v_\zeta\, dt_2\right|_{p_z = p_z^{\rm ext}}$$
$$v_\zeta(t_0') = v_\zeta(t_0'') = 0, \quad dv_\zeta/dt_0' > 0, \quad dv_\zeta/dt_0'' < 0, \quad (1.1)$$

where t is the period of orbital revolution of an electron in a magnetic field; **p**, $\mathbf{v} = \nabla\epsilon$, and ϵ are the quasi-momentum, velocity, and energy of an electron; ζ is the direction of the normal to the surface of the metal; and z is the direction of the constant magnetic field **H** (which makes an angle Φ with ζ).

It was shown that under the conditions of the anomalous skin effect, when the effective skin depth δ_{eff} is small in comparison with the mean free path $l = vt_0$, with v/ω, and with the radius of the Larmor orbit r.

$$\Delta j_i^{\text{qu}} = \sum \frac{h^3}{2\pi} H^2 \chi_i \left(\frac{d \ln S}{d\varepsilon}\right)^2 \frac{\partial \Delta M^z}{\partial H}\bigg|_{\varepsilon=\varepsilon_0,\ p_z = p_z^{\text{ext}};\ 0}, \quad (1.2)$$

$$\chi_i = \frac{4\pi e^2}{h^3 \overline{t_0^{*-1}}} \Bigg\{ v_i(t_1)\, s\left(\zeta - \int_{t_0'}^{t_1} v_\zeta dt_2\right) \overline{\times\, s\left(D - \zeta - \int_{t_1}^{t_0''} v_\zeta dt_2\right) v_j(t_1')\, E_j\left(\zeta - \int_{t_1'}^{t_1} v_\zeta\, dt_2\right)}\Bigg\}_{\text{av}},$$

$$s(w) = \begin{cases} 1 & (w > 0) \\ 0 & (w < 0) \end{cases}; \quad \frac{1}{t_0^*} = \frac{1}{t_0} + i\omega. \quad (1.3)$$

Here the averages (denoted by the bar and by "av") are taken with respect to t_1 and t_1'; the summation is taken over all extreme cross sections of the limiting Fermi surface $\epsilon(\mathbf{p}) = \epsilon_0$, if $\pi/2 - \Phi \ll \delta_{\text{eff}}/l$, and only over central cross sections $\epsilon(\mathbf{p}) = \epsilon_0$, $p_z = 0$ if $\pi/2 - \Phi \gg \delta_{\text{eff}}/r$. In the latter case equation (1.3) is valid only for a sufficiently strong magnetic field:

$$\Omega = \frac{2\pi |e| H}{c\, |\partial S / \partial\varepsilon|} \gg \omega, \frac{1}{t_0}; \quad \frac{r}{\delta_{\text{eff}}} \left(\frac{\mu H}{\varepsilon_0}\right)^{1/2} \ll 1. \quad (1.4)$$

We note that Eqs. (1.2) and (1.3) give correctly the zero approximation $(\mu H/\epsilon_0)^{1/2}$ also for $D < d$, but this approximation reduces to zero, and it is the next one which is of interest. The functions s in Eq. (1.3) appeared because, as it turns out, the approximation under consideration depends entirely on electrons whose average velocity is very small in the bulk metal and which do not collide with the surfaces of the film.

Equations (1.2) and (1.3) permit us to obtain the quantum correction to the total surface impedance under the conditions indicated.

2. CALCULATION OF THE TOTAL SURFACE IMPEDANCE

In order to determine the total surface impedance it is necessary to solve Maxwell's equations, which in our case reduce to

$$E_\alpha''(\zeta) = 4\pi i\omega c^{-2} j_\alpha(\zeta) \quad (\alpha = x,\ \xi), \quad (2.1)$$

$$j_\zeta(\zeta) = 0 \quad (2.2)$$

with the following relation between the current density and the electric field intensity:

$$j_i(\zeta) = j_i^{\text{cl}}(\zeta) + \Delta j_i^{\text{qu}}(\zeta) \quad (2.3)$$

(ξ is the direction in the plane of the metal perpendicular to x). We will consider first the case of a half space. We write $j_i(\zeta)$ in the form $j_i(|\zeta|)$ and extend $E_j(\zeta)$ as an even function into the region $\zeta < 0$, just as was done in Refs. 2 and 3.

Taking a Fourier transform of both sides of (2.1) and (2.2), we obtain

$$-k^2 \mathcal{E}_\alpha(k) - 2E_\alpha'(0) = 4\pi i\omega c^{-2} \{j_\alpha^{\text{cl}} + \Delta j_\alpha^{\text{qu}}(k)\}; \quad (2.4)$$

$$j_\zeta^{\text{cl}}(k) + \Delta j_\zeta^{\text{qu}}(k) = 0. \quad (2.5)$$

The expression for $j_i^{cl}(k)$ was obtained in Ref. 3 (the results of which we shall use repeatedly in what follows), and $\Delta j_i^{\text{qu}}(k)$ has the following form for the case of a half space:

$$\Delta j_i^{\text{qu}}(k) = \sum \frac{h^3}{2\pi} H^2 \chi_i \left(\frac{d \ln S}{d\varepsilon_0}\right)^2 \frac{\partial \Delta M^z}{\partial H}\bigg|_{\varepsilon=\varepsilon_0,\ p_z = p_z^{\text{ext}};\ 0} \quad (2.6)$$

$$\chi_i(k) = (4\pi e^2 / h^3 \overline{t^{*-1}}) \Bigg[v_i(t) \Bigg[v_j(t') \Bigg\{ \cos\left(\frac{kc}{eH} \int_t^{t'} v_\zeta dt_2\right) \mathcal{E}_j(k) - \frac{1}{\pi} \int_{-\infty}^{\infty} \sin \frac{c}{|eH|} \left(k \int_{t_0'}^{t} v_\zeta dt_2 + k' \int_{t_0'}^{t'} v_\zeta dt_2\right) \frac{\mathcal{E}_j(k')\, dk'}{k + k'} \Bigg\} \Bigg]_{\text{av}} \Bigg]_{\text{av}},$$

$$\mathcal{E}_j(-k) = \mathcal{E}_j(k) = 2 \int_0^\infty E_j(\zeta) \cos k\zeta\, d\zeta;$$

$$j_i(k) = 2 \int_0^\infty j_i(\zeta) \cos k\zeta\, d\zeta. \quad (2.7)$$

For the anomalous skin effect only large k and k′ play a substantial role;[3] by the method of steepest descents with respect to k and k′ we find

$$\chi_i \approx \frac{8\pi^2 e^2}{h^3 \overline{t_0^{*-1}}} \frac{v_i v_j}{|v_\zeta'(t)|\, T^2} \frac{1}{k} \times \left\{ \mathcal{E}_j(k) - \frac{1}{\pi} \int_0^\infty \frac{\sqrt{k}\, \mathcal{E}_j(k')\, dk'}{\sqrt{k'}\,(k + k')} \right\}\Bigg|_{\varepsilon=\varepsilon_0;\ p_z = p_z^{\text{ext}};0;\ v_\zeta=0} \quad (2.8)$$

It is clear that $\chi_\zeta(\zeta)$ reduces to zero in this approximation (since $v_\zeta = 0$). The same thing happens also for $j_\zeta^{cl}(k) \sim v_\zeta$ because of the fact that, for the anomalous skin effect, only electrons travelling almost parallel to the surface are important at all times. According to the same principle, the term with $\mathcal{E}_\zeta(k)$ drops out in the equations for j_α, and it is sufficient to carry out the summation only over two indices j, x and ξ.

Thus Eq. (2.5) is satisfied automatically, and there remains only the solution of Eq. (2.4).

In the case of a magnetic field parallel to the surface of the metal, the asymptote of $j_\alpha^{cl}(k)$ with respect to k has the form[3]

$$j_\alpha^{\mathrm{cl}}(k) = \frac{3\pi}{4k}\left\{A_{\alpha\beta}^{\mathrm{cl}}\mathcal{E}_\beta(k) - \frac{2}{\pi^2}C_{\alpha\beta}^{\mathrm{cl}}\int_0^\infty \frac{k\ln(k/k')\,\mathcal{E}_\beta(k')\,dk'}{k^2-k'^2}\right.$$
$$\left. - \frac{1}{\pi}(A_{\alpha\beta}^{\mathrm{cl}} - C_{\alpha\beta}^{\mathrm{cl}})\int_0^\infty \sqrt{\frac{k}{k'}}\frac{\mathcal{E}_\beta(k')\,dk'}{(k+k')}\right\}, \qquad (2.9)$$

where

$$A_{\alpha\beta}^{\mathrm{cl}} = \frac{8e^2}{3h^3}\int_0^{2\pi}\frac{n_\alpha n_\beta}{K}\cdot\frac{d\varphi}{1-\exp\left\{-2\pi i\frac{\omega}{\Omega}-\frac{2\pi}{\Omega t_0}\right\}};$$
$$B_{\alpha\beta}^{\mathrm{cl}} = \frac{8e^2}{3h^3}\int_0^{2\pi}\frac{n_\alpha n_\beta}{K}d\varphi; \qquad (2.10)$$
$$C_{\alpha\beta}^{\mathrm{cl}} = B_{\alpha\beta}^{\mathrm{cl}} - \frac{2e^2}{3h^3}\int_0^{2\pi}\frac{n_\alpha n_\beta}{K}\left(1-\exp\left\{-2\pi i\frac{\omega}{\Omega}-\frac{2\pi}{\Omega t_0}\right\}\right)d\varphi;$$
$$\mathbf{n} = \mathbf{v}/v = (n_x, n_y, n_z) = (\sin\theta\sin\varphi,\ \cos\theta,\ \sin\vartheta\cos\varphi);$$

all quantities are evaluated at $\epsilon = \epsilon_0$, $\theta = \pi/2$ ($v_y = 0$); K is the Gaussian curvature at the corresponding point; $1/t_0$ is averaged over the time of revolution of an electron in its orbit.

In the case most important to us, that of a strong magnetic field ($\Omega \gg \omega$, $1/t_0$) and resonance ($\omega \approx \pm\Omega$, $\pm 2\Omega, \ldots$) it is sufficient to retain in (2.9) only terms with $A_{\alpha\beta}$. Then from (2.9), (2.6), and (2.8) it is clear that the quantum correction changes only $A_{\alpha\beta}$:

$$j_\alpha(k) = \frac{3\pi}{4}A_{\alpha\beta}\frac{1}{k}\left\{\mathcal{E}_\beta(k) - \frac{1}{\pi}\int_0^\infty\frac{\sqrt{k}\mathcal{E}_\beta(k')\,dk'}{\sqrt{k'}(k+k')}\right\}; \qquad (2.11)$$

$$A_{\alpha\beta} = A_{\alpha\beta}^{\mathrm{cl}} + \Delta A_{\alpha\beta}^{\mathrm{qu}}; \qquad \Delta A_{\alpha\beta}^{\mathrm{qu}} = \sum\frac{16}{3}\frac{e^2}{t_0^{*-1}}H^2\frac{v_\alpha v_\beta}{|v_y'(t)|T^2}$$
$$\times\left(\frac{d\ln S}{d\varepsilon}\right)^2\frac{\partial\Delta M^z}{\partial H}\Big|_{\varepsilon=\varepsilon_0,\ p_z=p_z^{\mathrm{ext}},\ v_\zeta=0}. \qquad (2.12)$$

In Ref. 3 it was shown how one can find, in terms of $A_{\alpha\beta}$, the total surface impedance $Z_{\alpha\beta}$, which is determined by the equation

$$E_\alpha(0) = Z_{\alpha\beta}I_\beta, \quad I_\beta = -(c^2/4\pi i\omega)E_\beta'(0), \qquad (2.13)$$

I_β is the total current in the direction β. In the special case when the complex tensor $A_{\alpha\beta}$ is reduced to principal axes, the principal values of Z_α are expressed in terms of the principal values of A_α:

$$Z_\alpha \approx \frac{16}{9}\left(\frac{\sqrt{3}\pi\omega^2}{c^4}\right)^{1/3}e^{i\pi/3}A_\alpha^{-1/3}. \qquad (2.14)$$

In the general case of a constant magnetic field of arbitrary magnitude and direction, the exact expression for $j_\alpha(k)$ has a very complex form. However, assuming only a small error (of the order of several percent) in the numerical factor in the expression for the impedance, one can write $j_\alpha(k)$ approximately as

$$j_\alpha(k) = \frac{3\pi}{4k}a_{\alpha\beta}; \quad a_{\alpha\beta} = a_{\alpha\beta}^{\mathrm{cl}} + \Delta a_{\alpha\beta}^{\mathrm{qu}}, \qquad (2.15)$$

$\Delta a_{\alpha\beta}^{qu}$ is determined from equation (2.12), where v_ξ' replaces v_y' and $a_{\alpha\beta}^{cl}$ coincides with $A_{\alpha\beta}^{cl}$ [see (2.10)] for $\pi/2 - \Phi \ll \delta_{\mathrm{eff}}/l$ and with $B_{\alpha\beta}^{cl}$ for $\pi/2 - \Phi \gg \delta_{\mathrm{eff}}/r$. The impedance $Z_{\alpha\beta}$ is easily found in terms of $a_{\alpha\beta}$:

$$Z_{xx,\,\xi\xi} = \frac{16}{9}\left(\frac{\sqrt{3}\pi\omega^2}{c^4}\right)^{1/3}e^{i\pi/3}\frac{(x_1+x_2)x_1x_2 + a_{\xi\xi,\ xx}}{x_1x_2(x_1^2+x_1x_2+x_2^2)};$$
$$Z_{x\xi} = Z_{\xi x} = -\frac{16}{9}\left(\frac{\sqrt{3}\pi\omega^2}{c^4}\right)^{1/3}e^{i\pi/3}\frac{a_{x\xi}}{x_1x_2(x_1^2+x_1x_2+x_2^2)};$$
$$x_{1,2} = \{{}^1\!/_2(a_{xx}+a_{\xi\xi}) \pm [{}^1\!/_4(a_{xx}-a_{\xi\xi})^2 + a_{x\xi}^2]^{1/2}\}^{1/3}. \qquad (2.16)$$

We choose the cubic root whose argument lies in the interval $(-\pi/6, +\pi/6)$.

It is clear from the equations derived that the case of equal numbers of "holes" and electrons ($N_1 = N_2$) is not a special one; this result differs from the static case, but is the same as for the classical part of the high-frequency $Z_{\alpha\beta}$.

As already pointed out, the equations derived give the zero- and first-order approximations for the impedance with respect to $(\mu H/\epsilon_0)^{1/2}$ and the zero-order approximation with respect to δ_{eff}/r. This approximation can reduce to zero only in the exceptional cases when v_x or v_ξ equals zero at the point $\epsilon = \epsilon_0$, $p_z = p_z^{ext}$, $v_\zeta = 0$. Such a case occurs, for example, for an isotropic quadratic law of dispersion in a constant magnetic field parallel to the surface and having the same direction as the electric field.

Furthermore, Eqs. (2.15) are inapplicable if the equations $p_z = p_z^{ext}$, $v_\zeta = 0$ coincide, or $\epsilon = \epsilon_0$, $p_z = p_z^{ext}$, $v_\zeta = 0$ determines a line instead of an isolated point. This happens, for instance, when the magnetic field is perpendicular to the surface and the dispersion law is isotropic. Since electrons with $p_z = 0$ are at all times in the skin layer, the skin effect is normal for them, consequently $\bar{v}_x$ or $\bar{v}_y$ becomes zero; the zeroth approximation of the impedance with respect to $(\mu H/\epsilon_0)^{1/2}$ reduces to zero; and in the sums in Eq. (4.4) of Ref. 1 it is necessary to take terms with $l \neq 0$ into account. Since electrons with a vanishing v_z do not generally collide with the surface, the calculation can be carried out also in this exceptional case (see Sec. 3).

Naturally, because of the "normality" of the skin effect for the electrons which are responsible for the quantum oscillations, and the "anomaly" of the skin effect for the remaining electrons, the relative magnitude of the quantum oscillations turns out to be considerably greater in this case than in the static one.

We note in conclusion that all the equations derived above are valid also for films which are not too thin (according to the criterion $D > d$ given above) struck on one side by an electromagnetic wave. In this case, because of the anomaly of the skin effect, the electrons with $\bar{v}_\zeta = 0$ (which are only important for Δj_i^{qu}), which do not get into the "skin-layer," in general make no contribution to Δj_i^{qu}, and the second surface of the film has absolutely no effect on Δj_i^{qu}.

If the film thickness is $D < d$, then the amplitude of the quantum oscillations with the corresponding period is zero to the approximation under consideration, since $\chi_i = 0$ [see Eq. (1.3)]. This circumstance permits one, by studying $Z_{\alpha\beta}(H)$ in films of width $D < l$, to determine the d and S_{ext} corresponding to exactly this period in terms of the value of the magnetic field for which an abrupt increase occurs in the amplitude of the quantum oscillation with a given period and for a given direction of the constant magnetic field.

3. QUANTUM OSCILLATIONS OF THE SURFACE IMPEDANCE IN A MAGNETIC FIELD PERPENDICULAR TO THE SURFACE FOR A QUADRATIC DISPERSION LAW

We assume that $(\mu H/\epsilon_0)^{1/2} l/\delta_{eff} \ll 1$ and $l > r$. Then it is possible to regard the electrons making the essential contribution to the quantum oscillations and having $v_z \sim v(\mu H/\epsilon_0)^{1/2}$ as generally not through the spatial distribution of the electric field **E**, setting $\mathbf{E} = 0$ outside the metal. Since such an extrapolation of the electric field into the region outside the metal gives simultaneously the correct classical result,[4,5] we shall obtain the exact quantum-mechanical formula for **j**.

In the case under study the quasi-classical matrix elements are*

$$[\Phi(y, p_y, z, p_z)]_{nn', p_z p_z'} = \frac{1}{Th}\int_{-\infty}^{\infty} \exp\left\{-\frac{i}{\hbar}(p_z - p_z')z\right\} dz \int_0^T e^{-i\Omega(n-n')t}\,\Phi(y(t), p_y(t)z, p_z)\,dt; \quad (3.1)$$

$$\Omega = 2\pi/T = |e|H/m^* c.$$

Once the quasi-classical matrix elements are known, one can calculate $j_{x,y}$ from the equations derived in the preceding sections. It is convenient to introduce

$$j_\pm = j_x \pm i j_y;\quad E_\pm = E_x \pm i E_y. \quad (3.2)$$

Then

$$E_\pm''(z) = 4\pi i\omega c^{-2} j_\pm(z); \quad (3.3)$$

$$j_\pm(z) = \int_0^\infty K_\pm(|z - z'|)\,E_\pm(z')\,dz'; \quad (3.4)$$

$$K_\pm(w) = -\frac{|e|^3 H}{ch^3}\int_{-\infty}^{\infty} dp_z \sum_{n=0}^{\infty} v_n^2 \times \frac{f^0(\varepsilon_{np_z} \mp \hbar\Omega + v_z p_z') - f^0(\varepsilon_{np_z})}{\mp\hbar\Omega + v_z p_z'}\,\frac{\exp\{-ip_z' w/\hbar\}}{1/t_\pm - ip_z' v_z/\hbar}, \quad (3.5)$$

where

$$v_n^2 = 2(n + 1/2)\hbar\Omega/m;\quad \varepsilon_{np_z} = (n + 1/2)\hbar\Omega + p_z^2/2m;$$

$$1/t_\pm = 1/t_0 + i(\omega \pm \Omega). \quad (3.6)$$

From (3.5)

$$\Delta K_\pm^{qu}(w) = \sum_{s=1}^{\infty} I_s;\quad I_s = -\frac{2|e|^3 H}{ch^3}\int_0^\infty e^{2\pi i n s} dn \int_{-\infty}^{\infty} dp_z' v_n^2 \times \frac{f^0(\varepsilon_{np_z} \mp \hbar\Omega + v_z p_z') - f^0(\varepsilon_{np_z})}{\mp\hbar\Omega + v_z p_z'}\,\frac{e^{-ip_z w/\hbar}}{1/t_\pm - ip_z' v_z/\hbar}. \quad (3.7)$$

Setting $n = n' \pm 1 - v_z p_z'/\hbar\Omega$ and noting that, by hypothesis,

$$|v_z p_z'| \sim (\mu H/\varepsilon_0)^{1/2} r/\delta_{eff} \ll 1,$$

we obtain the following result (the lower limit for

*It is a simple matter to obtain this equation by writing Φ in the form

$$\Phi = \sum_s \varphi_s(y, p_y)\,\psi_s(z, p_z)$$

and observing that in the quasi-classical approximation

$$(np_z|\varphi_s|n'p_z') = \delta(p_z - p_z')\frac{1}{T}\int_0^T e^{-i\Omega(n-n')t}\varphi_s(t)\,dt;$$

$$(np_z|\psi_s|n'p_z') = \frac{1}{h}\delta_{nn'}\int_{-\infty}^{+\infty}\exp\left\{-\frac{i}{\hbar}(p_z - p_z')z\right\}\psi_s(z, p_z)\,dz.$$

n has no effect on the oscillating part):

$$I_s = \frac{4|e|^3 t_\pm H}{m\, ch^2} \delta(w) \int^{\infty} e^{2\pi isn} dn \int_{-\infty}^{\infty} dp_z \cdot f^0(\varepsilon_{np_z}).$$

We now easily find

$$\Delta K_\pm^{\mathrm{qu}}(w) = -\frac{2e^2 t_\pm H}{m\varepsilon_0} \delta(w)\, \Delta M^z = -\frac{2H}{\varepsilon_0 N} \sigma_\pm \delta(w)\, \Delta M^z$$

$$\sigma_\pm = Ne^2 t_\pm / m, \qquad (3.8)$$

where N is the electron density. Here

$$\Delta j_\pm^{\mathrm{qu}}(z) = -\frac{2H}{\varepsilon_0 N} \sigma_\pm \Delta M^z E_\pm(z) = \Delta s_\pm^{\mathrm{qu}} E_\pm(z),$$

i.e., as was assumed, the skin effect is normal for the electrons responsible for $\Delta j_\pm^{\mathrm{qu}}$, and $\Delta j_\pm^{\mathrm{qu}}$ does not depend on the form of the boundary condition. In particular, such a result would also have been obtained for specular reflection, for which Eq. (3.4) has the form

$$j_\pm(z) = \int_{-\infty}^{\infty} K_\pm(|z - z'|)\, E_\pm(z')\, dz'. \qquad (3.9)$$

Since the character of the reflection has only a weak effect on the classical equation for the surface impedance,[3,4] we shall not solve the exact Wiener-Hopf equation, but shall consider the case of specular reflection.

Writing down the Fourier transform of (3.9) and taking into consideration the fact that, according to Ref. 4,

$$K_\pm^{\mathrm{cl}}(k) \sim 3\pi\sigma / 4kl, \qquad (3.10)$$

we obtain

$$Z_\pm = -\frac{4\pi i\omega}{c^2} \frac{E_\pm(0)}{E'_\pm(0)} = \frac{8i\omega}{c^2} \int_0^{\infty} \frac{dk}{k^2 + 3\pi i/2\delta^2 lk + \Delta s_\pm^{\mathrm{qu}} 4\pi i\omega/c^2}$$

$$= Z_\pm^{\mathrm{cl}}(0) \left\{1 - e^{-\pi i/3} \frac{28\sqrt{3}\sigma_\pm}{c^2 \varepsilon_0 N} \omega H \left(\frac{2\delta^2 l}{3\pi}\right)^{2/3} \Delta M^z\right\};$$

$$Z_\pm^{\mathrm{cl}}(0) \approx \frac{16}{9} \left(\frac{\sqrt{3}\pi\omega^2 l}{c^4 \sigma}\right)^{1/3} e^{i\pi/3}. \qquad (3.11)$$

Thus the quantum correction to the surface impedance in the zero approximation with respect to l/δ is purely real; its relative amplitude for $kT < \hbar\Omega$ is of the order of $(k\Omega/\epsilon_0)^{3/2} (l/\delta)^{2/3}$.

The fact that the calculations were carried out in terms of the variables ζ_{chem}, H (ζ_{chem} is the chemical potential) and not N, H, as already mentioned,[1] is also unimportant here because $(l/\delta)^{2/3} \gg 1$.

4. POSSIBILITY OF CONSTRUCTING THE FERMI SURFACE

It has been shown by I. Lifshitz and Pogorelov[6] that the study of quantum oscillations is a very convenient method for ascertaining the form of the Fermi surface in metals. The periods of the quantum oscillations

$$\Delta(1/H) = |e|\hbar / cS_{\mathrm{ext}}\ (\varepsilon_0)$$

permit one to determine directly the extreme areas S_{ext} of cross sections of the Fermi surface. Knowledge of these areas for different directions makes it possible to construct $\epsilon(\mathbf{p}) = \epsilon_0$.

From this point of view the study of quantum oscillations of the high-frequency conductivity is particularly convenient for two reasons. First, the quantum oscillations are, generally speaking, $\epsilon_0/\mu H$ times larger at high frequencies than in the static case. In particular, in strong magnetic fields the amplitude of the oscillations of dR/dH and dX/dH ($Z = R + iX$), which can be directly measured experimentally, exceeds the classical values of dR/dH and dX/dH by a factor of $(\epsilon_0/\mu H)^{1/2} \gg 1$ [this conclusion follows at once from (2.15)], which facilitates the observation of the quantum oscillations.

Second, in an inclined magnetic field the oscillations do not yield all the extreme cross sections, as in the static case or in the case of a parallel magnetic field, but only the central sections. Consequently, a study of the oscillations for different angles of inclination of the magnetic field with respect to the surface of the metal can materially simplify the analysis of the experimental curves, which in the static case is quite difficult because of the superposition of a large number of harmonics with different periods.

Up to the present time quantum oscillations of the high-frequency conductivity have not been observed experimentally. However, a study of the classical and quantum-mechanical variation of the surface impedance in a constant magnetic field is of special interest, since in principle it permits a completely determination of the form of the Fermi surface and of the velocities of the electrons over it. (The quantum-mechanical variation can be easily distinguished from the classical one by the sizes of the periods.)

A study of quantum oscillations in films of width $D < l$ is particularly convenient, since by determining $Z_{\alpha\beta}(\mathbf{H})$ one can find all the extreme cross sections without harmonic analysis. For this it would be sufficient to increase the magnetic field and find values of H for which there was an abrupt increase in the amplitude of an oscillation with a given period (different periods would appear one after the other with increasing field). Furthermore, one can obtain d directly: knowledge of this quantity for all directions makes it possible to determine its form, at least for a convex surface. In particular in a parallel field on a central section, which is easily distinguished from others by the inclination of the field, $d = |(2c/eH)\, p_x^{max}|$ (the maximum is taken with respect to t), so that by rotating **H** in the plane of the metal we establish the shape of the cross-section $\epsilon(\mathbf{p}) = \epsilon_0$, $p_y = 0$. If we use films whose surfaces are oriented in various ways with respect to the crystallographic axes, we can plot $\epsilon(\mathbf{p}) = \epsilon_0$ directly.

Finding the magnitudes of the velocities of the electrons on the Fermi surface requires a knowledge of the extreme effective masses $(\partial S/\partial\epsilon)_{ext}/2\pi$ for different directions,[6] which can be determined from the amplitudes of the quantum oscillations of the magnetic susceptibility. However, the magnitude of the amplitude is extremely sensitive to various incidental effects, such as, for example, a mosaic crystal structure. Consequently the determination of $(\partial S/\partial\epsilon)_{ext}$ from quantum oscillations appears to be very unreliable.

$(\partial S/\partial\epsilon)_{ext}$ can be determined with considerable accuracy from the classical variation of the impedance in a magnetic field aligned parallel to the surface. The extremal masses are determined directly from the resonant frequencies of the cyclotron resonance.[3]

$$(\partial S/\partial\varepsilon)_{\text{ext}} = 2\pi|e|H_{\text{res}}/\omega c,$$

in which the interpretation of the superposition of the resonance curves is naturally simpler than the interpretation of the superposition of the quantum harmonics.

It is not difficult to distinguish the classical variation of $Z_{\alpha\beta}$ from the quantum-mechanical one. There are differences between the classical and quantum oscillations both with respect to their relative magnitudes,

$$(|\Delta Z^{\text{qu}}| \ll |Z^{\text{cl}}|, \quad |d\Delta Z^{\text{qu}}/dH| \gg |dZ^{\text{cl}}/dH|),$$

and their periods (the periods of the quantum oscillations are, as a rule, considerably smaller); in a strong magnetic field $(\Omega \gg \omega)$ the classical oscillations (cyclotron resonance) usually vanish, while the quantum-mechanical ones of course remain; for an inclined magnetic field only the quantum oscillations corresponding to central sections are conserved (in this case it is obviously expedient to study dZ/dH, since, in the next approximation with respect to the anomaly, the cyclotron resonance is preserved in an inclined field as well[3]). Beyond this, a change of the frequency ω of the high-frequency field can give a well-defined result: the periods of the quantum oscillations will not change at all, but the periods of the cyclotron resonance will vary directly as ω.

The most convenient way to observe the quantum oscillations of dZ/dH is in strong magnetic fields $\Omega \gg \omega$.

5. QUANTUM OSCILLATIONS OF THE STATIC CONDUCTIVITY IN FILMS

We shall make several remarks relating to quantum oscillations in films in the static case.

We note first of all that in films which are not too thin (according to the criterion given above, D > d), but also not too thick $(D \sim d)$, only those electrons with $\overline{v}_\zeta = 0$ which do not collide with the surface are important for determining Δj_i^{qu}, even in the static case. The point is that the energy spectrum of all the remaining electrons which, clearly, do collide with the surface, is not quasi-classically degenerate with respect to P_x, and therefore such electrons give a contribution in the next approximation with respect to $(\mu H/\epsilon_0)^{1/2}$, their contribution being $(\mu H/\epsilon_0)^{1/2} \gg 1$ times smaller than the contribution of the "sub-surface" electrons with $\overline{v}_\zeta = 0$ (see Ref. 1). In those cases when the quantum correction, necessitated by those electrons which do not collide with the surface, does not reduce to zero, the problem can be solved in light of the above remark.

This pertains first of all to the de Haas-van Alphen effect in films.[7]

For a film in static fields, the quantum correction to the current density is determined by Eqs. (1.2) and (1.3). It is to be understood that these formulas are not exact in the static case, since it is impossible to introduce a free-path time even in the classical case at low temperatures. One can convince himself, however, that all of the results obtained below are qualitatively correct for any form of the collision integral.

Averaging the current density over the thickness we have

$$\overline{\Delta j_i^{\text{qu}}} = \frac{1}{D}\int_0^D \Delta j_i^{\text{qu}}(\zeta)\, d\zeta$$

$$= \sum \frac{h^3}{2\pi} H^2 h_i \left(\frac{d\ln S}{d\varepsilon}\right)^2 \frac{\partial \Delta M^z}{\partial H}\bigg|_{\varepsilon=\varepsilon_0, p_z=p_z^{\text{ext}}}; \qquad (5.1)$$

$$h_i = \frac{4\pi e^2}{h^3 \overline{t_0^{-1}}}\, \overline{v}_i \left\{ v_j \frac{1}{D} \int_{Q_1}^{D-Q_2} E_j(\mu)\, d\mu \right\}_{\text{av}};$$

$$Q_1 = \int_{t_0'}^{t_1'} v_\zeta dt_2; \quad Q_2 = \int_{t_1'}^{t_0''} v_\zeta dt_2. \qquad (5.2)$$

In an inclined field in this approximation the summation in (5.1) is carried out only over central cross-sections, on which (naturally, if they are closed) $\bar{\mathbf{v}} = 0$ and $\Delta j_i^{qu} = 0$.

The effect is possible in the first approximation with respect to $(\mu H/\epsilon_0)^{1/2}$ only in a magnetic field parallel to the surface of the film, if the Fermi surface has non-central sections. Then, since E_x and E_z do not depend on y,

$$E_y(\mu) = E_y(D - \mu); \ \bar{v}_x = \bar{v}_y = 0, \ v_y = -|c/eH|\, dp_x/dt;$$

$$\overline{\Delta j_x^{qu}} = \overline{\Delta j_y^{qu}} = 0; \ \overline{\Delta j_z^{qu}}$$

$$= \sum \frac{h^3}{2\pi} H^2 h_z \left(\frac{d \ln S}{d\varepsilon}\right)^2 \frac{\partial \Delta M^z}{\partial H}\bigg|_{\varepsilon=\varepsilon_0,\ p_z = p_z^{ext}};$$

$$h_z = \frac{4\pi e^2}{h^3 \overline{t_0^{-1}}} \bar{v} \left\{ \left(1 - \frac{d}{D}\right) \bar{v}_z E_z \right.$$

$$\left. - \left[\frac{v_y}{D}\left(\int_0^{Q_1} E_y(\mu)\, d\mu + \int_0^{Q_2} E_y(\mu)\, d\mu\right)\right]_{av} \right\},$$

where $E_y(y)$ is determined from the condition $j_y(y) = 0$. It is easy to see that in the case of the bulk metal, only $\overline{\Delta\sigma_{zz}^{qu}}$ is different from zero in the approximation under consideration. Thus practically the only case for which there are, generally speaking, "large" quantum oscillations is that of high frequencies. The oscillations of the static conductivity in films apparently do not exceed the oscillations in the bulk metal, in spite of the presence of a substantial anisotropy of the current density.

In order to determine a non-vanishing $\overline{\Delta\sigma_{ik}^{qu}}$ in films it is necessary to solve the very difficult problem of finding the energy spectrum of the electrons in the film for diffuse reflection of electrons from the boundary.

CONCLUSIONS

1. Quantum oscillations of the high frequency conductivity are predicted, whose relative amplitudes (the amplitudes as compared with the classical quantities) are, generally speaking, $\epsilon_0/\mu H \gg 1$ times larger than the relative amplitudes of the quantum oscillations of the static conductivity.

2. The amplitudes of the quantum oscillations of dR/dH, dX/dH exceed the classical values of dR/dH, dX/dH by a factor of $(\epsilon_0/\mu H)^{1/2} \gg 1$.

3. In a magnetic field aligned parallel to the surface $(\pi/2 - \Phi \ll \delta_{eff}/l)$, the periods of the quantum oscillations are determined by all the extreme cross-sections $p_z = p_z^{ext}$ of the Fermi surface $\epsilon(\mathbf{p}) = \epsilon_0$;

$$\Delta(1/H) = |e| h / cS(\varepsilon_0, p_z^{ext});$$

in an inclined magnetic field $(\pi/2 - \Phi \gg \delta_{eff}/r)$, they are determined only by the central sections $p_z = 0$, $\epsilon(\mathbf{p}) = \epsilon_0$.

4. The tensor of the surface impedance is expressed in Eqs. (2.16) in terms of the tensor $a_{\alpha\beta} = a_{\alpha\beta}^{cl} + \Delta a_{\alpha\beta}^{qu}$, where

$$a_{\alpha\beta}^{cl} = \begin{cases} \dfrac{8e^2}{3h^3}\displaystyle\int_0^{2\pi} \frac{n_\alpha n_\beta}{K} \frac{d\varphi}{1 - \exp(-2\pi i\omega/\Omega - (2\pi/\Omega)\overline{t_0^{-1}})}; & \left(\dfrac{\pi}{2} - \Phi \ll \delta_{eff}/l\right) \\ \dfrac{8e^2}{3h^3}\displaystyle\int_0^{2\pi} \frac{n_\alpha n_\beta}{K}\, d\varphi; & \left(\dfrac{\pi}{2} - \Phi \gg \delta_{eff}/r\right) \end{cases}$$

$$\Omega = 2\pi/T = 2\pi |e| H / c(\partial S/\partial\varepsilon),$$

where all quantities are taken at $v_\zeta = 0$; $\mathbf{n} = \mathbf{v}/v$; φ is the angle read along the curve $\epsilon(\mathbf{p}) = \epsilon_0$, $v_\zeta = 0$; K is the Gaussian curvature of the Fermi surface; S is the area of the cross-section $\epsilon(\mathbf{p}) = \epsilon_0$, $p_z = \text{const}$; and $t_0(\mathbf{p})$ is the free-path time, which can always be introduced for the anomalous skin effect, and

$$\Delta a_{\alpha\beta}^{qu} = \sum \frac{16}{3} \frac{e^2}{\overline{t_0^{*-1}}} H^2 \frac{v_\alpha v_\beta}{|v'_\zeta| T^2} \left(\frac{d \ln S}{d\varepsilon}\right)^2 \frac{\partial \Delta M^z}{\partial H}\bigg|_{\varepsilon=\varepsilon_0,\ p_z = p_z^s,\ v_\zeta = 0},$$

$$p_z^s = \begin{cases} p_z^{ext} & (\pi/2 - \Phi \ll \delta_{eff}/l), \\ 0 & (\pi/2 - \Phi \gg \delta_{eff}/r). \end{cases}$$

5. A determination of the periods of the oscillations (both classical and quantum-mechanical) of the surface impedance with a magnetic field makes it possible to find S_{ext} (from the quantum periods) and $(\partial S/\partial\epsilon)_{ext}$ (from the resonant frequencies of the cyclotron resonance). Knowledge of these quantities for different directions of the magnetic field makes it theoretically possible to construct completely the form of the Fermi surface and to find the velocities of the electrons on it. A study of $Z_{\alpha\beta}(H)$ in a monocrystalline film permits one, first, to determine S_{ext} and $(\partial S/\partial\epsilon)_{ext}$ without a complicated harmonic analysis, and second, to plot the Fermi surface and the velocities of the electrons on it directly.

I am grateful to I. M. Lifshitz and L. D. Landau for valuable discussions.

Note added in proof (April 5, 1958). We add two more comments.

1. A substantial increase in the amplitude of the quantum oscillations occurs at very low frequencies, corresponding to the normal skin effect $(r \gg \delta)$

$$\Delta j_i^{qu}(t) = \Delta\sigma_{ik}^{qu} E_k(t) - a_{ik} \partial E_k / \partial t;$$

$$a_{ik} = -\sum_s \frac{8\pi e^2}{c^2 \overline{t_0^{-1}}} H^2 \left(\frac{d \ln S}{d\varepsilon}\right)^2$$

$$\times \overline{\frac{\partial \Delta M^z}{\partial H} v_i(t_1) \int_0^{t_1} v_\eta dt_2} \; \overline{v_j(t_1') \int_0^{t_1'} v_\eta dt_2} \Bigg|_{\varepsilon=\varepsilon_0, \; p_z=p_z^s},$$

where $\Delta\sigma_{ik}^{qu}$ is the quantum correction to the static conductivity, found in Ref. 9, and a is the positive solution of Det $|\sigma_{ik} - a\delta_{ik}| = 0$. For $r \gg \delta$ the ratio of the second term to the first is of the order of $(\epsilon_0/\mu H)(r/\delta)^2$.

2. In an inclined field, for $(r/\delta)(\mu H/\epsilon_0)^{1/2} < 1 < (l/\delta)(\mu H/\epsilon_0)^{1/2}$, cyclotron resonance occurs in the amplitude of the quantum oscillations. This make is possible to measure, on one sample of a metal, the cyclotron frequencies [and consequently also $(\partial S/\partial\epsilon)_{ext}$] for all directions of **H**. An investigation of a monocrystalline film makes it possible to determine all the $(\partial S/\partial\epsilon)_{ext}$ one after the other without additional harmonic analysis. Thus by determining $Z_{ik}(\mathbf{H})$ one can, in principle, construct from experiments with one sample the form of the Fermi surface (from the periods of the quantum oscillations) and determine the velocities of the electrons on it (from the cyclotron frequencies).

[1] M. Ia. Azbel', J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 969 (1958), Soviet Phys. JETP **7**, 669 (1958).

[2] M. Ia. Azbel', Dokl. Akad. Nauk SSSR **100**, 437 (1955).

[3] M. Ia. Azbel' and E. A. Kaner, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 811 (1956) and **32**, 896 (1957); Soviet Phys. JETP **3**, 772 (1956) and **5**, 730 (1957); Journ. Phys. Chem. Solids **4**, 1958.

[4] M. Ia. Azbel' and M. I. Kaganov, Dokl. Akad. Nauk SSSR **95**, 41 (1954).

[5] M. Ia. Azbel' and M. I. Kaganov, Тр. физич. фак-та ХГУ (Trans. Phys. Faculty Khar' kov State University) **6**, 59 (1955).

[6] I. M. Lifshitz and A. V. Pogorelov, Dokl. Akad. Nauk SSSR **96**, 1143 (1954).

[7] M. Ia. Azbel', J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 754 (1958), Soviet Phys. JETP **7**, 518 (1958).

[8] I. M. Lifshitz, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 814 (1956) and **32**, 1509 (1957); Soviet Phys. JETP **3**, 774 (1956) and **5**, 1227 (1957); I. M. Lifshitz and A. M. Kosevich, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 88 (1957); Soviet Phys. JETP **6**, 67 (1958).

Translated by W. M. Whitney
238