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TWO-PHOTON ANNIHILATION OF POSITRONIUM IN THE P-STATE

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A relativistically invariant expression for the probability amplitude of two-photon annihilation of positronium has been obtained by summing an infinite number of diagrams of a certain particular class. We have calculated the nonrelativistic limit of two-photon positronium annihilation in the S- and P-states, as well as the selection rules for these processes.

In a previous work[1] the author extended the methods of quantum field theory to the problem of annihilation (or creation) of particles in bound states. In the present work, as a specific example of the results there obtained, we shall consider the annihilation of positronium in the P-state, a problem which has not been treated by the usual quantum field theory.* In the present communication we shall not take account of radiative corrections, and shall calculate the probability amplitude for two-photon annihilation simply by summing diagrams. The same results can be obtained, on the other hand, by making use of the Green's function which describes two-photon positronium annihilation.[1] In fact it is advantageous to use this Green's function for calculating the radiative corrections, since the summation of an infinite number of diagrams becomes extremely complicated when virtual particle annihilation is taken into account (see, for instance, the author's above-cited work).

1. PROBABILITY AMPLITUDE FOR TWO-PHOTON POSITRONIUM ANNIHILATION

In the lowest approximation the probability amplitude A_f for two-photon annihilation of the free particles is written†

$$A_f = e^2\Phi^*_{kk'}(\xi\xi')\, C(23')\,\gamma(\xi, 3'3)\, G(31')\,\gamma(\xi', 1'1)\,\Psi_f(12). \tag{1}$$

Here $\Psi_f(12)$ is the wave function of the electron and positron in the free state, $G(31')$ is the Green's function of the electron,[3] and

$$\Phi_{kk'}(\xi\xi') = (2\pi/\sqrt{k_0k_0'})\,[l_\nu l'_{\nu'} \exp i(-k\xi - k'\xi') + l_{\nu'} l'_\nu \exp i(-k\xi' - k'\xi)]$$

is the symmetrized function of two photons with momenta k and k' and polarization l and l'. The numbers (see Ref. 4) denote the sets of all coordinates and spin indices of the particles, whereas the symbol ξ (or ξ', ξ'', . . .) denotes the set of all coordinates and components of the polarization vector of the photon. A repeated symbol denotes summation (for spin indices and polarization vector components) and integration (for the coordinates). Further,

$$\gamma(\xi, 12) = (\gamma_\nu)_{\alpha_1\alpha_2}\delta(\xi - x_1)\,\delta(x_1 - x_2),$$
$$C(12) = C_{\alpha_1\alpha_2}\delta(x_1 - x_2), \tag{2}$$

where $\gamma_{1,2,3} = \beta\alpha_{1,2,3}$ and $\gamma_0 = \beta$, while C is a matrix which transforms an electron-positron field operator to its charge conjugate. We shall set $C = \alpha_2$.

Figure 1 shows the diagram* corresponding to Eq. (1). Since we wish to obtain the probability amplitude for two-photon annihilation of bound particles, we add to the reducible diagram of Fig. 1 all "ladder-type" diagrams (Fig. 2). When this is done, the probability amplitude A for two-photon annihilation is written

$$A = e^2\Phi^*_{kk'}(\xi\xi')\, C(67')\,\gamma(\xi, 7'7)\, G(75')\,\gamma(\xi', 5'5)\,[\Psi_f(56) + e^2 G(51')\, G(62')\,\gamma(\bar\xi, 2'2)\, D(\bar\xi\bar\xi')\,\gamma(\bar\xi', 1'1)\,\Psi_f(12)$$
$$+ e^4 G(53')\, G(64')\,\gamma(\bar{\bar\xi}, 4'4)\, D(\bar{\bar\xi}\bar\xi')\,\gamma(\bar\xi', 3'3)\, G(31')\, G(42') \times \gamma(\bar{\bar\xi}, 2'2)\, D(\bar{\bar\xi}\bar{\bar\xi}')\,\gamma(\bar\xi', 1'1)\,\Psi_f(12) + \ldots], \tag{3}$$

where $D(\xi\xi')$ is the photon Green's function.† This kind of an approximation for A means that

*K. Tumanov[2] has calculated the probability of positronium annihilation in the P-state using quantum electrodynamics in configuration space, as suggested by Iu. Shirokov.

†We shall use a system of units in which $\hbar = c = 1$, and the summation convention:

$$ab = a_\nu b_\nu = a_0b_0 - a_1b_1 - a_2b_2 - a_3b_3.$$

*For simplicity, in Figs. 1, 2, and 3 we have omitted similar graphs in which the k and k' photons are interchanged.

†This function is defined as $D(\xi\xi') = \delta_{\nu\nu'}D(x - x')$, where $\Box D(x - x') = -4\pi i\,\delta(x - x')$.

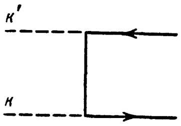

FIG. 1

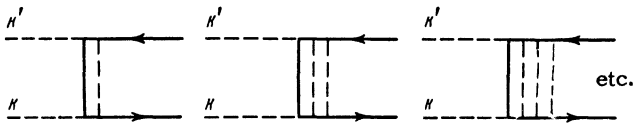

FIG. 2

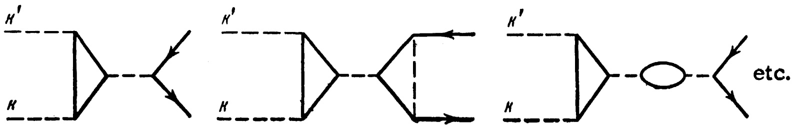

FIG. 3

some graphs of higher powers in e^2 are included, while others are not. As is known, the physical meaning of this method may be explained as follows.[5] In the bound state, particles interact for a very long (infinite) time. If $e^2 \ll 1$, the probability of finding one virtual quantum in the field is small and the probability of finding two quanta simultaneously is even smaller. Although the probability for the exchange of one quantum during a small time interval is fairly small, during the infinite time of existence of the bound state an indefinite number of quanta may be exchanged successively. It is just such processes that the "ladder-type" graphs deal with. Omitted diagrams of higher powers of e^2 refer to processes in which two or more quanta are in the field simultaneously. If $e^2 \ll 1$, such graphs are not important in the bound state if we restrict ourselves to the first nonzero approximation.

We note that Fig. 2 omits diagrams (Fig. 3) which refer to the electron-positron exchange interaction involving one-photon virtual annihilation. This interaction is also of the type in which at every instant of time there is only a single virtual photon in the field. According to Furry's theorem,[6] the total contribution of such diagrams to the probability amplitude of Eq. (3) must vanish. In general, summation of the "ladder-type" diagrams of Fig. 2 together with those of Fig. 3 will lead to a particle bound state whose wave function is a solution of the Bethe-Salpeter type, which includes, in addition to the usual interaction, the electron-positron exchange interaction which results from their single-photon virtual annihilation.[1,4]

The infinite sum in square brackets in (3) is a solution to the Bethe-Salpeter equation obtained by successive approximation. To show this, let us write the Bethe-Salpeter equation in the integral form

$$\Psi(12) = \Psi_f(12) + e^2 G(13') G(24') \gamma(\xi, 3'1') D(\xi\xi') \gamma(\xi', 4'2') \Psi(1'2'), \quad (4)$$

where $\Psi_f(12)$ satisfies the free-particle equation

$$G^{-1}(11') G^{-1}(22') \Psi_f(1'2') = 0$$

and is the zeroth approximation to the exact wave function $\Psi(12)$. We obtain the first correction to the zeroth approximation by replacing $\Psi(1'2')$ in the right side of (4) by its zeroth approximation $\Psi_f(1'2')$. The second approximation is obtained by replacing $\Psi(1'2')$ on the right side of (4) by its first approximation, etc. Continuing this iteration process ad infinitum, we obtain a representation of the solution of (4) in the form of the infinite sum of Eq. (3), so that we can now write

$$A = e^2 \Phi^*_{kk'}(\xi\xi') C(25) \gamma(\xi, 53') G(3'3) \gamma(\xi', 31) \Psi(12), \quad (5)$$

where $\Psi(12)$ is the positronium wave function satisfying the Bethe-Salpeter equation.[5]

Equation (5) can also be written in terms of $\overline{G}_{ep}(\xi\xi', 21)$, the Green's function describing two-photon positronium annihilation.[1] Indeed, according to the author's previously-cited work we have

$$\overline{G}_{ep}(\xi\xi', 21) = e^2 (D(\xi\bar{\xi}) D(\xi'\bar{\xi}') + D(\xi'\bar{\xi}) D(\xi\bar{\xi}')) C(2'5) \times \gamma(\bar{\xi}, 53') G(3'3) \gamma(\bar{\xi}', 31') K(1'2', 12), \quad (6)$$

in the first nonvanishing approximation. Here $K(1'2', 12)$, which is the Green's function of the interacting electron and positron,[4] and the photon and electron Green's functions D and G, are taken in the lowest approximation in e^2. Equation (5) follows directly from (6).

2. NONRELATIVISTIC APPROXIMATION FOR THE PROBABILITY AMPLITUDE

Let us rewrite (5) in terms of the relative momentum p. Then in the positronium center-of-mass coordinate system we have

$$A = i\frac{2\pi e^2}{m}\int\left(\frac{C\hat{\text{l}}'(\hat{p}+\hat{\text{k}}+m)\hat{\text{l}}}{p_0^2-[(\text{p}-\text{k})^2+m^2]}+\frac{C\hat{\text{l}}(\hat{p}-\hat{\text{k}}+m)\hat{\text{l}}'}{p_0^2-[(\text{p}+\text{k})^2+m^2]}\right)_{\alpha_2\alpha_1} \times \psi_{\alpha_1\alpha_2}(p)\,d^4p\,\delta(K-k-k'), \quad (7)$$

Here $\psi(\text{p})$ is the positronium wave function in relative momentum space, m is the electron mass, and K is the total positronium momentum. For any vector a we write $\hat{\text{a}} = a_\nu\gamma_\nu$, and $\hat{\mathbf{a}} = a_1\gamma_1 + a_2\gamma_2 + a_3\gamma_3$.

In calculating the amplitude as given by (7), we make use of the fact that the relative velocity v of the particles in the positronium atom is small ($v \sim e^2$), so that we shall henceforth neglect all terms of order v^2 and higher. For convenience, we shall rewrite (7) in terms of the two-by-two Pauli matrices σ rather than the Dirac matrices γ, and shall consider only those of the small components of $\psi(\text{p})$ to be nonzero which are of order v. We then obtain

$$A = -i\frac{2\pi e^2}{m}\Bigg[\int\left(\frac{\sigma_2(\text{l}'\sigma)[(\text{p}-\text{k})\sigma](\text{l}\sigma)}{p_0^2-[(\text{p}-\text{k})^2+m^2]} + \frac{\sigma_2(\text{l}\sigma)[(\text{p}+\text{k})\sigma](\text{l}'\sigma)}{p_0^2-[(\text{p}+\text{k})^2+m^2]}\right)_{\alpha_2\alpha_1}\psi^{\text{L}}_{\alpha_1\alpha_2}(p)\,d^4p$$
$$+\,2m\int\left(\frac{\sigma_2(\text{l}'\sigma)(\text{l}\sigma)}{p_0^2-[(\text{p}-\text{k})^2+m^2]}+\frac{\sigma_2(\text{l}\sigma)(\text{l}'\sigma)}{p_0^2-[(\text{p}+\text{k})^2+m^2]}\right)_{\alpha_2\alpha_1} \times \psi^{\text{S}}_{\alpha_1\alpha_2}(p)\,d^4p\Bigg]\delta(K-k-k'), \quad (8)$$

where $\psi^{\text{L}}(\text{p})$ is the large (two-row) component of $\psi(\text{p})$, and $\psi^{\text{S}}(\text{p})$ is one of the two small (two-row) components of $\psi(\text{p})$, which are of order v. The small components (of order v) give equal contributions to (8), which explains the factor 2 in the second integral of that equation.

When integrating over the fourth component p_0 of the momentum in Eq. (8), it is convenient to expand the coefficient of ψ^{L} or ψ^{S} in powers of p_0^2. This corresponds to expanding the integral in powers of v^4. For our purposes the zeroth order term of this series is sufficient, namely

$$\int\frac{\psi^{\text{L}}(\text{p}, p_0)\,dp_0}{p_0^2-[(\text{p}-\text{k})^2+m^2]} = \frac{-1}{(\text{p}-\text{k})^2+m^2}\int\psi^{\text{L}}(\text{p}, p_0)\,dp_0 = \frac{-1}{(\text{p}-\text{k})^2+m^2}\,2\pi\psi^{\text{L}}(\text{p}, t=0). \quad (9)$$

A similar expression can be obtained for $\psi^{\text{S}}(\text{p})$. Thus after integrating over p_0, the probability amplitude of Eq. (8) contains the functions ψ^{L} and ψ^{S} evaluated at the same time $t_1 = t_2$ for both particles. In our approximation $\psi^{\text{L}}(\mathbf{p})$ evaluated at $t = 0$ is just the nonrelativistic two-component positronium wave function in momentum space. We shall evaluate $\psi^{\text{S}}(\mathbf{p})$ at $t = 0$ in the following way.

Up to terms of order v inclusive, the wave function of a single electron (or positron) in an external potential field is of the form

$$\begin{pmatrix}\chi \\ (\text{p}\sigma)\chi/2m\end{pmatrix} f(\text{x}), \quad (10)$$

where χ is a two-component spinor depending only on the spin indices, and $\mathbf{p} = -i\nabla$ is a differential operator acting on the nonrelativistic wave function $f(\mathbf{x})$ which depends on the coordinates and describes the motion of the particle in the external potential field (the expression in (10) can be replaced by a superposition of similar four-component functions). Bearing in mind Eq. (10), it is easy to find as accurate a positronium wave function with $t_1 = t_2$. This wave function is

$$\begin{pmatrix}\Phi & -\Phi\frac{(\text{p}\sigma^{\text{T}})}{2m} \\ \frac{(\text{p}\sigma)}{2m}\Phi & 0\end{pmatrix}\psi_{\text{nr}}(\text{x}). \quad (11)$$

Here Φ is a two-row spin wave function of two particles, and $\mathbf{p} = -i\nabla$ is a differential operator which acts on the nonrelativistic positronium wave function $\psi_{\text{nr}}(\mathbf{x})$. The numbers 1 and 2 in the spin indices $\Phi_{\alpha_1\alpha_2}$ as well as in the relative coordinates $\mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2$ denote the electron and positron, respectively. The superscript T denotes the transposed matrix.

If the positronium is in an eigenstate of the total angular momentum and perhaps of other physical quantities, it is described by a superposition of functions of the form $\Phi\psi_{\text{nr}}$. Then the four-row positronium wave function is a superposition of the four-row functions of Eq. (11), and can be written in general as

$$\begin{pmatrix}\varphi(\text{x}) & -\varphi(\text{x})\frac{(\text{p}\sigma^{\text{T}})}{2m} \\ \frac{(\text{p}\sigma)}{2m}\varphi(\text{x}) & 0\end{pmatrix}, \quad (12)$$

where the nonrelativistic two-row positronium wave function $\varphi(\mathbf{x})$ is an eigenfunction of the total angular momentum of the system (as well as of other physical observables making up a complete set). In Eq. (12) the differential operator $\mathbf{p}$ multiplying $\boldsymbol{\sigma}^{\text{T}}$ acts on the function $\varphi(\mathbf{x})$ on its left.

It follows from (9) and (12) that the remaining integrals in Eq. (8) are of the form

$$\int F(\text{p})\,\varphi(\text{p})\,d^3p, \quad (13)$$

where $\varphi(\mathbf{p})$ is defined in (12), and $F(\mathbf{p})$ denotes all the other functions of $\mathbf{p}$ which enter into the integral. If we now make use of the fact that the positronium wave function $\varphi(\mathbf{p})$ differs significantly from zero only in the small-momentum region where $p/m = v \ll 1$, we may calculate (13) up to terms of order v inclusive. We then obtain

$$\int F(\mathbf{p})\,\varphi(\mathbf{p})\,d^3p = \int\left(F(0) + p_n\frac{\partial F(0)}{\partial p_n}\right)\varphi(\mathbf{p})\,d^3p$$

$$= (2\pi)^3 F(\mathbf{p}=0)\,\varphi(\mathbf{x}=0) + \frac{(2\pi)^3}{i}\frac{\partial F(\mathbf{p}=0)}{\partial p_n}\frac{\partial\varphi(\mathbf{x}=0)}{\partial x_n}. \qquad (14)$$

3. ANNIHILATION OF POSITRONIUM IN THE S-STATE

Using (8), (9), and (14) the probability amplitude for two-photon annihilation of positronium in the S-state is given by

$$A = i\,\frac{(2\pi)^5 e^2}{2m^3}\left(\sigma_2\left[(\mathbf{l}\sigma)(\mathbf{k}\sigma)(\mathbf{l}'\sigma)\right.\right.$$

$$\left.\left. - (\mathbf{l}'\sigma)(\mathbf{k}\sigma)(\mathbf{l}\sigma)\right]\right)_{\alpha_2\alpha_1}\varphi_{\alpha_1\alpha_2}(\mathbf{x}=0)\,\delta(K-k-k').$$

$$= \frac{(2\pi)^5 e^2}{m^3}\mathbf{k}\,[\mathbf{l}\times\mathbf{l}']\,\mathrm{Sp}\,(\sigma_2\varphi(0))\,\delta(K-k-k'). \qquad (15)$$

Since σ_2 is an antisymmetric matrix, this probability amplitude differs from zero only for states whose wave function is antisymmetric in the spin indices. This means that the probability for two-photon positronium annihilation differs from zero only for states with total spin $s = 0$ (parapositronium), and vanishes if $s = 1$ (orthopositronium).[7] Finally, it follows from (15) that when positronium annihilates in the S-state giving off two photons, the polarizations of these photons are mutually perpendicular.

According to (15), the probability W for two photon annihilation of positronium in the S-state is

$$W = \frac{e^4}{4m^4}\int\sum_{\mathbf{l},\mathbf{l}'}(\mathbf{k}\,[\mathbf{l}\times\mathbf{l}'])^2\; d\Omega_k\;|\mathrm{Sp}\,(\sigma_2\varphi(0))|^2$$

$$= \frac{4\pi e^4}{m^2}|\varphi(0)|^2 = \frac{1}{2n^3}(e^2)^5 m. \qquad (16)$$

If the principal quantum number n takes on its minimum value $n = 1$, we have $W = (e^2)^5 m/2 = 0.8 \times 10^{10}\,\mathrm{sec}^{-1}$, which agrees with the well known results of Pomeranchuk.[7] We have introduced the factor $1/2$ into the formula for the probability of two-photon annihilation in order to take account of the two identical states of the system in which the photon momenta are interchanged.

4. ANNIHILATION OF POSITRONIUM IN THE P-STATE

Equation (5) can be used to calculate the probability of two-photon annihilation of positronium in any excited state. If, in particular, the positronium is in the P-state and its wave function is such that $\varphi(\mathbf{p}) = -\varphi(-\mathbf{p})$, the first term in (14) vanishes and the probability amplitude for two-photon annihilation of positronium in the P-state is given by the second term of the sum, namely

$$A = \frac{(2\pi)^5 e^2}{m}\left(\frac{\partial}{\partial k_n}\frac{\sigma_2\left[(\mathbf{l}'\sigma)(\mathbf{k}\sigma)(\mathbf{l}\sigma) + (\mathbf{l}\sigma)(\mathbf{k}\sigma)(\mathbf{l}'\sigma)\right]}{\mathbf{k}^2 + m^2} + \frac{(\mathbf{l}\mathbf{l}')\,\sigma_2\sigma_n}{m^2}\right)_{\alpha_2\alpha_1}$$

$$\times\frac{\partial\varphi_{\alpha_1\alpha_2}(0)}{\partial x_n}\,\delta(K-k-k')$$

$$= \frac{(2\pi)^5 e^2}{m^5}\left[m^2(l_n l'_m + l_m l'_n)\right.$$

$$\left. + (\mathbf{l}\mathbf{l}')\,k_n k_m\right]\frac{\partial}{\partial x_n}\mathrm{Sp}\,(\sigma_2\sigma_m\varphi(0))\,\delta(K-k-k'). \qquad (17)$$

The matrix $\sigma_2\sigma_m$ is symmetric, so that (17) fails to vanish only for states whose total spin $s = 1$ (orthopositronium). If, on the other hand, the total spin $s = 0$ (parapositronium), the probability for two-photon annihilation of positronium in the P-state vanishes by Landau's theorem,[8] which asserts that a two-photon system has no states whose total angular momentum is one. From the symmetry of (17) with respect to interchange of l and l' we may conclude that the polarizations of the photons are parallel in the two-photon annihilation of positronium in the P-state.

Equation (17) can be used to find the probability of two-photon annihilation of orthopositronium in the P-states, namely

$$W = \frac{e^4}{4m^8}\int\sum_{\mathbf{l},\mathbf{l}'}\left|\left[m^2(l_n l'_m + l_m l'_n) + (\mathbf{l}\mathbf{l}')\,k_n k_m\right]B_{nm}\right|^2 d\Omega_k$$

$$= \frac{2\pi e^4}{15m^4}\left[11\,|\mathrm{Sp}\,B|^2 + 6\,\mathrm{Sp}\,B^*\,(B + B^T)\right], \qquad (18)$$

where we have written

$$\partial\,\mathrm{Sp}\,(\sigma_2\sigma_m\varphi(0))/\partial x_n = B_{nm}.$$

In order to calculate B we must know the total angular momentum eigenfunctions $\varphi_{J,M}(\mathbf{x})$ of orthopositronium with $J = 0, 1$, and 2 and with the z-component of the total angular momentum $M = 0, \pm 1, \ldots, \pm J$. Solving the eigenvalue problem in the usual way, we obtain

$$\varphi_{0,0}=\frac{1}{\sqrt{3}}\begin{pmatrix}\psi_{-1} & -1/\sqrt{2}\,\psi_0\\ -1/\sqrt{2}\,\psi_0 & \psi_1\end{pmatrix},\quad \varphi_{1,M}=\frac{1}{2}\begin{pmatrix}-\sqrt{(1+M)(2-M)} & \psi_{M-1}M\psi_M\\ M\psi_M & \sqrt{(1-M)(2+M)}\,\psi_{M+1}\end{pmatrix},$$

$$\varphi_{2,M}=\frac{1}{2}\begin{pmatrix}\sqrt{(1+M)(2+M)/3}\,\psi_{M-1} & \sqrt{(2-M)(2+M)/3}\,\psi_M\\ \sqrt{(2-M)(2+M)/3}\,\psi_M & \sqrt{(1-M)(2-M)/3}\,\psi_{M+1}\end{pmatrix}, \qquad (19)$$

where $\psi_m \equiv R_{nl}(r)Y_{1m}(\theta,\varphi)$ is the Schroedinger wave function of positronium with principal quantum number n, orbital quantum number $l = 1$, and magnetic quantum number m. The gradient of $\psi_n(\mathbf{x})$ at $\mathbf{x} = 0$ is equal to the gradient of

$$\left(\frac{R_{n1}}{r}\right)_{r=0} rY_{1m}(\theta,\varphi)\equiv\frac{2}{3}\left(\frac{n^2-1}{a^5n^5}\right)^{1/2}\cdot rY_{1m}(\theta,\varphi). \qquad (20)$$

Here r, θ, and φ are spherical coordinates, $a = 2/me^2$ is the Bohr radius, and the $Y_{1m}(\theta,\varphi)$ are defined in terms of the associated Legendre polynomials $P_1^m(\cos\theta)$ by[9]

$$Y_{1m}(\theta,\varphi)=\frac{1}{\sqrt{2\pi}}\Theta_{1m}(\theta)\,e^{im\varphi}, \qquad (21)$$

where

$$\Theta_{1m}(\theta)=(-1)^m\sqrt{\frac{3(1-m)!}{2(1+m)!}}P_1^m(\cos\theta)\quad\text{for } m\geqslant 0, \qquad (22)$$

$$\Theta_{1,-|m|}(\theta)=(-1)^m\Theta_{1,|m|}(\theta)\quad\text{for } m<0. \qquad (23)$$

Finally, in agreement with Tumanov[2] we obtain the following result for the probability W of two-photon annihilation of orthopositronium in the P-state.

(a) For J = 0 and M = 0 we have

$$W=\frac{n^2-1}{8n^5}(e^2)^7m,$$

and in particular for n = 2, we obtain $W = 1.0 \times 10^4\ \text{sec}^{-1}$.

(b) For J = 1 and M = 0 or ±1, we have

$$W=0,$$

which follows also from the work of Landau.[8]

(c) For J = 2 and M = 0, or ±1, or ±2 we have

$$W=\frac{n^2-1}{30n^5}(e^2)^7m,$$

and in particular for n = 2, we obtain $W = 0.26 \times 10^4\ \text{sec}^{-1}$.

As is seen from the above results, the lifetime of positronium with respect to annihilation in the P-state is long enough for its visual spectrum to be observed.

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