USE OF THE OPTICAL MODEL FOR THE ANALYSIS OF $\pi - p$ AND $p - p$ SCATTERING AT HIGH ENERGIES

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π-p and p-p scattering at energies above 1 Bev in the laboratory system has been analyzed on the basis of a model in which the nucleon is regarded as an optically homogeneous sphere with sharp boundaries and a complex refractive index. It is shown that the available experimental data can be fitted by choosing a sphere of radius $R = (1.08 \pm 0.07) \times 10^{-13}$ cm, which is independent of the type of the interacting particles and of their energy. The optical parameters of the sphere are computed. The contributions to the elastic scattering cross section from the real and imaginary parts of the scattering amplitude are estimated. One can assume that the contribution from the real part of the scattering amplitude is small for π-meson energies of 1.37 Bev and for proton energies above ~ 5 Bev, and thus at higher energies the scattering can be analyzed using the general scattering theory without spin, or else on the basis of the model of a purely absorbing sphere. The possible behavior of the cross sections for π-p and p-p interaction with increasing energy of the colliding particles is discussed.

1. INTRODUCTION

IN 1949, Fernbach, Serber, and Taylor[1] made use of an optical analogy for the analysis of neutron scattering from nuclei at high energies. Nuclear matter was regarded as a refractive and absorptive medium. During the passage of the neutron through the medium its wave vector, which is equal to k_0 outside the nucleus, takes on the complex value

$$k = k_0 + k_1 + iK/2, \qquad (1)$$

where K denotes the absorption coefficient of the medium, and k_1 is the change in the real part of the wave vector of the neutron. Then, in analogy with optics, one can characterize the nuclear matter by a complex refraction index $n = k/k_0$. This optical model received wide attention in the analysis of the experimental data on the scattering of fast particles from nuclei, and led to many valuable results.

The formalism of the optical model of the nucleus was subsequently used in the analysis of the scattering of π mesons and protons from protons at energies of order 1 Bev and higher.[2-4] This approach may be called the optical model of the nucleon. It was shown that the experimental data on π-p scattering at 1.37 Mev (Ref. 3) could be fitted by a model in which the nucleon is represented as a homogeneous sphere of radius $R = (1.18 \pm 0.1) \times 10^{-13}$ cm, characterized by a coefficient of absorption $K = 0.67 \times 10^{-13}\,\text{cm}^{-1}$ and $k_1 = 0$.

Thus the sphere can be considered purely absorbing.

In an analogous manner the data on p-p scattering for the energies 0.8, 1.5, and 2.75 Bev (Ref. 2) were analyzed. The proton was regarded as a homogeneous, purely absorbing sphere ($k_1 = 0$) of radius R, which is independent of the energy. The incoherent scattering was considered insignificant. With these assumptions the experimental data could be fitted by taking for the proton model a sphere of radius $R = 0.93 \times 10^{-13}$ cm with absorption coefficients $K = 4.3$, 3.7, and $2.7 \times 10^{13}\,\text{cm}^{-1}$ for proton energies 0.8, 1.5, and 2.75 Bev, respectively. A comparison of the results of the analysis shows that the proton is more "transparent" for the π meson than for a proton, and that the range of the π-p interaction is greater than that of the p-p interaction. Recently published data on the elastic scattering of protons by protons at 2.24, 4.40, and 6.15 Bev (Ref. 4) were analyzed from the point of view of the extremely simple optical model. In all variants of this model, the nucleon is represented by a disc with different optical parameters. With a certain choice of the parameters of this disc, it was possible to achieve

TABLE I

E_p, Bev	$10^{14}\lambda$, cm	σ_t, mb	σ_e, mb	σ_i, mb
1.5	2.35	47.2±0.9[10]	20±2[2]	27±2[2]
2.24	1.92	44.1±4[8]	17±3[4]	26±2[8]
2.75	1.73	41 ±1[10]	15±2[2]	26±2[2]
4.40	1.37	34 ±2[8]	9.7±1.5[4]	24.2±2[8]
6.15	1.16	31.3±1.5[8]	7.5±1.5[4]	23.8±2[8]
E_π, Bev				
1.37	2.7	30.6±2.8[9]	6.6±1[3]	24.0±1.5[3]

agreement with experiment.

We made an attempt to analyze all available experimental data on p-p and π-p scattering, in the energy region of several Bev, on the basis of the optical model of the nucleon. The analysis was carried through under the assumption that the range of interaction is determined by an optically homogeneous sphere with sharp boundary, and that the incoherent elastic scattering can be neglected. In light of this analysis, we examined several aspects concerning the use of the general scattering formalism in the problem under consideration, in analogy to the work done earlier.[5-8]

2. EXPERIMENTAL DATA AND FORMULAE USED IN THE ANALYSIS

Table I summarizes the experimental data on the scattering cross sections at high energies, including the values of the total cross section (σ_t), the elastic cross section (σ_e), and the inelastic cross section (σ_i) for p-p and π-p interactions. The second column of the table gives the value of the wavelength λ of the impinging particle in the center-of-mass system for the corresponding energy E of the impinging particle in the laboratory system. The σ_i for $E_p = 2.24$, 4.40, and 6.15 Bev have not been experimentally measured. They have been obtained using the ideas reported in Ref. 8, and appear to be perfectly reasonable, since there are grounds to assume that σ_i changes slowly in this energy region. One can judge this from the results of the estimate for the average value of the cross section for the inelastic interactions in the nucleon-nucleon collisions at energies of the order of 50 Bev, (Ref. 11) giving for σ_i the value (21 ± 4) mb. Moreover, as will be seen from the following, the results of the analysis change very little for variations of σ_i over a wide interval of values. The choice of the value of the cross section for the inelastic interaction thus does not appear to be critical.

Apart from the data in Table I, we have available experimental differential cross sections for the elastic scattering of π mesons and protons from protons at the indicated energies. These data have also been used in the analysis.

We make a few remarks about the formulae with which the experimental results quoted above can be analyzed.

It is known that, neglecting the incoherent scattering and regarding the nucleon as an optically homogeneous sphere, the scattering cross sections can be expressed in terms of the parameters of this sphere.[1] In this case the inelastic scattering cross section is

$$\sigma_i = \pi R^2 \{1 - [1-(1+2KR)\exp(-2KR)]/2K^2R^2\} \quad (2)$$

and the differential cross section for the elastic scattering (here $\sigma_e = \sigma_d$),

$$d\sigma_d/d\omega = |f(\vartheta)|^2, \quad (3)$$

where the scattering amplitude is

$$f(\vartheta) = ik_0 \int_0^R [1 - \exp\{(-K + 2ik_1)s\}] J_0(k_0\rho \sin\vartheta)\rho d\rho. \quad (4)$$

In these formulae ϑ is the scattering angle; $s = \sqrt{R^2 - \rho^2}$.

The integration of (3) leads to a rather complicated expression for the total cross section of the elastic diffractional scattering (see, e.g., formula (6) in Ref. 1). It is known, however, that for an opacity $\sigma_i/\pi R^2 \leq 0.9$, which corresponds to $KR \leq 2.3$ and $k_1/K \leq 1$, the expression for the total cross section of the diffractional scattering can be brought into the simpler form[12]

$$\sigma_d = \sigma_d(K, k_1 = 0)\left\{1 + 4\left(\frac{k_1}{K}\right)^2\left[1 - \frac{1}{18}(KR)^2 + \cdots\right]\right\}, \quad (5)$$

where

$$\sigma_d(K, k_1 = 0) = \frac{\pi R^2}{B^2}\{B^2 - 14 - 2(1+B)e^{-B} + 8(2+B)e^{-B/2}\}, \quad B = 2KR, \quad (6)$$

which gives a result differing by not more than 1% from that obtained with the exact expression for

σ_d. We use expression (5), since the experimental data satisfy the corresponding requirements. The quoted expressions are justified for energies for which the condition $\lambda \ll R$ is true in the c.m.s.

3. PARAMETERS OF THE OPTICAL MODEL OF THE NUCLEON FOR THE DESCRIPTION OF THE SCATTERING AT HIGH ENERGIES

The parameters of the optical model of the nucleon to be determined from the experimental data are the radius of the homogeneous sphere R and its optical parameters K and k_1. The radius R of the interaction sphere was taken as the starting quantity in the determination of the parameters. For every given radius R, the values of K and k_1 were calculated at different energies of the interacting particles, using formulae (2) and (5) and the data in Table I. These values were determined for the average values of the cross section as well as for their limiting values corresponding to the quoted experimental errors.

It is seen from the curves of Bethe and Wilson[12] that for the available values of σ_e and σ_i the case $k_1 = 0$ gives maximum values for $\sigma_e/\pi R^2$ $\sigma_i/\pi R^2$, i.e., minimum values for R. Therefore, for each energy, the interval for the values of the radius considered was bounded from below by the radius of the purely absorbing "gray" sphere. The minimum values for the radii for the energies E_p are listed in Table II. For E = 1.37 Bev, $R_{min} = 1.01 \times 10^{-13}$ cm.

TABLE II

E_p, Bev	1.5	2.24	2.75	4.4	6.15
$10^{13} \cdot R_{min}$, cm	0.90	0.89	0.90	0.92	0.95

For each set of values R, K, and k_1 the differential cross sections for elastic scattering were calculated using formulae (3) and (4). Expression (3) was integrated numerically using Simpson's rule, with an accuracy not worse than 1% for small scattering angles and several percent for large ones. In the course of the calculation, it appeared that upon a 20% change in σ_i, with constant R and σ_d, the change in the value of $d\sigma_d/d\omega$ is still within the limits of error of the numerical integration. Thus the choice of the value of σ_i does not appear to be critical for this analysis.

The angular distributions for the elastic scattering were calculated for the limiting values of σ_d with fixed values of R. In the same way, the range of possible angular distributions was determined in accordance with the experimental accuracy of the total cross sections. Comparison of

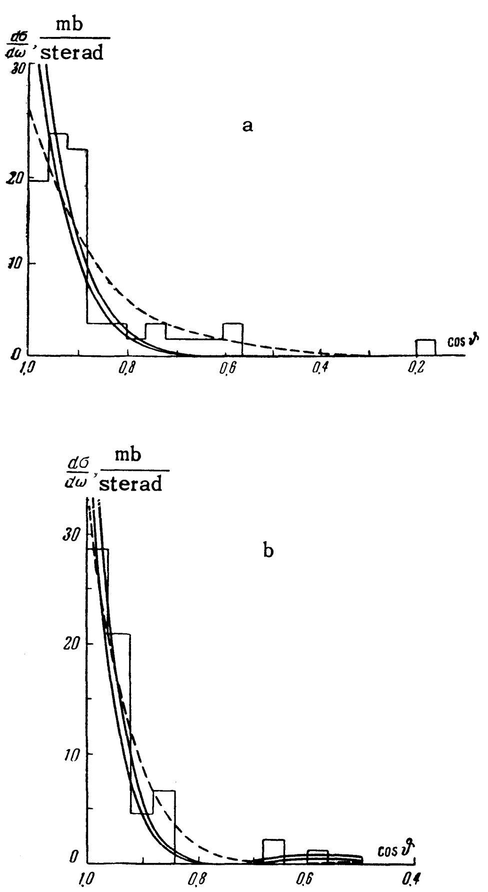
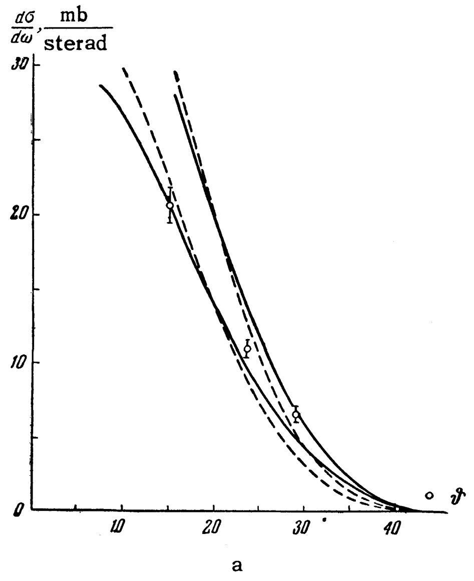


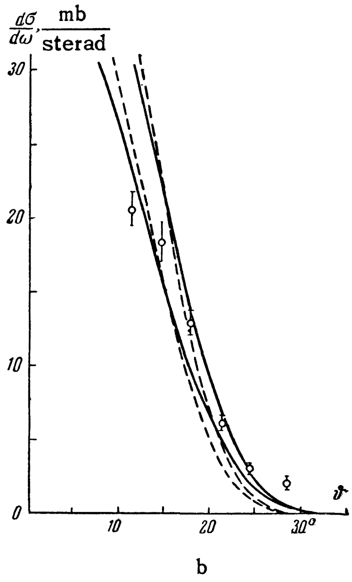
FIG. 1. The solid curves describe the region of possible angular distributions for the elastic p – p scattering in the center of mass system for the radii: (a) $E_p = 1.5$ Bev, $R = 1.3 \times 10^{-13}$ cm, and (b) $E_p = 2.75$ Bev, $R = 1.2 \times 10^{-13}$ cm. The dotted curves describe the angular distributions calculated with the model of a purely absorbing sphere with radius $R = 0.93 \times 10^{-13}$ cm for both energies. The histograms represent the experimental angular distributions of Ref. 2.

these ranges with the experimental data on the differential cross sections allowed us to choose the values of R (together with the corresponding values of k_1 and K) satisfying the experimental data with respect to the total as well as with respect to the differential scattering cross sections. We list below the results of the calculations and their comparison with the experimental data.

(a) p-p scattering at 1.5, 2.24, and 2.75 Bev.

The experimental data at 1.5 and 2.75 Bev have been obtained with a diffusion chamber[3] and have poor statistical accuracy. Therefore the experimental distributions are reconciled to an equal degree with an optical model of the nucleon having parameters that fluctuate over a very wide range. (see Fig. 1). The available counter data of Cork et al.[4] at $E_p = 2.24$ Bev, however, allow one to narrow down the range of the fluctuations of the radius of the sphere for this energy region. It





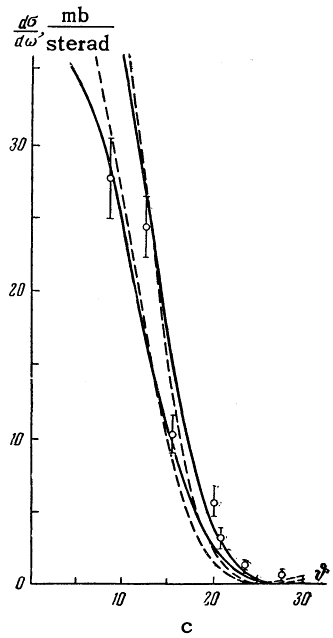


FIG. 2. The solid curves describe the region of possible angular distributions for the elastic $p-p$ scattering in the center of mass system for $R = 1.10 \times 10^{-13}$ cm. The dotted curves describe the region of angular distributions for $R=1.20 \times 10^{-13}$ cm. The experimental points are taken from Ref 4. a – $E_p = 2.24$ Bev, b – $E_p = 4.40$ Bev, c – $E_p = 6.15$ Bev.

turns out that the homogeneous sphere may have a maximum radius $R = 1.15 \times 10^{-13}$ cm, since definite disagreement with the experimental data occurs even for $R = 1.20 \times 10^{-13}$ cm (see Fig. 2a). For these values of the radius of the sphere, the contribution to the cross section of the elastic interaction from the real part of the scattering amplitude, due to values of k_1 different from zero, can reach 35%. It should be noted that for $R \leq 1.15 \times 10^{-13}$ cm agreement with experiment exists only at angles ϑ not above 30°. The minimum radius for $E_p = 2.24$ Bev is in accordance with the value 0.93×10^{-13} cm, which was given in Ref. 2 for the energies 1.5 and 2.75 Bev. Furthermore, the purely absorbing sphere ($k_1 = 0$) may only have a radius below 1.0×10^{-13} cm.

It should be noted that the considerations of Rarita[13] in connection with the applicability of the optical model of the nucleon to the p-p scattering at $E_p \approx 1.0$ Bev are equally justified for the energies considered above.

(b) p-p scattering at 4.40 and 6.15 Bev.

The experimental data at $E_p = 4.40$ Bev[4] for $\vartheta < 30°$ are satisfactorily fitted with a homogeneous sphere with a radius from 0.95×10^{-13} cm to 1.15×10^{-13} cm. The values $R = 0.92$ and 1.20×10^{-13} cm lead to definite disagreement with the experimental results (see Figs. 2b and 3a). The purely absorbing sphere may have a radius of not more than 1.10×10^{-13} cm. The measured angular distribution of the elastic scattering at $E_p = 6.15$ Bev is satisfactorily fitted with a homogeneous

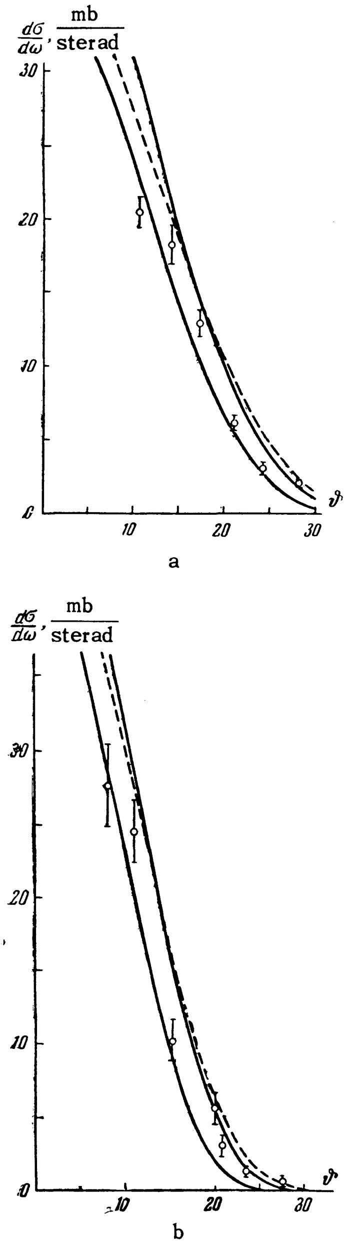


FIG. 3. The solid curves describe the region of possible angular distributions for the elastic p – p scattering in the center-of-mass system, calculated using the model of a purely absorbing sphere, based on the following data: (a) $E_p = 4.40$ Bev, $\sigma_i = 24.2$ mb, $\sigma_d = (9.7 \pm 1.5)$ mb, $R = (0.97 - 1.05) \times 10^{-13}$ cm; (b) $E_p = 6.15$ Bev, $\sigma_i = 23.8$ mb, $\sigma_d = (7.5 \pm 1.5)$ mb, $R = (1.0 - 1.15) \times 10^{-13}$ cm. The dotted curves give the angular distributions for the case of a purely absorbing sphere with minimum radius: (a) $R = 0.92 \times 10^{-13}$ cm, (b) $R = 0.95 \times 10^{-13}$ cm. The experimental points are taken from Ref. 4.

sphere with a radius from 1.0×10^{-13} cm to 1.15×10^{-13} cm. The disagreement with experiment for $R = 0.95 \times 10^{-13}$ cm and $R = 1.20 \times 10^{-13}$ cm is seen in Figs. 3b and 2c.

In addition, for $E_p = 6.15$ Bev and all values of the radius inside the indicated interval, the sphere may be purely absorbing.

(c) π-p scattering at $E_\pi = 1.37$ Bev.

The available experimental data on the scattering of π mesons by protons[3] were obtained with a diffusion chamber and have therefore bad statistical accuracy. It was found that these data could be fitted with a model of a homogeneous sphere whose radius takes on arbitrary values within a wide interval. Therefore the restriction of the range of values R was achieved with the use of the dispersion relations, which show that the contribution from the real part of the scattering amplitude to the elastic scattering cross section at the angle 0° and at $E_\pi = 1.37$ Bev amounts to about 7%.[9] For the application of the optical model, this means that the radius of the homogeneous sphere may range from 1.01×10^{-13} cm to 1.25×10^{-13} cm.

4. ON THE APPLICABILITY OF THE OPTICAL MODEL OF THE NUCLEON WITH $K_1 = 0$

Let us consider the optical model of the nucleon with $k_1 = 0$, in which the form of interaction range is not specified, in other words, we assume: (1) the elastic interaction has diffraction character and there is no potential scattering; (2) the scattering cross sections are independent of the spins of the interacting particles. These assumptions are equivalent to those made in Refs. 5 to 8. In connection with a discussion of p-p and π-p interactions from the point of view of the general scattering theory, which permits a more consistent description of the scattering at high energies. It is therefore of interest to investigate the limits of applicability of these notions.

We start from the known formula for the scattering amplitude of spinless particles (see, e.g., Ref. 14):

$$f(\vartheta) = \frac{i\lambda\!\!\!^{-}}{2} \sum_{l=0}^{\infty} (2l+1)(1-\beta_l) P_l(\cos\vartheta),$$

$$\beta_l = \exp\{2i\eta_l\}. \tag{7}$$

Here $\hbar l$ denotes the orbital angular momentum, $P_l(\cos\vartheta)$ are the Legendre polynomials, and η_l is the scattering phase. The optical model parameters are combined with the scattering phase in the following way:[1]

$$\eta_l = (k_1 + {}^1\!/_2\, iK)\, s_l. \tag{8}$$

Here $2s_l$ is the path length of the incident particle with orbital angular momentum $\hbar l$ in nuclear matter.

For the case $k_1 = 0$ the quantity $\beta_l = \exp(-Ks_l)$

must be real and positive, and the scattering amplitude pure imaginary.

We have calculated β_0 for all the interaction energies mentioned with the help of the analytic expression for $d\sigma/d\omega$ from Ref. 8. It was assumed that the angular distribution is known with an accuracy of $\pm 15\%$ for all angles.

The calculation makes use of the formula (see, e.g., Refs. 7 and 8)

$$(1-\beta_l)\lambda\!\!\!^{-} = \int_{-1}^{+1} |f(x)| P_l(x)\,dx, \quad |f(x)|^2 = \frac{d\sigma}{d\omega}. \tag{9}$$

The results of the calculation are listed in Table III.

Thus the analysis of the p-p scattering under the assumptions enumerated above leads to negative values of β_0 for energies of 1.5 to 4.4 Bev. This implies that the original conditions of this investigation are not valid for this range of proton energies. Consequently, an investigation of the type of Refs. 5 to 8 and the optical model with $k_1 = 0$ are applicable to the description of the p-p scattering only for energies above ~ 5 Bev.

An analogous consideration of the π-p scattering at $E_\pi = 1.37$ Bev gives $\beta_0 = 0.15 \pm 0.08$. This implies that the assumptions made above do not contradict the experimental data for these π-meson energies.

TABLE III

E_p, Bev	1.5	2.24	2.75	4.4	6.15
β_0(S wave)	-0.46 ± 0.13	-0.52 ± 0.13	-0.39 ± 0.12	-0.11 ± 0.10	$+0.06\pm0.08$

5. CONCLUSIONS

All available experimental data on π-p and p-p scattering in the energy region of several Bev can be fitted satisfactorily with the optical model of the nucleon, where the range of interaction is represented in the form of a homogeneous sphere with sharp boundary and complex refractive index. In the framework of this investigation, there are reasons to assume that the radius for the interaction is independent of the types of the colliding particles and of their energies. This radius ranges from 1.01×10^{-13} cm to 1.15×10^{-13} cm. Table IV gives the values of the absorption coefficient K as well as the contributions of the real parts of the scattering amplitude for three values of the radius of interaction. It is seen from the data in Table IV that the contribution to the elastic cross section from the real part of the scattering amplitude decreases with increasing energy. This means that k_1 decreases, going to zero in the limit, i.e., the homogeneous sphere becomes purely absorbing. In this limiting case the optical model drops the concept of a range of interaction, and is equivalent to an investigation of the scattering at high energies on the basis of the general scattering theory under the assumptions made in the papers of Refs. 5 to 8. On the basis of these considerations we have investigated, in Sec. 4, the limits of applicability of these concepts for the analysis of the scattering at high energies. It was found that the analysis of the experimental data under the assumptions mentioned is valid for the p-p interactions only for energies above ~ 5 Bev. The results of the π-p scattering at 1.37 Bev can be similarly fitted. The question of the applicability of this method for the analysis of the π-p scattering is still open, since we do not have as yet the necessary experimental data for smaller energies.

If, as seems entirely possible, the "form factor" of the nucleon is independent of the energy, we can draw the following conclusions about the behavior of the cross sections for the π-p and p-p interactions with increasing energy of the colliding particles. It is known that, for high energies, the cross sections for the interactions of π mesons and protons with nucleons tend towards a constant limiting value. This appears to be a consequence of the finite dimensions of the nucleon. (We neglect the Coulomb interaction.) If this limiting value for π mesons is ≈ 30 mb, then the total cross section at the π-meson energy 1.37 Bev, as considered above, is already equal to its limit calculated with the help of the dispersion relations.[15] There are grounds to expect that the values of the elastic and inelastic cross sections for the π-p interaction will not change with increasing energy, since the elastic cross section can at these energies be regarded as a consequence of the inelastic cross section. An increase or a decrease of the inelastic interaction entails a corresponding increase or decrease of the elastic interaction, leading to a change in σ_t. If these considerations are right then, for $E_\pi \to \infty$, $\sigma_t \approx 30$ mb, $\sigma_i \approx 24$ mb, and $\sigma_e \approx 6$ mb.

Similarly, it is known for the nucleon-nucleon interaction that $\sigma_i = 21 \pm 4$ mb at $E_N = 50$ Bev. This implies that the inelastic cross section

TABLE IV

E_p, Bev	$K\cdot 10^{13}$ cm^{-1}	$\lvert \mathrm{Re}\, f(\vartheta)\rvert^2/\lvert f(\vartheta)\rvert^2$, %		
		$R=1.05\cdot 10^{-13}$, cm	$R=1.1\cdot 10^{-13}$, cm	$R=1.15\cdot 10^{-13}$, cm
1.5	0.64÷2.6	6÷21	12÷27	20÷35
2.24	0.60÷2.1	8÷23	15÷30	22÷35
2.75	0.60÷2.0	5÷22	9÷28	15÷35
4.4	0.53÷1.3	0÷21	0÷29	5÷35
6.15	0.51÷1.0	0÷16	0÷23	0÷30
$E_\pi = 1.37$ Bev	0.53÷1.0	~7	~7	~7

changes slowly with energy. As our considerations above showed, one can interpret the elastic p-p interaction at $E_p = 6.15$ Bev as a consequence of the inelastic interaction. Then one may expect that, for $E_p \to \infty$, $\sigma_i \approx 24$ mb and $\sigma_e \approx 7$ mg, with $\sigma_t = 30$ to 31 mb. We thus arrive at the conclusion that for high energies of the colliding particles the total cross sections of the elastic and inelastic interactions of π mesons and nucleons with nucleons have the same values.

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