The only really interesting values of the parameter s are those much greater than unity. In fact, it follows from (23) that

$$s = \Theta_1 - \Theta_2 = l(T_1)/l(T_2), \tag{39}$$

and the ratio of the free paths must be greater than unity for the very existence of the CW, as already mentioned at the beginning of this section. (On the other hand, s is bounded from above by the condition that the wave must be weak.) In the case when $s \gg 1$, all the formulas become substantially simplified and an approximate relation can be established in explicit form between the lower temperature of the CW and the value of the adiabatic cooling. In this case the temperature T_0 about which the range is expanded drops out entirely from the equation.

Using the asymptotic expression $\overline{\mathrm{Ei}}(s) \approx e^s/s$ for $s \gg 1$, and noting that when $s \gg 1$ the root of (35) is $\beta \approx 1$, ($\tau_f \approx \ln s$), we obtain from (31) and (35)

$$\Theta_f \approx \ln \overline{\mathrm{Ei}}(s) - 1 \approx s - \ln s - 1. \tag{40}$$

From this we obtain from (38)

$$\Theta_2 \approx -\ln s. \tag{41}$$

Returning to the true temperature with the aid of (25) and taking (27) and (23) into account, we obtain the desired relation

$$Al(T_2) = S_2 = \sigma T_2^4, \tag{42}$$

It must be noted that according to (37) $\Theta_1 > \Theta_f > 0$, and according to (41) $\Theta_2 < 0$, i.e., the free path is expanded in accordance with (23) about the intermediate temperature in the CW:

$$T_2 < T_0 < T_f < T_1.$$

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EXCITATION OF VIBRATIONAL AND ROTATIONAL STATES OF NUCLEI DUE TO SCATTERING OF NUCLEONS

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Scattering of fast nucleons by black nuclei possessing vibrational or rotational levels is considered in the adiabatic approximation. It is shown that, in the diffraction region of scattering angles, the shape of the angular distributions of nucleons of definite energy, scattered with excitation of a given collective level of an even-even nucleus, does not depend on whether the level is a rotational or vibrational one.

We consider the scattering of fast neutrons or protons from nuclei possessing vibrational or rotational excited states.[1] We shall assume that the wavelength of the incident particle k^{-1} is much smaller than the nuclear dimension R ($kR \ll 1$), that the energy of the proton significantly exceeds the value of the Coulomb barrier ($Ze^2/RE \ll 1$), and that the nucleus absorbs all particles incident upon it (black nucleus). These assumptions correspond to neutron energies $E \gtrsim 10$ Mev and proton energies $E \gtrsim 20$ Mev. In this case, it is appropriate to make use of the adiabatic approximation, according to which the nucleus can be regarded as fixed during the scattering process. The condition of applicability of this approximation[2] can be written in the form $(\Delta\epsilon/E)kR \ll 1$, where $\Delta\epsilon$ is the energy of the collective excitation.

As is known, the determination of the effective cross sections in the adiabatic approximation reduces to the calculation of the amplitude of elastic scattering of particles by a nucleus of fixed orientation $f(\alpha_{lm}, \Omega)$. This amplitude depends not only on the direction of the scattering $\Omega = (\vartheta, \Phi)$, but also on the parameters α_{lm} which determine the shape of the nucleus (in the fixed system of reference):

$$r(\mathbf{n}) = R\Big(1 + \sum_{lm} \alpha_{lm} Y_{lm}(\mathbf{n})\Big),$$

where $Y_{lm}(\mathbf{n})$ are the normalized spherical harmonics. In the case of nuclei possessing vibrational levels,[1] the quantities α_{lm} can be connected with the operators of creation and annihilation of excitation quanta (phonons) having a momentum l with a projection m on the fixed axis:

$$\alpha_{lm} = \sqrt{\frac{4\pi}{2l+1}}\,\frac{p_l}{R}\,(b_{lm} + (-)^m b^*_{l,-m}), \tag{1}$$

where p_l is the amplitude of the zero vibrations about the equilibrium sphere of radius R. The operators b_{lm}, b^*_{lm} act as usual on the wave functions of the vibrational states $\psi_n(\alpha_{lm})$:

$$b\psi_n(\alpha) = \sqrt{n}\,\psi_{n-1}(\alpha); \quad b^*\psi_n(\alpha) = \sqrt{n+1}\,\psi_{n+1}(\alpha). \tag{2}$$

If, on the other hand, the nucleus possesses rotational levels, and its surface in the characteristic system of reference of the nucleus is described by the equation

$$R(\mu) = R\Big(1 + \sum_l \alpha_l P_l(\mu)\Big),$$

then the parameters α_{lm} depend in the following way on the angles $\omega(\theta, \varphi)$ which define the direction of the axis of symmetry of the nucleus in the fixed system of coordinates:

$$\alpha_{lm}(\omega) = \frac{4\pi\alpha_l}{2l+1} Y^*_{lm}(\omega). \tag{3}$$

In small-angle scattering $\theta < 1$ and for the conditions given above the amplitude of the elastic scattering of nucleons from a fixed nucleus can be given in the form[2]

$$f(\omega, \Omega) = f_e(\omega, \Omega) + f_d(\omega, \Omega). \tag{4}$$

Here $f_e(\omega, \Omega)$ is the scattering amplitude of particles in the electric field of the fixed nucleus. Considering scattering at small angles, we can neglect the finite charge distribution in the nucleus. For a sufficiently small departure of the nuclear shape from spherical, when the conditions

$$\alpha_2 Ze^2/\hbar v \ll 1 \quad \text{or} \quad (p_2/R)\,Ze^2/\hbar v \ll 1 \tag{5}$$

are satisfied for nuclei with rotational or vibrational levels, respectively, the effect of multipole electric interaction on the scattering can be taken into account by perturbation theory. In this case, we can write down the scattering amplitude in the electric field of the nucleus in the form

$$f_e(\omega, \Omega) = f_c(\theta) + \sum_{lm} \alpha_{lm}(\omega)\,\langle \mathbf{k}' | V_{lm} | \mathbf{k}\rangle;$$

$$f_c(\theta) = -\frac{2\eta}{k}\frac{\Gamma(1+i\eta)}{\Gamma(1-i\eta)} \exp\left\{-2\eta i \ln\frac{\theta}{2}\right\}; \quad \eta = \frac{Ze^2}{\hbar v}, \tag{6}$$

$$\langle \mathbf{k}' | V_{lm} | \mathbf{k}\rangle = -\frac{\mu}{2\pi\hbar^2}\frac{3Ze^2R^l}{(2l+1)}\int \psi^+_{-\mathbf{k}'}(\mathbf{r})\, r^{-l-1} Y_{lm}\left(\frac{\mathbf{r}}{r}\right)\psi^+_{\mathbf{k}}(\mathbf{r})\,d\mathbf{r},$$

where $\mathbf{k}$ and $\mathbf{k}'$ are the wave vectors of the incident and scattered particle, respectively, and the $\psi^+_{\mathbf{k}}(\mathbf{r})$ are the wave functions that describe the scattering in the Coulomb field Ze^2/r (Ref. 3).

According to Ref. 2, the second term in Eq. (4) has the form

$$f_d(\omega, \Omega) = i\,\frac{(kR)^{2(1+i\eta)}}{k}\,\xi(\omega)\,t^{-2(1+i\eta)} \int_0^t x^{1+2i\eta} J_0(x)\,dx;$$

$$t = kR\theta\,[\xi^2(\omega)\cos^2(\varphi - \Phi) + \sin^2(\varphi - \Phi)]^{1/2}; \tag{7}$$

$$\xi(\omega) = [1+\varepsilon]^{-1}; \quad \varepsilon = \sum_l \sqrt{\frac{2l+1}{4\pi}}\,\alpha_{l0}(\omega).$$

When $\eta = 0$, the amplitude of $f_d(\omega, \Omega)$ describes the diffraction scattering of neutrons; it is therefore natural to call it the diffraction part of the amplitude of $f(\omega, \Omega)$.

We shall assume that the nucleus is in the ground state prior to scattering, and shall limit ourselves below to a consideration of elastic scattering with a transition of the nucleus from the ground to the first excited state. We shall consider only quadrupole deformations of the nucleus, assuming that the only α_{lm} different from zero are those with $l = 2$. In this case it is appropriate, for calculation of the effective cross section, to expand the amplitude $f(\omega, \Omega)$ in a series of powers of the deformation parameter α_{lm}. If condition (5) is satisfied, then Eq. (6) already gives essentially this expansion for the amplitude $f_e(\omega, \Omega)$. In order to obtain a similar expansion of the diffraction part $f_d(\omega, \Omega)$, it is useful to expand in powers of ϵ in Eq. (7). Denoting $kR\theta = a$, we get

$$f_d(\omega, \Omega) = \frac{i}{k}(kR)^{2(1+i\eta)}\left\{a^{-2(1+i\eta)}\int_0^a x^{1+2i\eta} J_0(x)\,dx\right.$$

$$+\alpha_{20}(\omega)\sqrt{\frac{5}{4\pi}}\left[i\eta a^{-2(1+i\eta)}\int_0^a x^{1+2i\eta}J_0(x)\,dx-\frac{1}{2}J_0(a)\right]$$
$$-\left(e^{i2\Phi}\alpha_{22}(\omega)+e^{-i2\Phi}\alpha_{2-2}(\omega)\right)\frac{1}{4}\sqrt{\frac{5}{6\pi}} \quad (8)$$
$$\times\left[(1+i\eta)\,a^{-2(1+i\eta)}\int_0^a x^{1+2i\eta}J_0(x)\,dx-\frac{1}{2}J_0(a)\right]\Big\}+\ldots$$

When calculating the effective cross sections, we can limit ourselves to the expansion terms written down, if

$$\alpha_2 kR\theta \ll 1 \text{ or } (p_2/R)\,kR\theta \ll 1. \quad (9)$$

We can obtain these conditions if we compute the terms of order α_{2m}^2 and α_{2m}^3 and compare their contribution to the scattering cross section with the contribution from terms of zeroth and first order in α_{2m}, putting $\eta \sim 1$.

In adiabatic approximation, as is well known, the differential scattering cross section, for which the nucleus undergoes a transition from the state $\varphi_\nu(\omega)$ to the state $\varphi_{\nu'}(\omega)$, is determined by the square of the modulus of the matrix element $\langle\varphi^*_{\nu'}(\omega)\,f(\omega,\Omega)\,\varphi_\nu(\omega)\rangle$. In particular, the excitation cross section of the rotational state (I, M) of the even-even nucleus has the form

$$\sigma_{IM}(\Omega)=|\langle Y^*_{IM}(\omega)\,f(\omega,\Omega)\,Y_{00}(\omega)\rangle|^2. \quad (10)$$

Similarly, in the case of scattering from a nucleus with vibrational levels, the excitation cross section of n phonons (l, m) is equal to

$$\sigma^{(n)}_{lm}(\Omega)=|\langle\psi^*_n(\alpha_{lm})\,f(\alpha_{lm},\Omega)\,\psi_0(\alpha_{lm})\rangle|^2. \quad (11)$$

In this case the matrix elements of the amplitude $f(\alpha_{2m}, \Omega)$, which determine the elastic and inelastic scattering cross sections, have in accord with Eqs. (1, 2, 4, 6, and 8), the form:

$$\langle\psi^*_0(\alpha_{2m})\,f(\alpha_{2m},\Omega)\,\psi_0(\alpha_{2m})\rangle=\frac{i}{k}(kR)^{2(1+i\eta)}a^{-2(1+i\eta)}\int_0^a x^{1+2i\eta}J_0(x)\,dx+f_c(\theta);$$
$$\langle\psi^*_1(\alpha_{20})\,f(\alpha_{2m},\Omega)\,\psi_0(\alpha_{20})\rangle=\frac{p_2}{R}\Big\{i\frac{(kR)^{2(1+i\eta)}}{k}\Big[i\eta a^{-2(1+i\eta)}\int_0^a x^{1+2i\eta}J_0(x)\,dx$$
$$-\frac{1}{2}J_0(a)\Big]+\sqrt{\frac{4\pi}{5}}\langle\mathbf{k}'|V_{20}|\mathbf{k}\rangle\Big\}, \quad (12)$$
$$\langle\psi^*_1(\alpha_{2\pm1})\,f(\alpha_{2m},\Omega)\,\psi_0(\alpha_{2\pm1})\rangle=-\frac{p_2}{R}\sqrt{\frac{4\pi}{5}}\langle\mathbf{k}'|V_{2\mp1}|\mathbf{k}\rangle;$$
$$\langle\psi^*_1(\alpha_{2\pm2})\,f(\alpha_{2m},\Omega)\,\psi_0(\alpha_{2\pm2})\rangle$$
$$=\frac{p_2}{R}\Big\{-\frac{e^{\pm i2\Phi}}{2\sqrt{6}}\,i\,\frac{(kR)^{2(1+i\eta)}}{k}\Big[(1+i\eta)\,\alpha^{-2(1+i\eta)}\int_0^a x^{1+2i\eta}J_0(x)\,dx$$
$$-\frac{1}{2}J_0(a)\Big]+\sqrt{\frac{4\pi}{5}}\langle\mathbf{k}'|V_{2\mp2}|\mathbf{k}\rangle\Big\}.$$

Making use of the relation (3), we note that the matrix elements that determine the scattering cross section from a nucleus with rotational levels differ from the matrix elements (12) only by a factor which is independent of the scattering angle, namely,

$$\langle Y^*_{00}(\omega)\,f(\omega,\Omega)\,Y_{00}(\omega)\rangle$$
$$=\langle\psi^*_0(\alpha_{2m})\,f(\alpha_{2m},\Omega)\,\psi_0(\alpha_{2m})\rangle;$$
$$\langle Y^*_{2m}(\omega)\,f(\omega,\Omega)\,Y_{00}(\omega)\rangle$$
$$=\frac{R}{p_2}\frac{\alpha_2}{\sqrt{5}}\langle\psi^*_1(\alpha_{2m})\,f(\alpha_{2m},\Omega)\,\psi_0(\alpha_{2m})\rangle. \quad (13)$$

It is seen from Eqs. (10) to (13) that in a region of sufficiently small angles (9), the form of the angular distribution of the nucleons of a given energy, which scatter with the excitation of a given collective state of the black nucleus, does not depend on whether this state is rotation or vibrational. The forms of the angular distributions

$$\sigma_2(\Omega)=\sum_M\sigma_{2M}(\Omega) \quad\text{and}\quad \sigma^{(1)}_2(\Omega)=\sum_m\sigma^{(1)}_{2m}(\Omega),$$

of the corresponding excited levels are also independent of the nature of the level.

It is curious to note that the collective states of an even-even nucleus with odd projections of the moment are excited only because of the electrical interaction, i.e., they do not arise in neutron scattering. We also note that the equations in (4), which describe the scattering of neutrons on nuclei with vibrational levels, follow from Eqs. (11) and (12) for $\eta = 0$.

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KINETIC THEORY OF MAGNETOHYDRODYNAMIC WAVES

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We take account of thermal motion of electrons and ions in considering the propagation of magnetohydrodynamic waves in an ionized gas.

As has been shown by Åström[1,2] and Ginzburg,[3] magnetohydrodynamic waves in an ionized gas are nothing more than low-frequency ordinary and extraordinary electromagnetic waves, familiar from the theory of the propagation of radio waves in the ionosphere. The frequency of these waves is much less than the Larmor frequency of the ions. In the above-cited works the electron and ion motions were described by equations for their mean velocities. The phase velocity V_Φ of a magnetohydrodynamic wave is usually much less than the velocity of light c, and may be comparable with the mean thermal velocity v_T^e and v_T^i of the electrons and ions. One can therefore expect that if $V_\Phi \lesssim v_T^e$, the thermal velocity of the charged particles will strongly influence the propagation of the magnetohydrodynamic waves.

If the frequency ω of the magnetohydrodynamic waves is much less than the frequency ν_c of "short-range" collisions, and if the wavelength λ is large compared with the mean free path, a local Maxwell distribution is established during a time on the order of $2\pi/\omega$. In this case, as is well known, the equations of hydrodynamics can be used, and it follows that in addition to magnetohydrodynamic waves of the Alfven type, two mixed magnetosound waves may propagate in the plasma. If, on the other hand, $\omega \gg \nu_c$, the thermal motion of the charged particles can be taken into account by finding the magnetohydrodynamic wave propagation using the kinetic equation with self-consistent interaction.[4]

The present work is devoted to the kinetic theory of magnetohydrodynamic waves propagating in a plasma at any angle θ with respect to an external magnetic field. "Short-range" collisions leading to damping of the waves are not included. The case $\theta = 0$ has been treated by Gershman[5] (see also Dungey[6]). It is found that if $\theta = 0$, the "short-range" collisions give only a small contribution even if it is not true that $\nu_c \ll \omega$.[3,5,6] In any case, the effect of "short-range" collisions will be small for arbitrary θ if $\nu_c \ll \omega$.

1. DISPERSION EQUATION

Consider electromagnetic waves propagating in a plasma of electrons and singly ionized ions. Let $f_{0\alpha}$ be the equilibrium value of the distribution function for particles of type α ($\alpha = e$ denotes electrons, and $\alpha = i$ denotes ions). We shall write a kinetic equation for $f_\alpha(\mathbf{v}, \mathbf{r}, t)$, the small difference between the actual value of the distribution function and $f_{0\alpha}$, assuming that the frequency of the waves is so high that we may neglect the collision integral in this equation. We then have

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\frac{\partial f_\alpha}{\partial \mathbf{r}} + \frac{e_\alpha}{m_\alpha}\mathbf{E}\frac{\partial f_{0\alpha}}{\partial \mathbf{v}} - \omega_H^\alpha \frac{\partial f_\alpha}{\partial \vartheta} = 0, \quad (1)$$

$$\omega_H^\alpha = e_\alpha H_0 / m_\alpha c, \quad f_{0\alpha} = n_0 (m_\alpha / 2\pi T_\alpha)^{3/2} \exp(-m_\alpha v^2 / 2T_\alpha).$$

Here e_α and m_α are the charge and mass of the particles of type α (with $e_i = e > 0$), $\mathbf{H}_0$ is the external magnetic field strength, ϑ is the polar angle in velocity space ($\mathbf{v}$ is the velocity of par-