rents and densities the formula has to be modified.

This way one can give a mathematical description of the treated physical picture of the capture process. The theory then gives good agreement with experiment, both qualitatively and quantitatively.

[1] V. I. Logunov and S. S. Semenov, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1513 (1957), Soviet Phys. JETP **6**, 1168 (1958).

Translated by M. Danos
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ANALYTICITY OF THE NONRELATIVISTIC SCATTERING AMPLITUDE AND THE POTENTIAL

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As is well known, the amplitude for scattering of a particle of given angular momentum l by a central field of force cannot be analytically continued into the upper half plane of the variable k. An interesting proof of this is connected with the inverse problem of scattering theory (Gelfand and Levitan,[1] Marchenko[2]).

For brevity we assume that there are no bound states and that scattering takes place in the s state; the generalization of the problem is obvious.

Following Marchenko,[2] the solution of the equation

$$d^2\psi(x,k)/dx^2 + (k^2 - V(x))\psi(x,k) = 0 \tag{1}$$

can be given as an expansion in integrals over the system of functions

$$\varphi(y,k) = (2/\pi)^{1/2}\sin[ky + \delta(k)] \tag{2}$$

[$\delta(k)$ is the scattering phase, known from experiment] in the following way

$$\psi(x,k) = \varphi(x,k) + \int_x^\infty A(x,y)\varphi(y,k)\,dy. \tag{3}$$

In this equation $A(x,y)$ is determined from the integral equation, the kernel and inhomogeneity of which are expressed through the Fourier component of the scattering amplitude $M(k) = \exp\{2\pi i\delta(k)\} - 1$:

$$m(z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} M(k)e^{ikz}dk, \quad \text{for } z > 0. \tag{4}$$

If $m(z) = 0$ for $z > 0$, then it follows from the equation of Marchenko that $A(x,y) = 0$ and, according to Eq. (1), the wave function $\psi(x,k)$ coincides with the solution of the free equation everywhere except at the origin (contact interaction). On the other hand, $m(z)$ going to zero for $z > 0$ means, according to Eq. (3), that the scattering amplitude does not have poles for $\mathrm{Im}\,k > 0$ and grows with $k \to \infty$ ($\mathrm{Im}\,k > 0$) no faster than as a power of k, since in this case the contour of integration can be closed around a half circle of large radius lying in the upper half plane.

From this it follows that, if the scattering is described by a potential, then either the amplitude has a pole (a so-called spurious pole, since we assumed that the system did not have a level), or it grows faster than a polynomial for $k \to \infty$ ($\mathrm{Im}\,k > 0$).

If the potential is bounded in space [$V(x) = 0$ for $x > a$], then $m(z)$ goes to zero for $z > 2a$. This follows from the relation

$$\int \varphi(y,k)\varphi(x,k)\,dk = \delta(y-x) - m(x+y). \tag{5}$$

In fact, for (x and y) $> a$, $\varphi(x,k)$ and $\varphi(y,k)$ coincide with the solution of the Schrödinger equation and, therefore, should be orthogonal. But then it follows from Eq. (4) that the function

$$M_a(k) = M(k)e^{2ika} \tag{6}$$

is analytic in the upper half plane. This result was obtained by Van Kampen[3] from other considerations.

If the scattering amplitude is known for all energies, then, as was shown most rigorously in the work of Khuri,[4] the function

$$g(E) = M(E,\tau) - V_\tau/4\pi \tag{7}$$

(where $M(E,\tau)$ is the scattering amplitude, viewed as a function of energy E and given momentum τ, and V_τ is the Fourier component of the potential) can be analytically continued in the complex E plane (or upper half plane of k) and a dispersion relation can be given for it.*

From the dispersion relation for the function (7) obtained by Khuri, it can be seen that if the scattering amplitude is known, then the potential is determined by the amplitude without solution of the integral equation.

We emphasize that this assertion is valid if the scattering amplitude is known for all energies. Since even the Schrödinger equation is valid only in a limited region of energy, then the scattering amplitude can be given only for some energy interval. This problem will be treated in another communication.

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*The conditions imposed on the potential by this coincide with the condition that for $k \to \infty$ the scattering amplitude be given by the first Born approximation. Then the dispersion relations are valid for $\tau < 2\alpha$, where α is the maximum positive number for which the integral

$$\int_0^\infty e^{2\alpha} \mid V(y) \mid dy.$$

exists. We note, in addition, that if this integral exists for arbitrary α, then the scattering amplitude does not have "spurious poles."

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[1] I. M. Gel' fand and B. M. Levitan, Izv. Akad. Nauk SSSR, Ser. Mat. **15**, 309 (1951).

[2] V. A. Marchenko, Dokl. Akad. Nauk SSSR **104**, 695 (1955).

[3] N. G. van Kampen, Phys. Rev. **91**, 1267 (1953).

[4] N. N. Khuri, Phys. Rev. **107**, 1148 (1957).

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TWO CLASSES OF INTERACTION LAGRANGIANS

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LET us consider Lagrangians of the strong interaction of baryons and mesons, retaining their isotopic structure as given by Salam.[1] We note that the wave functions of particles belonging to the same isotopic multiplet behave similarly under all transformations.

We divide strong interaction Lagrangians into two classes. To the first class we assign interactions that contain at least one vertex where the fermion does not change a single one of its fundamental characteristics: mass, electric charge, strangeness. These are the electromagnetic interactions, the interactions of π mesons with nucleons, of π mesons with Σ particles, and of π mesons with Ξ particles. To the second class we assign interactions that contain only vertices at which the fermion necessarily changes at least one of its fundamental characteristics: mass, electric charge, strangeness. These are the interactions of π mesons with Λ and Σ particles, and also all interactions of K particles with baryons.

Let φ, χ, ϕ, ξ be operators of fields of spin zero which transform in the following ways under the operations of space inversion P, charge conjugation C, and time reversal T:

$$\begin{array}{lllll} P: & \varphi' = \varphi & \chi' = \chi & \phi' = -\phi & \xi' = -\xi \\ C: & \varphi' = \varphi^* & \chi' = -\chi^* & \phi' = \phi^* & \xi' = -\xi^* \\ T: & \varphi' = \varphi^* & \chi' = -\chi^* & \phi' = -\phi^* & \xi' = \xi^*. \end{array} \tag{1}$$

The interaction Lagrangians of the first class for the fields φ, χ, ϕ, ξ (taken truly neutral for simplicity) have different forms which cannot be reduced to each other, namely:

$$L_\varphi = g_\varphi \bar{\psi}\psi\varphi_0, \tag{2}$$

$$L_\chi = f_\chi \bar{\psi}\gamma_\mu \psi\, \partial\chi_0 / \partial x_\mu, \tag{3}$$

$$L_\phi = g_\phi \bar{\psi}\gamma_5\psi\ \phi_0 + f_\phi \bar{\psi}\gamma_5\gamma_\mu\psi\, \partial\phi_0/\partial x_\mu. \tag{4}$$

A characteristic feature of the Lagrangians assigned to the second class is the presence, in addition to the usual terms, of interactions of the form $i(\bar{\psi}_1 O \psi_2 \theta - \bar{\psi}_2 O \psi_1 \theta^*)$. The Lagrangians of the second class describing the interaction of the bosons φ, χ, ϕ, ξ with the fermions ψ_1 and ψ_2, which have the same phase factors under the operations P, C, T, are written in the following form:

$$L_\varphi = g_\varphi (\bar{\psi}_1\psi_2\varphi + \bar{\psi}_2\psi_1\varphi^*) + if_\varphi\left(\bar{\psi}_1\gamma_\mu\psi_2 \frac{\partial\varphi}{\partial x_\mu} - \bar{\psi}_2\gamma_\mu\psi_1 \frac{\partial\varphi^*}{\partial x_\mu}\right), \tag{5}$$

$$L_\chi^* = ig_\chi (\bar{\psi}_1\psi_2\chi - \bar{\psi}_2\psi_1\chi^*) + f_\chi\left(\bar{\psi}_1\gamma_\mu\psi_2 \frac{\partial\chi}{\partial x_\mu} + \bar{\psi}_2\gamma_\mu\psi_1 \frac{\partial\chi^*}{\partial x_\mu}\right), \tag{6}$$

$$L_\phi = g_\phi\ (\bar{\psi}_1\gamma_5\psi_2\phi + \bar{\psi}_2\gamma_5\psi_1\phi^*) + f_\phi\left(\bar{\psi}_1\gamma_5\gamma_\mu\psi_2 \frac{\partial\phi}{\partial x_\mu} + \bar{\psi}_2\gamma_5\gamma_\mu\psi_1 \frac{\partial\phi^*}{\partial x_\mu}\right), \tag{7}$$

$$L_\xi = ig_\xi (\bar{\psi}_1\gamma_5\psi_2\xi - \bar{\psi}_2\gamma_5\psi_1\xi^*) + if_\xi\left(\bar{\psi}_1\gamma_5\gamma_\mu\bar{\psi}_2 \frac{\partial\chi}{\partial x_\mu} - \bar{\psi}_2\gamma_5\gamma_\mu\psi_1 \frac{\partial\chi^*}{\partial x_\mu}\right). \tag{8}$$