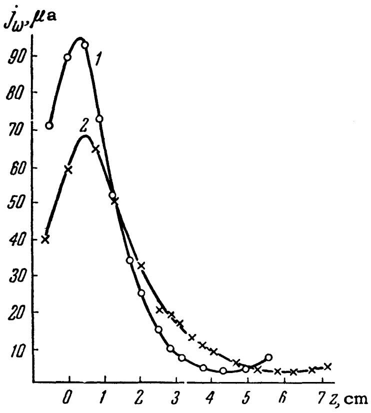

FIG. 1. Distribution of ion current on the walls: 1 — H = 0, 2 — H = 840 oersted.

an argon pressure of 0.7 mm Hg, is shown in Fig. 1. The electrons and ions diffuse farther along the axis, owing to the reduced diffusion towards the walls in the magnetic field. At a certain distance from the cathode, j_ω increases in the magnetic field, since it decreases near the cathode. The total value of the ion current remains almost constant at H = 840 gausses from z = 0 to the minimum, as shown on the curves of Fig. 1, even though the ratio $D_\parallel/D_\perp$ equals 2.5.

The variation of $D_\parallel/D_\perp$ obtained in the first experiments is shown in Fig. 2, where the abscissa

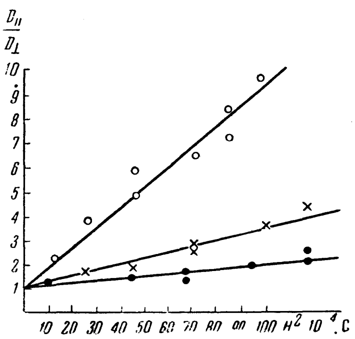

FIG. 2. Dependence of $D_\parallel/D_\perp$ on the square of the magnetic-field intensity. Pressure in argon: ○ — 0.25 mm Hg, × — 0.7 mm Hg, ● — 1.0 mm Hg.

represents the square of the magnetic field intensity. The value of $D_\parallel/D_\perp$ was determined in accordance with (1) from the slope of the lines on the semi-logarithmic graph. Figure 2 agrees fully with the classical formula

$$D_\perp = \frac{D_\parallel}{1 + k(b_e H / c_0)^2}; \qquad (2)$$

where b_e is the mobility of the electrons, c_0 the velocity of light, and k is the ratio of the ion and electron mobilities.

The advantage of the method proposed here over those described in the literature is that there is no need for measuring the electron concentration in the plasma. The results obtained show that when the plasma is no longer longitudinally homogeneous, a decrease in $D_\perp$ is not accompanied by the same decrease in diffusion current on the walls. A frequently-encountered masked case of an inhomogeneous column is a column with moving strata. Here charge diffusion along the tube is characteristic of the strata as well as of the low-voltage arc region.[3-5] Results of measurements in a positive column[6] must therefore be accepted with caution.

[1] B. N. Kliarfel'd and V. D. Sobolev, J. Tech. Phys. (U.S.S.R.) **17**, 319 (1947).

[2] A. V. Nedospasov and K. E. Torgonenko Радиотехника и электроника (Radio Engg. and Electronics) **2**, 494 (1957).

[3] B. N. Kliarfel'd, J. Exptl. Theoret. Phys. (U.S.S.R.) **22**, 66 (1952).

[4] A. B. Stewart, J. Appl. Phys. **27**, 911 (1958).

[5] W. H. Bostick and M. A. Levine, Phys. Rev. **97**, 13 (1955).

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ANOMALOUS GALVANOMAGNETIC PROPERTIES OF METALS AT LOW TEMPERATURES

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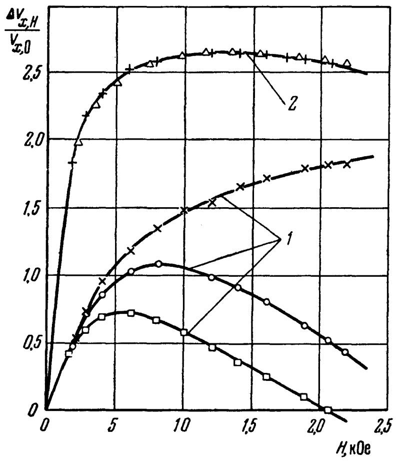
In many works devoted to the study of galvanomagnetic properties of metals not enough attention is paid to the technique used to bring in the current and potential leads. In strong effective fields [$H_{ef} = H_0\sigma_0(T)/\sigma_0(300°K)$] this can lead to a distortion of the observed phenomena. Thus, for example, in Refs. 1 to 4 the potential difference V_x, measured across the potential electrodes, increases as usual in weak magnetic field, passes through a maximum, diminishes to zero, and sometimes reverses its sign.

Since analogous anomalous changes in potential difference were observed by us in an investigation of the galvanomagnetic properties of bismuth in longitudinal and transverse magnetic fields,* supplementary experiments were performed to determine how the shapes of the electrodes and the methods by which they are interconnected influence the variation of V_x in a magnetic field. It becomes possible to attribute the anomalies in the variation of V_x observed in Refs. 1 to 4 to quadratic effects, particularly, the "quadratic" Hall effect, observed up to now only in germanium,[5] HgSe,[6,7] and Ga.[8] This effect is the appearance of a transverse potential difference in specimens located in a magnetic field. This potential difference, V_y, is in the plane passing through the current and magnetic field vectors, and is a quadratic function of the magnetic field. For isotropic specimens it attains its maximum at an angle of 45° between the current and field. Investigations of the "quadratic" Hall effect in bismuth of varying purity, in aluminum, and in tin[9] have shown that V_y increases sharply with decreasing temperature and with increasing purity of the specimens. In sufficiently pure specimens, at a temperature of 4.2°K and at $H = 13{,}000$ Oe, V_y of the "quadratic" Hall effect exceeded V_x, measured at $H = 0$, by thousands and sometimes tens of thousands times.

It is obvious that in those cases when the change in the resistance in the magnetic field is small (for example, in measurements in a longitudinal field), a slight V_y component along the specimen is sufficient to distort qualitatively the curve of the true variation of resistance in the magnetic field, for this additional component cannot be eliminated by double reversal of the directions of the current in field, as is usually done in the measurement of resistance. The results are particularly badly distorted when the current contact area is small compared with the specimen cross section area and the potential electrodes are located near to the specimen ends. In this case the current-density gradients on the ends of the specimen (which usually are not strictly symmetrical) will give rise to stray currents which may interact with the magnetic field to produce an additional potential difference at the potential electrodes, a difference which cannot be eliminated by reversing the current or the field.

Furthermore, if the line passing through the potential electrodes is not quite parallel with the axis of the specimen, an additional voltage component of the "quadratic" Hall effect may arise between the two. Even in the absence of such a deviation from parallelism, if the current electrodes are not symmetric about the specimen axis, an additional potential difference of either sign can occur, owing to either the first or the second reasons above.

Experiments with specimens of different geometries have shown that increasing the specimen length to diameter ratio, while retaining an identical relative placement of the potential electrodes, does not lead to a substantial reduction in the anomalous effects. However, increasing the distance between the ends of the specimens to the potential-electrodes, and also changing the ratio of the contact area to the specimen cross-section area, affects quite strongly the variation of the potential difference in a longitudinal magnetic field (see diagram).



Relative variation of V_x in a longitudinal magnetic field at $T = 77°K$ in Bi specimens of various shapes. Curves 1 – polycrystalline plate measuring 0.7 × 6.0 × 3.8 mm; ○ – current electrodes make point contact, × – flat copper current electrodes, □ – repeated fusing of point electrodes (flat contacts removed and specimen shortened somewhat). Curves 2 – monocrystalline cylindrical specimens of identical crystal orientation, 15 mm long and with diameters of 2 mm (+) and 0.5 mm (Δ); the distance from the end of either specimen to the potential-electrode lead-in is on the order of two diameters.

In connection with this it appears rational to perform measurements on a specimen having a cross section equal to those of the current electrodes. In this case the potential electrodes must be located as far away as possible from the ends of the specimen and as parallel as possible to its axis. Failure to observe these precautions in Refs. 1 to 4 has most probably caused the indicated "anomalies." This likelihood is emphasized by the fact that in Ref. 2 a substantial change in the magnitude of the observed effect was noted when the current and potential electrodes were interchanged.

One cannot exclude the existence of more complicated mechanisms leading to an added longitudinal potential difference in a magnetic field exist. These added effects are, however, apparently substantially less than the effects noted above.

We consider it our pleasant duty to thank Academician P. L. Kapitza for discussing this communication.

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[1] M. C. Steele, Phys. Rev. **97**, 1720 (1955).

[2] D. K. C. MacDonald, Phil. Mag. **2**, 97 (1957).

[3] C. J. Gorter, Progress in Low Temperature Physics **2**, 182 (1957).

[4] J. Babiskin, Phys. Rev. **107**, 981 (1957).

[5] Colman, Soldberg, and Davis, Phys. Rev. **94**, 1121 (1954).

[6] F. G. Bass and I. M. Tsidil'kovskii, J. Tech. Phys. (U.S.S.R.) **24**, 1834 (1954).

[7] I. M. Tsidil'kovskii, J. Tech. Phys. (U.S.S.R.) **27**, 12 (1957), Soviet Phys. JTP **2**, 9 (1957).

[8] J. Jahia and J. A. Marcus, International Conference on Low Temperature Physics and Chemistry, Wisconsin, 1957.

[9] Alekseevskii, Brandt, and Kostina, Вестник МГУ (Bull. Moscow State Univ.) (in press).

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THE DIFFERENTIAL FORM OF THE KINETIC EQUATION OF A PLASMA FOR THE CASE OF COULOMB COLLISIONS

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As is known,[1] the kinetic equation for the particle distribution function $f_\alpha(t, \mathbf{r}, \mathbf{v})$ of a completely ionized plasma ($\alpha = e$ and $\alpha = i$ denote electrons and ions, respectively) can be written for the case of Coulomb collisions in the form

$$\frac{\partial f_\alpha}{\partial t} + (\mathbf{v}\,\mathrm{grad}_r) f_\alpha + \frac{1}{m_\alpha}(\mathbf{F}_\alpha \nabla_v f_\alpha) = -\sum_\beta (\nabla_v \mathbf{j}_{\alpha\beta}), \tag{1}$$

$$(\mathbf{j}_{\alpha\beta})_i = \frac{2\pi\lambda e_\alpha^2 e_\beta^2}{m_\alpha}\int d\mathbf{v}' U_{ik}\left(\frac{f_\alpha}{m_\beta}\frac{\partial f'_\beta}{\partial v'_k} - \frac{f'_\beta}{m_\alpha}\frac{\partial f_\alpha}{\partial v_k}\right), \tag{2}$$

$$U_{ik} = \partial^2 \mid V \mid / \partial v_i \partial v_k = (V^2\delta_{ik} - V_i V_k)/V^3;$$

$$V_i = v_i - v'_i.$$

The present note shows that the particular structure of the integrals in Eq. (2) for the current allows one to introduce new variables in the form of new unknown "potential" functions

$$\Phi_\alpha(t, \mathbf{r}, \mathbf{v}) = \int d\mathbf{v}' f_\alpha(\mathbf{v}') \mid \mathbf{v} - \mathbf{v}' \mid, \tag{3}$$

which will transform the integro-differential equations (1) into pure differential form.

Let us consider the first integral in the expression for $\mathbf{j}_{\alpha\beta}$. If we make use of the expression for U_{ik}, we obtain

$$\int d\mathbf{v}' U_{ik}\frac{\partial f'_\beta}{\partial v_k} = 2\frac{\partial \varphi_\beta}{\partial v_i}, \qquad \varphi_\beta(t, \mathbf{r}, \mathbf{v}) = \int \frac{f_\beta(\mathbf{v}')\, d\mathbf{v}'}{\mid \mathbf{v} - \mathbf{v}' \mid}. \tag{4}$$

Further, noting the identity

$$V^{-3}\{V^2 \nabla_v f_\alpha - \mathbf{V}(\mathbf{V}\nabla_v f_\alpha)\} = (\nabla_v f_\alpha \nabla_v)\nabla_v \mid \mathbf{v} - \mathbf{v}' \mid, \tag{5}$$

we can transform the second integral in (2) according to

$$\int d\mathbf{v}' f'_\beta U_{ik}\frac{\partial f_\alpha}{\partial v_k} = \frac{\partial f_\alpha}{\partial v_k}\cdot\frac{\partial^2 \Phi_\beta}{\partial v_k \partial v_i}. \tag{6}$$

The other quantities entering into (1) are also easy to express in terms of the Φ_β. In particular,

$$\varphi_\beta = \frac{1}{2}\nabla_v^2 \Phi_\beta, \quad f_\beta = -\frac{1}{4\pi}\Delta_v \varphi_\beta = -\frac{1}{8\pi}\nabla_v^4 \Phi_\beta. \tag{7}$$

Inserting (4), (6), and (7) into (1), we obtain the differential equation

$$\frac{\partial}{\partial t}(\nabla_v^4 \Phi_\alpha) + (\mathbf{v}\,\mathrm{grad}_r)(\nabla_v^4 \Phi_\alpha) + \frac{1}{m_\alpha}(\mathbf{F}_\alpha \nabla_v^5 \Phi_\alpha)$$
$$= -\sum_\beta \frac{2\pi\lambda e_\alpha^2 e_\beta^2}{m_\alpha}\left(\nabla_v\left[\frac{1}{m_\beta}(\nabla_v^4 \Phi_\alpha)\nabla_v^3 \Phi_\beta - \frac{1}{m_\alpha}(\nabla_v^5 \Phi_\alpha \nabla_v)\nabla_v \Phi_\beta\right]\right). \tag{8}$$

for the "potential" functions. In the special case of a "moving" Maxwell distribution given by

$$f_\alpha^{(0)}(\mathbf{v}) = n_\alpha (m_\alpha / 2\pi T_\alpha)^{3/2} \exp\{-s_\alpha^2\}, \tag{9}$$

$$s_\alpha = \sqrt{\frac{m_\alpha}{2T_\alpha}}(\mathbf{v} - \mathbf{v}_\alpha^0)$$

(where $n_\alpha(t, \mathbf{r})$ is the density, $\mathbf{v}_\alpha^0(t, \mathbf{r})$ is the mean velocity, and $T_\alpha(t, \mathbf{r})$ is the temperature) we have