$$\Phi_\alpha^{(0)}(\mathbf{v}) = n_\alpha \left(\frac{2T_\alpha}{\pi m_\alpha}\right)^{1/2} M(s_\alpha),$$

$$M(s) = e^{-s^2} + (1 + 2s^2)\int_0^1 e^{-s^2x^2}dx. \quad (10)$$

It is easy to verify that $\Phi_\alpha^0(v)$ causes the self-current $\mathbf{j}_{\alpha\alpha}$ to vanish, which is as it should be.

From the definition (3) one easily obtains the asymptotic potential function (for $v \gg \langle v \rangle_\alpha$; we drop terms of order v^{-3}), namely

$$\Phi_\alpha(\mathbf{v}) = vn_\alpha\left\{1 - \frac{(\mathbf{v}\mathbf{v}_\alpha^0)}{v^2} + \left[\frac{T_\alpha}{m_\alpha v^2} - \frac{v_\alpha^{02}}{2v^2} - \frac{(\mathbf{v}\mathbf{v}_\alpha^0)^2}{2v^4} - \frac{(\mathbf{v}\Pi_\alpha\mathbf{v})}{2v^4}\right]\right\}, \quad (11)$$

where we make use of the tensor

$$\Pi_{\alpha ik} = \frac{1}{n_\alpha}\int d\mathbf{v}' f_\alpha'\left(u_{\alpha i}' u_{\alpha k}' - \frac{u_\alpha'^2}{3}\delta_{ik}\right), \quad \mathbf{u}_\alpha' = \mathbf{v}' - \mathbf{v}_\alpha^0.$$

If the distribution differs only slightly from a Maxwell distribution, so that we may write

$$\Phi_\alpha(\mathbf{v}) \approx \Phi_\alpha^0 + \Phi_\beta^{(1)}$$

$$= n_\alpha^{\ast}(2T_\alpha/\pi m_\alpha)^{1/2}[M(s_\alpha) + \chi(s_\alpha)], \quad \chi \ll M, \quad (12)$$

the linear approximation gives the following expression for the current $\mathbf{j}_{\alpha\alpha}$ due to collisions among particles only of type α:

$$\mathbf{j}_{\alpha\alpha} = (\lambda e_\alpha^4/4m_\alpha^2)\left[(\nabla_v^5\Phi_\alpha\nabla_v)\nabla_v\Phi_\alpha - (\nabla_v^4\Phi_\alpha)\nabla_v^3\Phi_\alpha\right]$$

$$\approx (\lambda e_\alpha^4 n_\alpha^2/4\pi m_\alpha^2)(m_\alpha/2T_\alpha)^{1/2}\left[16e^{-s^2}(\mathbf{s}\nabla_s)\nabla_s\chi + 8e^{-s^2}\nabla_s^3\chi + (\nabla_s^5\chi\nabla_s)\nabla_s M - (\nabla_s^4\chi)\nabla_s^3 M\right]. \quad (13)$$

In this case all the equations can be linearized.

Under certain special conditions, it is possible to lower the order of the differential equation. Such a situation may occur, for instance, when the distribution depends only on the absolute value of the velocity. Since the use of Eq. (8) for the "potential" functions may lead to extra solutions, the final result must be verified by inserting it into the initial equation (1).

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[1] L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) 7, 203 (1937).

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274

PROOF OF THE ABSENCE OF RENORMALIZATION OF THE VECTOR COUPLING CONSTANT IN BETA-DECAY

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GELL-MANN and Feynman have proposed[1] that the vector coupling constant of the β-decay interaction is not subject to renormalizations due to the strong meson-nucleon interaction, if a direct interaction between the π meson and electron-neutrino fields is introduced such as to make the vector part of the meson-nucleon β-interaction Hamiltonian of the form

$$H = G_V\left[\bar{\psi}\gamma_\mu\tau^+\psi + 2i\left(\Phi^+T^+\nabla_\mu\Phi - (\nabla_\mu\Phi^+)T^+\Phi\right)\right]J_\mu + \text{Herm. conj.}$$

$$J_\mu = \tfrac{1}{2}\bar{\psi}_e\gamma_\mu(1+\gamma_5)\psi_\nu, \quad (1)$$

where $\tau^+ = \frac{1}{2}(\tau_x + i\tau_y)$, $T^+ = \frac{1}{2}(T_x + iT_y)$ are the isotopic spin operators and $\Phi = (\varphi, \varphi^0, \varphi^+)$ is the meson wave function.

This assumption of Gell-Mann and Feynman may be rigorously proved if it is noted that in the presence of the β-interaction (1) the complete nucleon — π meson Lagrangian (in which the meson-nucleon interaction is included but interactions with the electromagnetic field are not) admits the group of infinitesimal transformations

$$\psi = [1 - i(\tau^+\chi + \tau\chi^*)]\psi'; \qquad \Phi = [1 - 2i(T^+\chi + T\chi^*)]\Phi';$$

$$J_\mu = J_\mu' + \partial\chi/\partial x_\mu \quad (2)$$

where χ is an infinitesimal numerical function. The existence of the group of transformations (2) makes possible the proof of a theorem analogous to the Ward theorem in quantum electrodynamics. To obtain the proof it is only necessary to calculate the nucleon Green's function $G(x, y, J_\mu)$ in the presence of a time and space independent external β-current J_μ, and to define the vertex part as

$$\Gamma_\mu^+(x, y; \xi) = \partial G^{-1}(x, y; J_\mu)/\partial J_\mu|_{J_\mu=0}\,\delta(\xi).$$

Putting $\chi(x) = J_\mu x_\mu$, one obtains from the definition of the Green's function

$$G(x, y; J_\mu) = \langle 0 \mid T\{\psi(x), \bar{\psi}(y)\} \mid 0\rangle$$

and from the relations (2):

$$\left.\frac{\partial G(x,\ y;\ J_\mu)}{\partial J_\mu}\right|_{J_\mu=0} = i\,\langle 0 \mid T\,\{\tau^+ x_\mu \psi(x),\ \overline{\psi}(y)\} \mid 0\rangle - i\,\langle 0 \mid T\,\{\psi(x), \overline{\psi}(y)\,\tau^+ y_\mu\} \mid 0\rangle. \qquad (3)$$

However, this quantity, when viewed as a matrix in isotopic spin space, should be of the form $F(x, y)\,\tau^+$. Taking this into account one may bring (3) into the form

$$\left.\frac{\partial G(x,\ y;\ J_\mu)}{\partial J_\mu}\right|_{J_\mu=0} = i\,(x_\mu - y_\mu)\,\langle 0 \mid T\,\{\psi(x),\ \overline{\psi}(y)\} \mid 0\rangle\,\tau^+ = i\,(x_\mu - y_\mu)\,G(x,\ y)\,\tau^+$$

and, consequently,

$$\Gamma_\mu^+(x,\ y;\ \xi) = -i\,(x_\mu - y_\mu)\,G^{-1}(x-y)\,\delta(\xi)\,\tau^+. \qquad (4)$$

Going over to the momentum representation, we obtain a relation analogous to the Ward theorem in quantum electrodynamics:

$$\Gamma_\mu^+(p,\ p;\ 0) = \tau^+ \partial G^{-1}(p) / \partial p_\mu. \qquad (5)$$

The remainder of the proof of the absence of charge renormalization is the same as in quantum electrodynamics when vacuum polarization is ignored.

If, in addition to the π meson — nucleon interactions, it is also desired to take into account the interactions of nucleons with K mesons and hyperons, the group of transformations (2) must be extended to include the strange particles, so as to have no renormalization of the vector coupling constant of the β-interaction. This can be achieved by assuming that the K meson and Ξ hyperon wave functions transform as the nucleon wave function, the Σ hyperon wave function transforms as the π meson wave function, and the wave function of the Λ^0 particle remains unchanged. The existence of such a group of transformations of the strange particles requires that the vector part of the β-interaction Hamiltonian of K mesons and hyperons be of the form

$$H = G_V[2\overline{\psi}_\Sigma \gamma_\mu T^+ \psi_\Sigma + i\,(\varphi_K^+ \tau^+ \nabla_\mu \varphi_K - (\nabla_\mu \varphi_K^+)\,\tau^+ \varphi_K) + \overline{\psi}_\Xi \gamma_\mu \tau^+ \psi_\Xi]\,J_\mu + \text{Herm. conj.} \qquad (6)$$

The Hamiltonian (6) describes β-decays of strange particles in which strangeness does not change* (e.g., $\Sigma^- \to \Sigma^0 + e^- + \nu$, $K^- \to K^0 + e^- + \nu$). The constant G_V in the Hamiltonian (6) corresponds to the constant in the Hamiltonian (1) and, just like the latter, does not get renormalized by strong interactions.

We note that radiation corrections due to the electromagnetic field have been neglected in the present proof. The interaction of the particles with the electromagnetic field is not invariant under the group of transformations (2) and should, in general, lead to a renormalization of the constant G_V.

*Processes involving a strangeness change, cannot, clearly, influence the magnitude of the renormalization in processes in which strangeness does not change.

[1] R. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

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275

LOWER EXCITED (ROTATIONAL) LEVELS OF T^{234}

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USING an ionization chamber with grid,[1] we investigated the energy spectrum of α-particles from U^{238}. The spectrum obtained is shown in Fig. 1

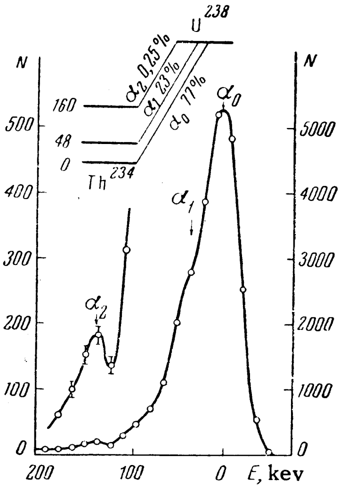

FIG. 1

where α_0 is the ground-state group of 4.182-Mev α particles from U^{238}. In our opinion, α_2 is the group of particles corresponding to the transition