THE MECHANICS OF FORMATION OF STRIATIONS IN THE POSITIVE COLUMN OF A GAS DISCHARGE

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It is shown that the appearance of striations in the positive column cannot be explained by the instability of a uniform column. With the assumption that the variation in density of charged particles along the column can be neglected, a nonlinear equation for the electron temperature and potential gradient has been derived from the equation of conservation of the particles. This equation admits of periodic solutions. Whether the column is uniform or striated depends upon the boundary conditions at the cathode end.

UNDER certain conditions, the uniform luminosity of the positive column in a discharge is broken up. Instead of a continuous luminosity distribution, of bright stripes or striations appear along the axis of the column. These can be either stationary or shift rapidly along the column (running striations).[1] The presence of striations is accompanied by periodic variations of the energy of the charged particles, of the potential gradient, and of other quantities along the column axis.

In attempting to explain this phenomenon it is natural to assume first that the breakdown of the uniform column when striations are present is due to instability in the uniform column.[2]

However, a test of column stability, carried out by the method of small perturbations,[3] using the system of equations recently developed by us,[4] does not support this hypothesis. In studying the stability of the column, we took the following into account: A stationary striation represents a steady-state phenomenon. For a transition to occur place from a uniform to a striated column, the small perturbations must be chosen of the form*

$$\varepsilon = \bar{\varepsilon} e^{ikx+\omega t}; \quad \nu = \bar{\nu} e^{ikx+\omega t} \text{ etc.,} \tag{1}$$

where $k = 2\pi/\lambda$ and ω is a real number.

Substituting $n_e = n_c(1 + v)$, $U_e = U_{ec}(1 + \epsilon)$ and so on, into our system of equations [reference 4, Eq. (9)], and using the definitions in (1), we obtain the following equation with respect to k and ω:

$$\frac{3}{4}\left(U_{ec}k^2 + \frac{\omega}{b_p}\right)\frac{1}{b_e}\frac{\partial H_1}{\partial U_{ec}} + E_x^2\frac{\mu^2}{a^2}U_e\left(\frac{U_i}{U_{ec}} - 1\right) = 0, \tag{2}$$

where

$$H_1 = \frac{4}{3}\sqrt{2e/\pi m}\,(\varkappa/\lambda_e)\,U_{ec}^{3/2},$$

and the dependence of the electron energy loss by collision, κ, on the mean electron energy U_{ec} for different gases in given in Engel' and Shtenbek's book.[5] Since $U_i/U_{ec} - 1 > 0$ and, for gases in which striations occur, $\partial H_i/\partial U_{ec} > 0$, it follows that Eq. (2) can be satisfied only by negative values of ω, i.e., our model of the positive column has no instability.

The purpose of the present work is to establish the following mechanism for the formation of striations. In the discharge regions adjacent to the ends of the positive column (e.g. the Faraday dark space) the mean electron energy, potential gradients etc. may have values different from those which must exist in the positive column under the given conditions (i.e., for the given value of pa). On the other hand, the equations which describe the electron-ion plasma possess, in addition to a solution independent of the x coordinate, another solution which is the sum of the x-independent solution and a certain oscillating function

$$U_e(pa, x) = U_{ec}(pa) + \varepsilon(pa, x)\,U_{ec}(pa).$$

Since our system of equations has a unique solution for the given boundary conditions, any other boundary condition involving a discontinuous change in the parameters at the boundary can be satisfied only by the periodic solution $U_{ec} + \epsilon U_{ec}$ (which corresponds to the presence of striations in the column). At the same time, boundary conditions without discontinuities can be satisfied only by the solution U_{ec} (in which case there are no striations). In some cases, because of fluctuations in

*The notation of our previous paper[4] will be used throughout this article.

the parameters, the discontinuity at the boundary of the positive column may vary periodically with time. Such oscillating boundary conditions are satisfied by a solution corresponding to a shifting of the periodic structure along the axis of the column (running striations)*.

Measurements[5,7] show that, in fact, there is a discontinuous change in electron temperature potential gradient and other parameters at the cathode end of the positive column. Shottky[8] developed a system of equations by which he was able to calculate, for different discharge conditions (different pa), values of electron temperature, potential gradient, and the radial distribution of electron density which were well confirmed by experiment. The system of equations which we have used appears to be a natural extension of Shottky's equations to the case of a non-uniform plasma.

In the system of equations used here (as in Shottky's) we disregard electron capture by gas molecules, ionization by collision, and recombination in the gas space.

A simple calculation shows that under the usual conditions (if there are no vapors of water, alcohol, or fatty acids in the tube[4b] (and if the current is not too large) the corresponding terms in the equations are small in comparison with the other terms, and may be neglected. For the last two processes one can reach the same conclusion even without any calculation, since the positive column, and hence also the striations in it, obey the similarity law very exactly.[9] These processes, which represent violations of the similarity law, can therefore not play an important part. We have taken the electron temperature to be constant along any radius of the tube, but variable along the axis. The electron and ion densities may vary with the radius, but are constant along the tube axis.†

Taking $n_e = n_p = N$ to be independent of the x coordinate, we have two continuity equations for the electrons and the ions (see reference 4):

$$\partial N/\partial t + \operatorname{div}(\mathbf{u}_e N) - ZN = 0, \tag{3}$$

$$\partial N/\partial t + \operatorname{div}(\mathbf{u}_p N) - ZN = 0, \tag{4}$$

$$u_{ex} = -b_e(E_x + \gamma_e \partial U_e/\partial x), \tag{5}$$

$$u_{px} = b_p(E_x - \gamma_p \partial U_p/\partial x), \tag{6}$$

$$u_{pr} = -\frac{D_a}{N}\frac{\partial N}{\partial r}, \quad Z = \beta e^{-U_i/U_e}. \tag{7}$$

Here γ_e and γ_p are the thermal diffusion coefficients for the electrons and ions.[10]

For the stationary case (fixed striations), after substituting (5), (6), and the first equation of (7) into (3) and (4), and dividing by N, we obtain

$$-b_e\frac{\partial E_x}{\partial x} - \gamma_e b_e \frac{\partial^2 U_e}{\partial x^2} - \frac{D_a}{N}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial N}{\partial r}\right) - Z = 0,$$

$$b_p\frac{\partial E_x}{\partial x} - \gamma_p b_p \frac{\partial^2 U_p}{\partial x^2} - \frac{D_a}{N}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial N}{\partial r}\right) - Z = 0. \tag{8}$$

Separating the variables in the usual way, we obtain an equation for the electron and ion densities

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial N_r}{\partial r}\right) + \frac{\lambda^2}{b_p}N_r = 0, \tag{9}$$

whose solution (under the conditions of Shottky's limiting case) have the form

$$N_r = N_{r0} I_0 (\lambda r/\sqrt{b_p}), \quad \lambda^2 = (\mu/a)^2 b_p.$$

This is the well-known Shottky solution for the distribution of charged particles along the radius of a discharge tube.

For the energies of the electrons and ions in this case, we obtain from (8) the following two equations:

$$-b_e\frac{\partial E_x}{\partial x} - \gamma_e b_e \frac{\partial^2 U_e}{\partial x^2} + \frac{\mu^2}{a^2} b_p U_e - Z = 0,$$

$$b_p\frac{\partial E_x}{\partial x} - \gamma_p b_p \frac{\partial^2 U_p}{\partial x^2} + \frac{\mu^2}{a^2} b_p U_e - Z = 0. \tag{10}$$

In the case of a uniform column ($\partial E_x/\partial x = \partial^2 U_e/\partial x^2 = \partial^2 U_p/\partial x^2 = 0$) these equations reduce to the Shottky equation for the electron temperature as a function of the discharge parameters:

$$(\mu/a)^2 b_p U_{ec} - Z_c = 0. \tag{11}$$

The term $(\mu/a)^2 b_p U_{ec}$ in Eqs. (10) and (11) describes the depletion of charged particles due to ambipolar diffusion.*

We now find a solution of the system of equations (10) in the form:

$$\begin{aligned} U_e(pa, x) &= U_{ec}(pa) + \varepsilon(pa, x)\, U_{ec}(pa), \\ U_p(pa, x) &= U_{pc}(pa) + \varepsilon_1(pa, x)\, U_{pc}(pa). \end{aligned} \tag{12}$$

Substituting (12) into the system (10), using the relations (7) and (11), and dividing the first of these equations by $b_e U_{ec}$ and the second by $b_p U_{ec}$, we obtain

*The part played by the stability of processes at the ends of the column is indicated in reference 6.

†The physical conditions under which these assumptions are realized are dealt with at the end of this paper.

*In references 11 and 12 this term is omitted without justification. If this term is included, it is easy to see that there is no periodic solution at all under these author's assumption that the electron temperature is constant along the axis.

$$-\frac{1}{U_{ec}}\frac{\partial E_x}{\partial x} - \gamma_e \frac{\partial^2\varepsilon}{\partial x^2} - \frac{\mu^2}{a^2}\frac{b_p}{b_e}\left(\exp\left\{\frac{U_i\varepsilon}{U_e(1+\varepsilon)}\right\} - 1 - \varepsilon\right) = 0;$$

$$\frac{1}{U_{ec}}\frac{\partial E_x}{\partial x} - \gamma_p \frac{\partial^2\varepsilon_1}{\partial x^2}\frac{U_{pc}}{U_{ec}} - \frac{\mu^2}{a^2}\left(\exp\left\{\frac{U_i\varepsilon}{U_e(1+\varepsilon)}\right\} - 1 - \varepsilon\right) = 0. \qquad (13)$$

Comparing these two equations, and noting that γ_e and γ_p are of the order of unity[10] while $U_{pc}/U_{ec} \ll 1$ and $b_p/b_e \ll 1$, we obtain (after dividing through by γ_e) a nonlinear equation describing the variation of ϵ along the axis of the positive column:

$$\frac{d^2\varepsilon}{dx^2} + \frac{\mu^2}{a^2}\left(\exp\left\{\frac{U_i\varepsilon}{U_e(1+\varepsilon)}\right\} - 1 - \varepsilon\right) = 0. \qquad (14)$$

If the deviation from a uniform distribution is very small, so that $\epsilon U_i/U_e \ll 1$, then (14) can be approximated by the linear equation

$$\frac{d^2\varepsilon}{dx^2} + \frac{1}{\gamma_e}\frac{\mu^2}{a^2}\left(\frac{U_i}{U_e} - 1\right)\varepsilon = 0, \qquad (15)$$

whose solutions are harmonic functions with the period

$$l_{\rm lin} \approx (2\pi a/\mu)(\gamma_e U_i/U_e)^{1/2}. \qquad (16)$$

This is the result we obtained previously.[4]* Experiment shows, however, that the amplitudes of the electron-temperature deviations are not small, and may amount,[7,13,14] for example, to $\epsilon \approx 0.5$. Since $U_i/U_e \approx 10$, then $\epsilon U_i/U_e \approx 5$, i.e., the use of Eqs. (15) and (16) is not rigorously correct. We are forced to consider the nonlinear equation (14). Equations of this type have been thoroughly studied in the theory of nonlinear oscillations.[15] These are the so-called oscillation equations of a system with a nonlinear restoring force, from which Eq. (14) differs only in the definitions of the variables. For convenience, we shall use the terminology of oscillation theory in the following discussion.

We shall write equation (14) in the form $d^2\epsilon/dx^2 + f(\epsilon) = 0$, where

$$f(\varepsilon) = \frac{1}{\gamma}\frac{\mu^2}{a^2}\left(\exp\left\{\frac{U_i\varepsilon}{U_e(1+\varepsilon)}\right\} - 1 - \varepsilon\right)$$

is the "restoring force". We then calculate the "potential energy" of the system, reckoning the ordinate from the point at which $\epsilon = 0$:

$$F(\varepsilon) - F(0) = \int_0^\varepsilon f(\varepsilon)\,d\varepsilon = \int_0^\varepsilon \frac{\mu^2}{\gamma a^2}\left(\exp\left\{\frac{U_i\varepsilon}{U_e(1+\varepsilon)}\right\} - 1 - \varepsilon\right)d\varepsilon$$

$$= \frac{\mu^2}{\gamma a^2}\left\{-\left(1 + \varepsilon + \frac{1}{2}\varepsilon^2\right) + (1+\varepsilon)\exp\left\{\frac{U_i\varepsilon}{U_e(1+\varepsilon)}\right\} - \frac{U_i}{U_e}e^{U_i/U_e}\left[-\operatorname{Ei}\left(\frac{-U_i}{U_e(1+\varepsilon)}\right) + \operatorname{Ei}\left(\frac{-U_i}{U_e}\right)\right]\right\}, \qquad (17)$$

$$-\operatorname{Ei}(-x) = \int_x^\infty e^{-t}dt/t.$$

Figure 1 shows the calculated "restoring force" and "potential energy" of our system (the constant factor $\mu^2/\gamma a^2$, which is not essential here, has been dropped). Beneath this are shown the energy curves determined by the equation

$$\dot\varepsilon^2 - \dot\varepsilon_0^2 = 2\,[F(\varepsilon) - F(0)], \qquad (18)$$

where $[F(\epsilon) - F(0)]$ is determined from Eq. (17).

It is evident from Fig. 1 that the restoring force is strong for positive deviations of the electron energy and weak for negative deviations. The shapes of the "potential energy" and "energy" curves provide a clue to the nature of the solution. At the point $\epsilon = 0$ there is a minimum in the "potential energy". The origin is a singular point. In the absence of an initial deflection (i.e., no discontinuity at the end of the column) the most stable state is that of equilibrium (a uniform distribution of electron temperatures — no striations). If there is an initial deflection of not too large a magnitude, there is a stable periodic solution (striations) in which the amplitude of the electron-temperature fluctua-

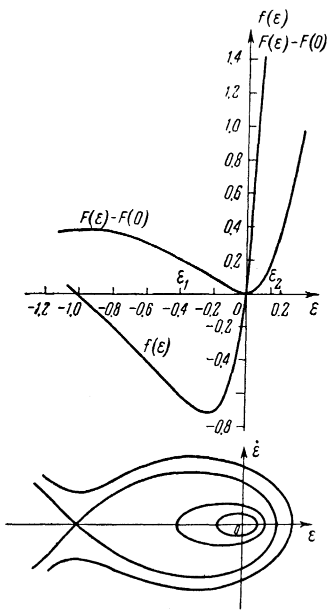

FIG. 1

*Objections have been raised in reference 12 to the fact that $l \approx \sqrt{\gamma_e}$, i.e., $l = 0$ when $\gamma_e = 0$. The quantity γ_e is equal to zero only if $1/\lambda_e \approx c_e$, but no gas is known with this property. For inert gases $1/\lambda_e \approx (e_e - c_0)$, but in this case $\gamma_e \neq 0$.

tions is determined by the size of the discontinuity at the boundary.

The curve of electron energy variation along the column axis can be obtained (by numerical integration, in our case) from the first integral of (14), which has the form:[15]

$$x = \int_0^{\varepsilon}\left\{\dot{\varepsilon}_0^2 + \frac{2\mu^2}{\gamma a^2}\left[\left(1+\varepsilon+\frac{1}{2}\varepsilon^2\right) - (1+\varepsilon)\times\exp\left\{\frac{U_i\varepsilon}{U_e(1+\varepsilon)}\right\} + \frac{U_i}{U_e}e^{U_i/U_e}\times\left\{-\mathrm{Ei}\left(\frac{-U_i}{U_e(1+\varepsilon)}\right) + \mathrm{Ei}\left(\frac{-U_i}{U_e}\right)\right\}\right]\right\}^{-1/2} d\varepsilon. \quad (19)$$

However, the shape of the curve can also be studied by considering the "potential energy" curve and Eq. (14) as it stands. This curve differs markedly from the sinusoidal one determined by the linear equation (15). Let us take the value of the maximum negative excursion to be $\epsilon_1 = -0.5$ (corresponding to the usual values of ϵ_1 for striations). From the "potential energy" curve we find the corresponding positive excursion to be $\epsilon_2 = 0.15$. Since the period of a system with a strong restoring force is less than the period of a weak force,[15] then the period (length of one striation) corresponding to a low electron temperature is longer than the period corresponding to a higher electron temperature. Furthermore, we can calculate from (14) the radius of curvature of the $\epsilon(x)$ curve at the minimum and maximum points (where $d\epsilon/dx = 0$). Taking $U_i/U_e = 10$, $a = 1.5$ cm, $\mu = 2.4$, and $\gamma = 1$, we get

$$R_{\varepsilon_1=-0.5} = \left|(d^2\varepsilon/dx^2)^{-1}_{\varepsilon=-0.5}\right| \approx 0.8 \text{ cm}.$$

Similarly, we obtain

$$R_{\varepsilon_2=0.15} \approx 0.13 \text{ cm}.$$

The $U_e = f_1(x)$ is shown in Fig. 2 (cf. reference 13).

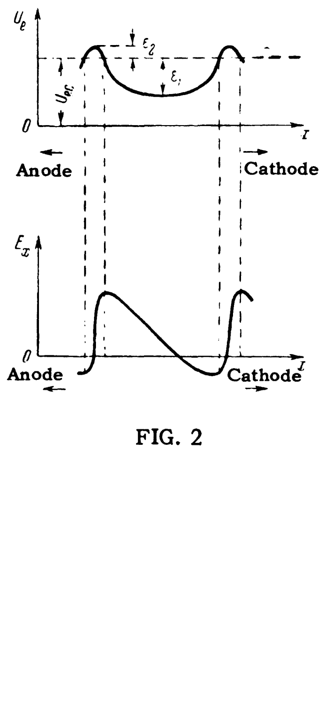

FIG. 2

The curve $E_x = f_2(x)$ can be obtained from the curve $U_e = f_1(x)$ and the first equation of (13) which, considering that $b_p \ll b_e$, can be written:

$$\frac{dE_x}{dx} \approx -\gamma_e U_{ec}\frac{d^2\varepsilon}{dx^2} = -\gamma_e\frac{d^2U_e}{dx^2}. \quad (20)$$

The curve $E_x = f_2(x)$ is also shown in Fig. 2. It is characterized by a very sharp rise of field intensity within the narrow luminous portion of the striation. The maximum field does not coincide with the maximum electron temperature (or luminosity), but is slightly displaced toward the cathode, while at the same time the minimum lies on the anode side of the luminous portion of the striation.

The period (i.e., the spacing of the striations) can be calculated from the formula:[15]

$$l = l_1 + l_2 = 2\int_0^{\varepsilon_1}\frac{d\varepsilon}{\sqrt{2[F(\varepsilon_1)-F(\varepsilon)]}} + 2\int_0^{\varepsilon_2}\frac{d\varepsilon}{\sqrt{2[F(\varepsilon_2)-F(\varepsilon)]}}, \quad (21)$$

where l_1 and l_2 are the lengths of the low-temperature and high-temperature portions of the striation respectively.

Inserting (17) into (21) we find for l_1

$$l_1 = \frac{a}{\mu}\sqrt{2\gamma}\int_0^{\varepsilon_1}\left\{(\varepsilon-\varepsilon_1) + \frac{1}{2}(\varepsilon^2-\varepsilon_1^2) - \left[(1+\varepsilon_1)\exp\left\{\frac{U_i\varepsilon_1}{U_e(1+\varepsilon_1)}\right\} - (1+\varepsilon)\exp\left\{\frac{U_i\varepsilon}{U_e(1+\varepsilon)}\right\} + \frac{U_i}{U_e}e^{U_i/U_e}\left\{-\mathrm{Ei}\left(\frac{-U_i}{U_e(1+\varepsilon_1)}\right) + \mathrm{Ei}\left(\frac{-U_i}{U_e(1+\varepsilon)}\right)\right\}\right]\right\}^{-1/2} d\varepsilon. \quad (22)$$

There is an analogous expression for l_2. The integral (22) apparently cannot be expressed in terms of elementary functions, but can be evaluated graphically or by numerical methods.

It is essential to ascertain whether the period calculated from (21) differs seriously from the value calculated from (16), which is based on a linear approximation. If in (22) we apply the Lagrange theorem to the terms in the square and curly brackets, we readily obtain the following estimates for l_1 and l_2:

$$l_{1\min} \approx l_{2\max} \approx \frac{1}{2}l_{\mathrm{lin}},$$

$$l_{1\max} \approx \frac{2a}{\mu}\sqrt{2\gamma}\sqrt{\frac{-\varepsilon_1}{1+\varepsilon_1}}, \quad (23)$$

$$l_{2\min} \approx \frac{2a}{\mu}\left[2\gamma\frac{U_i}{U_e}\exp\left\{\frac{-U_i\varepsilon_2}{U_e(1+\varepsilon_2)}\right\}\right]^{1/2}.$$

If $\epsilon_1 = 0.5$ and $\epsilon_2 = 0.15$ (which are typical for striated discharges) (23) gives

$$l_{1\max} \approx 1.5\, l_{1\mathrm{in}}, \quad l_{2\min} \approx {}^1/_3\, l_{1\mathrm{in}}. \tag{24}$$

From (23) it follows that the period calculated from (21) can never be less than $^1/_2 l_{\mathrm{lin}}$ or greater than $2l_{\mathrm{lin}}$, and if $\epsilon_1 > -0.5$ it is approximately equal to l_{lin}.

Before we turn to an experimental verification of the theory, let us return to the mechanical analogy of our system, which illustrates our conclusions so graphically. If we consider the variable ϵ in (14) as a displacement from the equilibrium position and the variable x as the time, this equation describes, for example, the motion of a mass suspended by a spring which is "strong" under tension and "weak" under compression. If the displacement and velocity are zero at the initial instant, there will be no oscillations; but if there is an initial velocity, oscillations will result. If there is a constant dissipative force, the vibrations will die out; if the dissipative forces act for only a short period of time, for instance during a half cycle, then after they cease to act the vibrations will continue with reduced amplitude, or cease altogether. Conversely, a short-lived injection of energy will lead to an increased amplitude. Returning now to the positive column, we note that here the "velocity" corresponds to the electron temperature gradient $\partial U_e/\partial x$.

Figure 3 shows the variation of U_e near the heat of the column.[7] It is immediately evident from the figure that in narrow region x_0x_1 at the head of the column there is an abrupt change in electron temperature from its value in the Faraday dark space to a value corresponding to the positive column (according to Schottky).

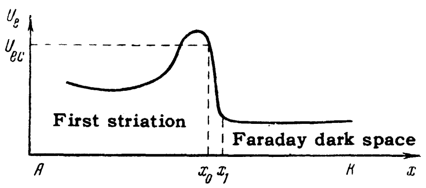

FIG. 3

This creates an initial gradient ("velocity") which in turn leads to stratification ("oscillations") that extends from the cathode end of the column right up to the anode. The conditions near the anode apparently do not affect the production of striations. Kliarfel' d[14] and Zaitsev[16] found that when the anode was moved or the gas pressure varied, the striations merely peeled off the anode. In a few cases the striations arising from the electron temperature gradient at the head of the column rapidly died out, whereupon the column became uniform.[14]*

By analogy with the mechanical system described above, it should be possible to affect the positive column, for instance to remove or add striations, by introducing charged probes into the corresponding portions of the striations.[14] The fact that it is possible by this method to suppress the striations in the region between the probe and the anode supports our conclusions about the stability of the positive column. Striations can also be removed or excited by transverse discharges of direct or high-frequency current (locally increasing U_e) or by local magnetic fields (lowering U_e).[14] For instance, if the electron temperature is raised in the dark portion of the striation, this must result in a weakening or destruction of the striations on the anode side of the disturbance. Conversely, a corresponding lowering of the electron temperature by a local magnetic field must lead to stronger stratification.

Such a conversion of the positive column from one type to another may be useful, for instance, when measuring U_{ec}, knowledge of which is indispensable for a comparison of the experimental and theoretical variations of the electron temperature. The mechanism with which we propose to explain the running striations can be verified by artificially creating an electron-temperature discontinuity within a discharge containing running striations. Such a discontinuity can be produced with the aid of an auxiliary transverse[14] or local high-frequency discharge, and must be greater than the value of the oscillation discontinuity. Instead of running striations we should then have standing ones. Apparently it is possible to create running striations in a uniform column by means of auxiliary transverse alternating-current discharges.

The assumption of an "incompressible" electron-ion plasma leads to a system of striations which do not decay. Taking account of the "compressibility" leads to the appearance in Eq. (14) of a supplementary term $(d\epsilon/dx)(d\nu/dx)$, describing the decay or build-up of the striations ("self-oscillation").†

In order to compare the theoretical and measured values of U_e and E_x and of the phase difference between them, it is necessary to produce striations with zero decay in the discharge. To

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*The effect of the atomic properties of the gas on the decay of the striations and the "compressibility" of the electron plasma will be considered in a separate paper.

†See preceding footnote.

accomplish this in a gas where the striations normally decay, it is possible, for instance, to mix with it a gas in which the striations build up (hydrogen), in the proper proportion to reduce the decay rate to zero.

Thus, Kliarfel'd[14] found that the addition of approximately 30% of hydrogen to neon reduced the decay to zero. It would seem to be possible to use corresponding mixtures of other gases as well, for instance, mixtures of two molecular gases.

In conclusion, I consider it a pleasure to express my thanks to Academician N. M. Bogoliubov for supervising this work, and to Prof. V. L. Granovskii, Prof. Ia. P. Terletskii, and A. A. Zaitsev for their encouragement and for discussion of the results.

[1] N. A. Kaptsov, Электрические явления в газах и вакууме (Electrical Phenomena in Gases in Vacuum), 1950.

[2] H. Robertson, Phys. Rev. **105**, 368 (1957).

[3] M. J. Druyvestein, Physica **1**, 273 (1934).

[4] I. M. Chapnik, Dokl. Akad. Nauk SSSR **107**, 529 (1956), Soviet Phys. "Doklady" **1**, 199 (1956); J. Tech. Phys. (U.S.S.R.) **27**, 978 (1957), Soviet Phys. JTP **2**, 894 (1957).

[5] A. von Engel and M. Steenbeck, Elektrische Gasentladungen, part II, (Russ. Transl.) 1936.

[6] A. A. Zaitsev and Kh. A. Dzherpetov, J. Exptl. Theoret. Phys. (U.S.S.R.) **24**, 516 (1953).

[7] D. Oettingen, Ann. Physik, **19**, 513 (1934).

[8] H. Schottky, Physik. Z. **25**, 342 (1924).

[9] H. Holm, Wiss. Veröff. a. d. Siemens-Konzern. **3**, 159 and 188 (1923).

[10] V. L. Granovskii, Электрический ток в газе (Electric Currents in Gases), 1952.

[11] S. Watanabe and W. Oleson, Phys. Rev. **99**, 1701 (1955).

[12] O. L. Prudkovskaia and M. F. Shirokov, Dokl. Akad. Nauk SSSR **112**, 1023 (1957), Soviet Phys. "Doklady" **2**, 96 (1957).

[13] H. Paul, Zeits. f. Phys. **97**, 330 (1935).

[14] B. H. Kliarfel'd, J. Exptl. Theoret. Phys. (U.S.S.R.) **22**, 66 (1952).

[15] J. Stoker, Nonlinear Vibrations in Mechanical and Electrical Systems, Interscience, N. Y., 1950.

[16] A. A. Zaitsev, Вестник МГУ (Bull. Moscow State Univ.) No. 9, 55 (1950).

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