DISPERSION RELATIONS AND THE DERIVATION OF THE EQUATIONS FOR K-MESON SCATTERING

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A study is made of the scattering of K mesons by nucleons, with inclusion of effects of the possible formation of Ξ particles. In addition to the other propositions of the standard theory, special use is made of the conditions of causality and of the unitary nature of the S matrix. These conditions, together with the additional hypothesis that the interaction is invariant with respect to rotations in four-dimensional isotopic spin space, enable us to obtain equations which are valid at not too high energies and are of the type of Low's equations[1,2] in the theory of π-meson scattering. The form of the interaction is involved in the equations through the inhomogeneous term. These equations are at the same time a generalization of the dispersion relations for scattering at arbitrary angles. Analogous equations for π-meson scattering have been obtained in reference 2, where, however, use was made of a special spectral representation of the scattering matrix.[3-5] This spectral representation is not used in the present paper.

THE fact that it is not a trivial problem to go from the scattering of π mesons to a treatment of K-meson scattering is mainly due to the special nature of the behavior of the K mesons with respect to transformations in ordinary space and isotopic spin space, and also to the fact that in the case of the K mesons it turns out to be necessary from the very beginning to take into account the interactions with particles that do not play any direct part in the scattering, in particular Σ and Λ particles, and also π mesons. In addition, one must give more careful attention to relativistic effects. The interactions are here taken to be the renormalized ones; effects of weak interactions are neglected. This last condition is formulated as the requirement that all interactions be invariant with respect to rotations in the four-dimensional isotopic space.[6] In this space nucleons, Ξ particles, and K mesons form the four-dimensional isotopic spinors:

$$\Psi_N = \begin{vmatrix} \psi_p \\ \psi_n \\ \psi_{\Xi^0} \\ \psi_{\Xi^-} \end{vmatrix} \text{ and } K = \begin{vmatrix} K^+ \\ K^0 \\ \tilde{K}^0 \\ K^- \end{vmatrix} = \| K^i \|, \quad (i = 1, 2, 3, 4); \qquad (1)$$

The following representation is chosen for the isotopic matrices:*

$$\gamma_i^i = \begin{vmatrix} 0 & \tau_i \\ -\tau_i & 0 \end{vmatrix} \ (i = 1, 2, 3); \quad \gamma_0^i = \begin{vmatrix} 0 & E_2 \\ E_2 & 0 \end{vmatrix}; \quad \gamma_5^i = \begin{vmatrix} iE_2 & 0 \\ 0 & -iE_2 \end{vmatrix}. \qquad (2)$$

This way of writing makes it possible to give a unified description of the dynamical behavior of all K mesons, and furthermore the "hypercharge" Y[6] of the heavy fermions is given apart from a factor i by the eigenvalue of the matrix γ_5^i. Unlike the convention of reference 6, we regard the isotopic space as pseudo-Euclidean, since only in this kind of a space can an isotopic spinor K that does not vanish identically be taken to be self-conjugate:

$$K = -i\gamma_2^i K^*. \qquad (3)$$

It is necessary, however, to impose the invariant requirement that K be self-conjugate, owing to the existence of only four different K mesons. In connection with the experimental facts that are interpreted in terms of the presence of different parities of K mesons, one can assume* that an

*Hereafter, where no special stipulation is made, the notations of reference 7 are used.

*One can try to give this fact a general explanation through the idea of a fusion of the ordinary and isotopic spaces.[8] We note also that the definition of the hypercharge is here an obvious four-dimensional way of writing the isotopic fermion number of d'Espagnat and Prendtki. The reflections in this four-dimensional isotopic space for the K mesons are analogous to the Pauli transformations[9] for spinor particles.

ordinary reflection results in multiplying each isotopic spinor by γ_5^i. To shorten the writing we shall also use the formal concept of the baryon space,[6]* in which Ψ_N and the real isotopic vector $\Psi_\Sigma(\Lambda_0, \Sigma_1 + i\Sigma_2, \Sigma_1 - i\Sigma_2, \Sigma_3)$ are represented as a two-component quantity $\Psi_B = \begin{vmatrix}\Psi_N\\ \Psi_\Sigma\end{vmatrix}$, on which the Pauli matrices ν_i act. After these remarks we can write the amplitude for the scattering of K mesons by nuclei in a form analogous to the amplitude for the scattering of π mesons:[7,10]

$$\delta(p' + q' - p - q) f(\mathbf{p}'s', \mathbf{q}'j; \mathbf{p}s, \mathbf{q}i) = \frac{\pi}{(2\pi)^3 i}\int dx dy e^{-i(qx - q'y)} \left\langle \mathbf{p}'s' \left| \frac{\delta^2 S}{\delta K_j^*(y)\, \delta K_i(x)} S^+ \right| \mathbf{p}s \right\rangle. \tag{4}$$

Here s′ and s are spinor and isotopic indices, including the value of the hypercharge, which is equal to +1 for the incident nucleon and ±1 for the scattered nucleon. The functional derivative $\delta/\delta K_j^*(y)$ appearing in Eq. (4) can be replaced, in virtue of the condition (3), by

$$\delta/\delta(i\gamma_2^i K^*)_j = (-1)^{\max(j, 5-j)} \delta/\delta K_{5-j}(y).$$

From this it follows that the amplitude for the scattering of K mesons is invariant with respect to the substitution

$$q \leftrightarrow -q' = -(p + q - p'),\quad i \leftrightarrow 5 - j \tag{5}$$

if one at the same time multiplies by the sign factor $a(ij) = (-1)^{\max(i,5-i)} \cdot (-1)^{\max(j,5-j)}$. This result is equivalent to a well known theorem of Gell-Mann and Goldberger on symmetry in the scattering of π mesons.[10] We further introduce, in analogy with reference 7, the K-meson current operator $j_i(x)$:

$$j_i(x) = i\frac{\delta S}{\delta K_i(x)} S^+ = -iS\frac{\delta S^+}{\delta K_i(x)}, \tag{6}$$

which, in virtue of the causality condition and the unitary property of the S matrix, satisfies the conditions

$$j_i^+(x) = (-1)^{\max(i,5-i)} j_{5-i}(x),$$
$$\delta j_i(x)/\delta K_j(y) = 0 \quad \text{for } y_0 < x_0, \tag{7}$$
$$\delta j_i(x)/\delta K_j(y) = 0 \text{ and } [j_i(x), j_j(y)] = 0 \text{ for } (x - y)^2 \leqslant 0.$$

Using these conditions, we can rewrite the scattering amplitude in the form

$$\delta(p + q - p' - q') f(\mathbf{p}'s', \mathbf{q}'j; \mathbf{p}s, \mathbf{q}i) = \frac{\pi}{(2\pi)^3 i}(-1)^{\max(j,5-j)}\int dx dy e^{-i(qx - q'y)} \tag{8}$$
$$\times \langle \mathbf{p}'s' | T(j_{5-j}(y)\, j_i(x)) + \Lambda_{5-j,i}\left(x, \frac{\partial}{\partial x}\right)\delta(x - y) | \mathbf{p}s \rangle.$$

*In reference 6 this space is called the nucleonic space.

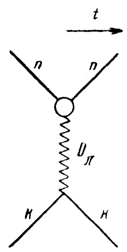

FIG. 1

To begin with let us consider separately the last term, which corresponds to the simultaneous production of the final K meson and annihilation of the initial K meson. Omitting effects of weak interactions, we find that this term is described by the diagram shown in Fig. 1, and is given by

$$\frac{1}{(2\pi)^5}\tilde{g}_{n\pi n}\tilde{g}_{K\pi K}\bar{v}^+(\mathbf{p}'s')\int dx\, dx' dy\, dy' d\xi\, d\xi' e^{i(q'y' + p'x' - px - qy)} \times \Gamma_{B\pi B}(x, x'; \xi) D_\pi(\xi, \xi')\Gamma^0_{K\pi K}(y, y'; \xi') v^-(\mathbf{p}s). \tag{9}$$

Here and in what follows Γ denotes as usual the total vertex part, Γ^0 denotes the vertex part in the first approximation of perturbation theory, determined by the form of the interaction in L_{int}, and D_π is the total propagation function of the π meson. Furthermore Γ^0 is proportional to the δ functions $\delta(y - y')\delta(y - \xi)$ or to finite derivatives of these functions; in virtue of the conservation of hypercharge or, what is equivalent, the conservation of strangeness, $\Gamma_{B\pi B}$ does not contain transitions from nucleons to χ particles. For an analogous reason, and also on account of the transformation properties of K and π mesons under reflections, in the case of parity conservation $\Gamma_{K\pi K} = 0$ and in the framework of the standard theory the entire term (9) must be set equal to zero. If, however, we require only invariance with respect to products of ordinary reflections and isotopic reflections (in particular charge conservation), and not with respect to the separate reflections,* $\Gamma_{K\pi K} \neq 0$ for $(i - j) \leq 2$.

Omitting the term (9) that has already been considered, after the usual transformations of the theory of dispersion relations,[1,7] we get from Eq. (8)

$$f(\mathbf{p}'s', \mathbf{q}'j; \mathbf{p}s, \mathbf{q}i) = (-1)^{\max(j,5-j)}(2\pi)^5/2 \times\int d\mathbf{k}\left\{\sum_n\left[\frac{\langle \mathbf{p}'s' | j_{5-i}(0) | n, \mathbf{k}\rangle\langle n, \mathbf{k} | j_j(0) | \mathbf{p}s\rangle}{p_0 + q_0 - E_n(\mathbf{k}) - i\varepsilon}\delta(\mathbf{p} + \mathbf{q} - \mathbf{k}) + \frac{\langle \mathbf{p}'s' | j_i(0) | n, \mathbf{k}\rangle\langle n, \mathbf{k} | j_{5-j}(0) | \mathbf{p}s\rangle}{p_0' - q_0 - E_n(\mathbf{k}) + i\varepsilon}\delta(\mathbf{p}' - \mathbf{q} - \mathbf{k})\right\}.\right. \tag{10}$$

We have further in the real case

$$p_0 + q_0 = p_0' + q_0', \quad p_0 \geqslant m_n \text{ and } q_0, q_0' \geqslant m_K > 0. \tag{11}$$

*This condition can be interpreted as the existence of rotations only in the combined space (ordinary × isotopic).

This last condition has allowed us to change the sign of $i\epsilon$ in the denominator of the second term of Eq. (10), since this denominator does not pass through zero. We now introduce the Hermitian and anti-Hermitian parts of the scattering amplitude:

$$D(\mathbf{p}'s', \mathbf{q}'j; \mathbf{p}s, \mathbf{q}i) = \frac{1}{2}[f(\mathbf{p}'s', \mathbf{q}'j; \mathbf{p}s, \mathbf{q}i) + f^*(\mathbf{p}s, \mathbf{q}i; \mathbf{p}'s', \mathbf{q}'j)],$$
$$A(\mathbf{p}'s', \mathbf{q}'j; \mathbf{p}s, \mathbf{q}i) = \frac{1}{2i}[f(\mathbf{p}'s', \mathbf{q}'j; \mathbf{p}s, \mathbf{q}i) - f^*(\mathbf{p}s, \mathbf{q}i; \mathbf{p}'s', \mathbf{q}'j)], \tag{12}$$

In virtue of the unitary property of the S matrix,

$$A|_{p+q=p'+q'} = -\frac{1}{2}\sum_n f^*(n, \mathbf{p}+\mathbf{q}; \mathbf{p}'s', \mathbf{q}'j) f(n, \mathbf{p}+\mathbf{q}; \mathbf{p}s, \mathbf{q}i)\,\delta(E_n - p_0 - q_0), \tag{13}$$

where $f(n, \mathbf{p}+\mathbf{q}; \mathbf{p}s, \mathbf{q}i)$ are the scattering amplitudes into states n with momentum $\mathbf{p}+\mathbf{q}$ and energy $E_n(\mathbf{p}+\mathbf{q}) > 0$. Here, as is usual in the theory of the dispersion relations, the system of states n is identified with a system of particles and complexes with the experimental values of the parameters at infinity, and is taken to be a complete set. On the other hand, from Eq. (10) with use of Eqs. (7) and (11) it follows that

$$A|_{p+q=p'+q'} = \pi\frac{(2\pi)^5}{2}(-1)^{\max(j,5-j)}\sum_n \langle \mathbf{p}'s'|j_{5-j}(0)|n, \mathbf{p}+\mathbf{q}\rangle\langle n, \mathbf{p}+\mathbf{q}|j_i(0)|\mathbf{p}s\rangle\,\delta(E_n(\mathbf{p}+\mathbf{q}) - p_0 - q_0). \tag{14}$$

It is essential that Eq. (14) is valid not only in the region $p_0 \geq m_n$, $q_0 \geq m_K$, but also in the region* $p_0 \geq m_n$, $q_0 > m \ \ - m_\Sigma$. We now consider the connection between D and A, following mainly the work of reference 7. For this purpose we introduce the function $f(z)$ of a complex variable z, obtained by replacing $q_0 - i\epsilon$ by z in Eq. (10). As follows from an examination of Eq. (10), when one takes into account the conservation laws the function $f(z)$ is analytic everywhere for $\mathrm{Im}\, z \neq 0$ and for prescribed p and p', at any rate on the segment of the real axis

$$-\sqrt{m_\Lambda^2 + (\mathbf{p}-\mathbf{q})^2} + p_0' < z < \sqrt{m_\Lambda^2 + (\mathbf{p}+\mathbf{q})^2} - p_0, \quad \mathrm{Im}\, z = 0.$$

We postulate further that $f(z)$ falls off at infinity† faster than z^{-1}. Using Eq. (7), one easily gets from Eq. (10) and (12) (for brevity the arguments other than q_0 are not written out):

$$D(q_0) = \frac{1}{2}\left[\lim_{z\to q_0 - i\varepsilon} f(z) + \lim_{z\to q_0 + i\varepsilon} f(z)\right];$$
$$A(q_0) = \frac{1}{2i}\left[\lim_{z\to q_0 - i\varepsilon} f(z) - \lim_{z\to q_0 + i\varepsilon} f(z)\right]. \tag{15}$$

Using the properties of the function $f(z)$ and applying the Cauchy integral theorem, taking the contours of integration shown in Fig. 2, one easily finds that

$$f(q_0 + i\varepsilon) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\tilde{q}_0 \frac{f(\tilde{q}_0 + i\varepsilon/2)}{\tilde{q}_0 - q_0 - i\varepsilon/2};$$
$$f(q_0 - i\varepsilon) = \frac{1}{2\pi i}\int_{+\infty}^{-\infty} d\tilde{q}_0 \frac{f(\tilde{q}_0 - i\varepsilon/2)}{\tilde{q}_0 - q_0 + i\varepsilon/2}. \tag{16}$$

Thereupon, taking the average of the expressions (16) and letting ϵ go to zero, we find that

$$D(q_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\tilde{q}_0 \frac{(\tilde{q}_0 - q_0)A(\tilde{q}_0)}{(\tilde{q}_0 - q_0)^2 + \varepsilon^2/4} = \frac{1}{\pi}\int_{-\infty}^{\infty} A(\tilde{q}_0)\,\mathrm{P}\frac{d\tilde{q}_0}{\tilde{q}_0 - q_0}. \tag{17}$$

*In using Eq. (13) in the region $|q_0| < m_K$ one must proceed in all the intermediate manipulations as is done in reference 7, replacing m_K^2 by a fictitious quantity $\tau < q_0^2, q_0'^2$, with respect to which the scattering amplitude is an analytic function, and continue the result to the value $\tau = m_K^2$.

†If the stated hypothesis does not hold and $f(z)$ behaves at infinity like z^n (evidently in reality $n = 0$), one could consider instead of $f(z)$ the quantity $f(z)/(z - z_0)^{n+1}$ in the usual way, with corresponding modifications of all subsequent relations. The standard way of solving the Low equations has meaning, however, only provided that intermediate states with large numbers of particles and high energies do not for any reason make an important contribution to the scattering, since only if this is true is it possible to justify dropping out the higher amplitudes in the infinite system of coupled equations. In this case one can also confirm the assumed decrease of $f(z)$ with increasing z, by arguing as follows from Eq. (10). Neglecting, for large z, all terms except z in the denominator of Eq. (10), by representing the δ functions as Fourier series one can carry out the summation over the intermediate states. After this the numerator of the integrand will be proportional to the matrix element of the commutator $[j_i(0, y), j_{5-j}(0)]$, which is zero in virtue of the causality condition. We emphasize that a complete treatment of this question is in general impossible in the framework of the existing theory, and that one must only hope that, as in the case of π-meson scattering, these last considerations are in some sense justified, at least for not too high energies of the scattered particles.

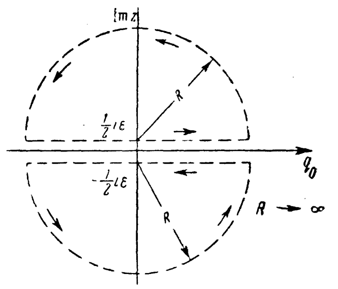

FIG. 2

It follows from Eq. (17) that for given $\mathbf{p}s$, $\mathbf{p}'s'$

$$f(\mathbf{p}'s', \mathbf{q}'j; \mathbf{p}s, \mathbf{q}i) = f_{ji}\left(q_0', \mathbf{n}' = \frac{\mathbf{q}'}{|\mathbf{q}'|};\ q_0, \mathbf{n} = \frac{\mathbf{q}}{|\mathbf{q}|}\right)$$
$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{A(p_0 - p_0' + \tilde{q}_0, \mathbf{n}', j; \tilde{q}_0, \mathbf{n}, i)}{\tilde{q}_0 - q_0 - i\varepsilon}\, d\tilde{q}_0. \qquad (18)$$

In Eq. (18) we carry out the change to observable quantities, recalling that A is expressed in terms of the scattering amplitude by Eq. (13) only for $q_0', q_0 \geq m_K$; $p_0, p_0' \geq m_N$. For this purpose we go over to Salam's reference system, in which $\mathbf{p} + \mathbf{p}' = 0$. In this system[7]

$$q_0 = q_0' = \sqrt{m_K^2 + p^2 + \lambda^2};\quad p_0 = p_0',\ \mathbf{q} = -\mathbf{p} + \lambda\mathbf{e},$$
$$\mathbf{q}' = \mathbf{p}' + \lambda\mathbf{e},\quad \mathbf{e}^2 = 1,\ \mathbf{ep} = 0 \qquad (19)$$

and the substitution (5) takes the form

$$i \leftrightarrow j;\ q_0 \leftrightarrow -q_0;\ \mathbf{e} \leftrightarrow -\mathbf{e}. \qquad (5')$$

In view of what has been said we can write, dropping from the list of arguments the prescribed values $\mathbf{p}s$, $\mathbf{p}'s'$,

$$f_{ji}(q_0, \mathbf{e}) = \frac{1}{\pi}\int_0^{\infty}\left[\frac{A_{ij}(\tilde{q}_0, \mathbf{e})}{\tilde{q}_0 - q_0 - i\varepsilon} - a(i,j)\frac{A_{ij}(\tilde{q}_0, -\mathbf{e})}{\tilde{q}_0 + q_0 + i\varepsilon}\right] d\tilde{q}_0. \qquad (20)$$

The region of integration in Eq. (20) can be divided into two parts: from zero to E_1 (the so-called unphysical part), and from E_1 to infinity, where in the chosen system of reference $E_1 = (m_K^2 + p^2)^{1/2}$. In the second region the expression (13) can be used for A_{ji}. We see, however, that owing to the presence in Eqs. (13) and (14) of δ functions of the energy, and in virtue of the conservation laws, if we have just the condition

$$p^2 < p_1^2 = \frac{{}^1\!/_4\left[(m_\Sigma + 2m_\pi)^2 - m_n^2 - m_K^2\right]^2 - m_n^2 m_K^2}{m_n^2 + m_K^2 + \left[(m_\Sigma + 2m_\pi)^2 - m_n^2 - m_K^2\right]} > 0,$$

and if we assume that the amplitude $f_{hi}(q_0, \mathbf{e})$ is itself determined by the corresponding relations for all $q_0 > 0$, then the contribution in the unphysical region will be given only by states with a single intermediate Σ particle, or also with a Σ particle and a π meson (in the case of an intermediate Λ particle the presence of two π mesons is possible). Therefore in Eq. (13) and (14) we break up the sum over the intermediate states into two parts

$$\sum_n = \sum_n{}' + \sum_n{}'', \qquad (21)$$

where $\sum_n''$ denotes the sum over the positive-energy intermediate states that can give a contribution in the unphysical region, except states with one nucleon and one K meson. We shall make a direct evaluation of the contribution to A corresponding to $\sum''$. From what has been said it follows that in the first place there will occur in the sum $\sum_n''$ the anti-hermitian component $A_{ji}^{(a)}$ of the part of the scattering amplitude given by the diagram shown in Fig. 3. The corresponding quantity can be determined in complete analogy with the theory of π-meson scattering in the following way:*

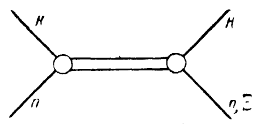

FIG. 3

$$A_{ji}^{(a)}\delta(p + q - p' - q') = -\frac{\tilde{g}_{BKB}\,\delta\,(-1)^{\max(j,5-j)}}{2i\cdot 2(2\pi)^5}$$
$$\times\left\{\overline{v}^{+}(p's')\int_0^{\infty} d\,p_0''\int d\mathbf{p}''\,\Gamma_{BK_{5-j}B}(p', p''; -q')\right.$$
$$\left.\times\, G_\Sigma(p'')\,\Gamma_{BK_iB}(p'', p'; q)\,v^{-}(p, s) - [\quad]\right\}. \qquad (22)$$

The second term in the curly brackets, indicated by square brackets, is obtained from the first term by the interchange $p' \longleftrightarrow p$, $q \longleftrightarrow q'$, $i \longleftrightarrow j$ and complex conjugation. Here Γ_{BKB} are vertex parts defined in the standard way:

$$\tilde{g}_{BKB}\,\Gamma_{BK_iB}(x, x'; \xi) = -\frac{\delta G_B(x, x')}{\delta\langle K_i(\xi)\rangle_0}$$

*To give to Eq. (22) an explicitly invariant form with respect to transformations in the four-dimensional isotopic space, one would have to replace $\overline{v}^{+}$ by $\overline{v^{+}\gamma_0^i}$ and make a corresponding change in the definition of G_B. To shorten the presentation we have not done this, but this circumstance must be kept in mind in examining Eq. (25) and in many other cases in which it is not sufficient to consider only invariance with respect to three-dimensional isotopic transformations.

and G_B is the total propagation function of the baryons

$$G_B = \begin{vmatrix} G_N \\ G_\Sigma \end{vmatrix}. \quad (23)$$

Here, for example, $G_\Sigma = i\langle T(\overline{\psi}_\Sigma \psi_\Sigma)\rangle_0$ has the form[7]

$$G_\Sigma(\mathrm{p}) = \frac{\gamma p s_1(p^2) + m_\Sigma s_2(p^2)}{m_\Sigma^2 - p^2 - i\varepsilon}, \quad s_1(m_\Sigma^2) = s_2(m_\Sigma^2) = 1, \quad (24)$$

where m_Σ is the experimental mass of the Σ particle.

By using the relation (7.15) of reference 7, the equation

$$\gamma_0 \Gamma^+_{BK_iB}(p', p''; -q')\gamma_0 = (-1)^{\max(i,5-i)}\Gamma_{BK_{5-i}B}(p'', p'; q') \quad (25)$$

which follows from the definition of Γ_{BKB} (see also the preceding note), and Eq. (24), we get from Eq. (24), the results

$$A_{ji}^{(a)}\delta(p+q-p'-q') = -\frac{\tilde{g}^2_{BKB}}{4(2\pi)^4}(-1)^{\max(j,5-j)}\bar{v}^+(\mathrm{p}'s')\int_0^\infty dp_0''\int d\mathrm{p}$$
$$\times \Gamma_{BK_{5-j}B}(p', p''; -q')(\gamma p'' + m_\Sigma)\delta(m_\Sigma^2 - p''^2)\Gamma_{BK_iB}(p'', p'; q)v^-(\mathrm{p}s)$$
$$= -\frac{\tilde{g}^2_{BKB}}{4(2\pi)^4}\delta(p'+q'-p-q)\int d\mathrm{p}'' \frac{\delta(p_0'' - \sqrt{\mathrm{p}''^2 + m_\Sigma^2})}{2\sqrt{\mathrm{p}''^2 + m_\Sigma^2}}\delta(p''-p-q)$$
$$\times \bar{v}^+(\mathrm{p}'s')\Gamma_{BK_{5-j}B}(p'; p'-p'')(\gamma p'' + m_\Sigma)\Gamma_{BK_iB}(p''; p''-p)v^-(\mathrm{p}, s), \quad (26)$$

where $\Gamma(p'', p''-p)\delta(p''-p-q) = \Gamma(p'', p; q)$.

As can be seen from Eq. (26), the vertex parts $\Gamma_{BKB}(p', p''; q)$ occur only with values of the momenta that satisfy the free-particle equations with the experimental masses; using the arbitrariness in the prescription of $\tilde{g}_{BKB}$, we can define the renormalized coupling constants of the interaction of baryons with K mesons by means of the condition

$$\tilde{g}_{BKB}\Gamma_{BKB}(m_B^2, m_B^2; m_K^2) = g_{BKB}\Gamma^0_{BKB}(1 + O(\mathrm{q}^2)), \quad (27)$$

where $O(0) = 0$ and Γ^0_{BKB} is the interaction corresponding to the first approximation of perturbation theory for the special L_{int} with the experimental parameters.* Inserting the expression (26) as the part A in Eq. (20) and performing the integration over $\tilde{q}_0$, we verify that $A^{(a)}$ gives two terms in the scattering amplitude. In the low-energy limit these terms agree respectively with the total term of the first approximation of perturbation theory for the diagram of Fig. 3 and with the term for the diagram obtained from Fig. 3 by the substitutions (5), for the experimental values of the parameters. We note that in consequence of the conservation of hypercharge one of these terms is always equal to zero. As has already been mentioned, besides the diagram of Fig. 3 there will also be contributions $A^{(b)}$ to $\sum_n''$ from diagrams in which at some stage in the intermediate states there are π mesons present as well as Σ particles. Even at low energies, however, the anti-Hermitian part $A^{(b)}$ constructed in the same way as Eq. (26) will be determined not only by the form of the interaction and the experimental parameters (masses and coupling constants) but also, in general, by the unknown functions $s_1(p^2)$, $s_2(p^2)$ of Eq. (24), and also by the analogous functions occurring in G_n and D_π. These functions must be further determined, either by a rigorous or approximate solution of the field equations, or by the study of other experiments.

Substituting Eqs. (13) and (21) into Eq. (20) and including the term (9), we finally obtain, in the reference system $\mathbf{p} + \mathbf{p}' = 0$ [Eq. (19)] for prescribed $\mathbf{p}s$, s' and $\mathbf{p}^2 < \mathbf{p}_1^2$,

$$f_{ji}(q_0, \mathrm{e}) = (1 + \mathrm{Int})\Big\{O_{ji}(1) + \frac{1}{2\pi}\sum_n{}' \frac{f^*(n, \lambda_n'\mathrm{e}; -\mathrm{p}s', \mathrm{p} + \lambda_n'\mathrm{e}j)\, f(n, \lambda_n'\mathrm{e}; \mathrm{p}s, -\mathrm{p} + \lambda_n'\mathrm{e}i)}{q_0 + p_0 - E_n - i\varepsilon}\Big\}. \quad (28)$$

Here Int means the substitution (5) with multiplication by $a(ij)$; λ_n' is defined by the relation $\lambda_n'^2 = (E_n(\lambda') - p_0)^2 - \mathbf{p}^2 - m_K^2 > 0$; and $O_{ji}(1)$ denotes terms calculated on the stated basis in the first approximation of perturbation theory, with the experimental parameters, from the diagrams

*For brevity we do not display distinctions between the interaction constants between K mesons and various baryons. These constants ($q_{\Sigma KN}$, q_{NKN}, etc.) can be introduced independently of each other, by considering separately the various processes of K-meson scattering.

of Figs. 1 and 3 and diagrams with a Σ particle and a π meson, possibly with inclusion of additional experimental information.

The equation that has been obtained is the first member of an infinite system of coupled equations for the scattering amplitudes, of the type of relativistic Low equations. In the first approximation, in which one usually solves the Low equations, all the higher amplitudes are omitted, and at sufficiently low energies we arrive at a closed integral equation.* As follows from our result (28), this equation is simply the corresponding dispersion relation, with the form of the interaction entering through the inhomogeneous term. A special property of the K mesons is the fact that a study of their scattering by means of equations of the type of the Low equations can provide a basis for important conclusions about the structure of the isotopic space. Moreover, qualitative conclusions already offer a possibility, by analogy with the π-meson scattering, of settling in which state the scattering will be largest.

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*By this approximate equation $f_{ij}(q_0, e)$ is determined not only in the physical region, but also for $0 < q_0 < (m_K^2 + p^2)^{1/2}$.

[1] F. E. Low, Phys. Rev. **97**, 1392 (1955).
[2] A. M. Brodskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 616 (1957), Soviet Phys. JETP **5**, 509 (1957).
[3] Y. Nambu, Phys. Rev. **100**, 394 (1955).
[4] A. Brodskii, Dokl. Akad. Nauk SSSR **111**, 787 (1956), Soviet Phys. "Doklady" **1**, 701 (1956).
[5] Проблемы современной физики (Problems of Contemporary Physics) No. 2, (1957), Introductory Article. 1957.
[6] J. Schwinger, Theorie des particules elementaires, l'Universite de Paris, 1957.
[7] N. N. Bogoliubov and D. B. Shirokov, Введение в теорию квантованных полей (Introduction to the Theory of Quantized Fields), GITTL, Moscow, 1957.
[8] D. Ivanenko and H. Sokolik, Nuovo cimento **6**, 226 (1957).
[9] G. Lüders, Nuovo cimento **7**, No. 2, 171 (1957).
[10] A. Klein, Phys. Rev. **99**, 998 (1955).

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ONE POSSIBLE MODE OF DEVELOPMENT OF EXTENSIVE AIR SHOWERS

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The development of extensive air showers is studied under the assumption that the fraction of energy lost in interaction of ultra-high energy particles with light nuclei is subject to strong fluctuations. It is shown that the main features of extensive air showers can be explained without recourse to the hypothesis that the nuclear component plays an important role in the development of showers in the depth of the atmosphere.

1. INTRODUCTION

It is well known that extensive air showers (EAS) consisting of 10^4 to 10^5 particles possess a lateral distribution which is independent of the altitude of observation (within the limits of 1000 to 640 g/cm^2 atmospheric depth) and that the number of such showers varies in the atmosphere exponentially, with an absorption coefficient $1/\mu = 130$ to 140 g/cm^2. These facts have been explained by several authors[1-3] who have assumed that the development of EAS is determined by the development of nuclear cascade. In that theory, the slow absorption of showers in the atmosphere is explained