THEORY OF THE DEVELOPMENT OF A SPARK CHANNEL

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The principal processes taking place in a spark channel at moderate currents are examined. Solutions are obtained for the motion of the gas outside the channel. A new type of hydrodynamic jump is considered — a strong discontinuity with external supply of heat. Certain solutions are found which describe the state of the gas inside the channel, and expressions are obtained for the characteristic parameters of the channel (radius, temperature, etc.).

1. INTRODUCTION

In the present paper we consider the development of a spark channel under comparatively high pressures and moderate currents. This process has been studied in detail by Mandel' shtam and his co-workers.[1-6] In reference 1, on the basis of experimental results, the idea was expressed that the rapid development of a spark channel is accounted for by the excitation of a shock wave. In subsequent papers, this phenomenon was studied in detail, both experimentally and theoretically. The theory of the development process was given by Drabkina; the results of her calculations are in good agreement with experiment. However, the theory advanced by Drabkina is not complete; the electrical conductivity and the temperature in the channel are not computed in this theory, so that it does not permit us to calculate the parameters of the spark directly, by starting from the law of current growth. Rather, it only relates the velocity of its growth with the energy released in the channel; this latter energy must be determined experimentally.

In the present research, an attempt is made to consider a specific mechanism of the discharge and to construct a step-by-step theory of the development of the channel, with account of the electrical conductivity and the thermal conductivity of the ionized gas in the channel.

In accord with the results of references 1 to 6, the picture of the development of the spark channel can be represented in the following form. A comparatively narrow current-carrying channel is formed in the gas, with high temperature and ionization. Joule heat is released in this channel, which then leads to an increase in the pressure and a thickening of the channel. The thickened channel acts like a piston on the remaining gas and, since the expansion takes place with supersonic speed, it produces a shock in the gas; this shock is propagated in front of the original "piston." The temperature in the vicinity of the shock (between the wave front and the "piston") is much higher than in the gas at rest, and the temperature in the channel itself is still many times higher than in the shock wave. Consequently, the density of the gas in the channel is very low, and the major part of the mass of the moving gas is displaced from it, which also makes it possible to consider the boundary of the channel as a piston.

The very fact of the formation of the narrow channel can evidently be understood by starting from the following considerations. After the gas sparks over and becomes conducting, Joule heat is released at points of flow. As is well known, the electrical conductivity of the gas increases rapidly with temperature. Thus, at a high degree of ionization, when the collisions of electrons with ions are important, the electrical conductivity is proportional to $T^{3/2}$, while at low ionization this dependence is even stronger, (because of the fact that the degree of ionization increases rapidly with temperature). As a consequence, a tendency appears toward a concentration of current in a comparatively narrow channel, so that at the places where the temperature is higher, the conductivity is also great, a large current exists there, and a large amount of heat is liberated, which leads to more heating, etc. The physical processes which determine the breadth of the channel and limit the concentration of current are the leakage of heat from the channel and the broadening of the heated region under the action of the pressure.

With some indefiniteness, we can consider as the channel the region from the axis to the point where the temperature becomes so low that ionization begins to fall off appreciably. In the channel, we can neglect the inertia of the gas, but it is necessary to take into consideration the release and transfer of heat. In the shock wave region, the inertia must be considered, but we can neglect the electrical and thermal conductivities. These two regions as separated by a transition layer, the "shell" of the channel. Heating and ionization of the gas that enters the channel take place in the shell.

2. FUNDAMENTAL EQUATIONS

The fundamental equations of the problem under consideration are the equation of continuity, the equation of motion, and the equation of energy transfer:

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial r} + \rho\frac{\partial (rv)}{r\partial r} = 0; \qquad (2.1a)$$

$$\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r}\right) + \frac{\partial p}{\partial r} = 0; \qquad (2.1b)$$

$$\frac{\partial}{\partial t}\left(\rho\varepsilon + \frac{\rho v^2}{2}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left\{r\rho v\left(\varepsilon + \frac{p}{\rho} + \frac{v^2}{2}\right)\right\} + \frac{\partial (rq)}{r\partial r} = jE. \qquad (2.1c)$$

Here ρ is the density, v the velocity, p the pressure, ϵ the internal energy per unit mass of gas, q the heat flow, f the current density, and E the electric field.

We shall write the equation of state in the form

$$p = (n_e + n_i)T = (Z+1)\rho T/m_a, \qquad (2.2)$$

where m_a is the average atomic mass, n_e and n_i the number of electrons and ions per unit volume, Z the average ionic charge, and $n_e = An_i$. The temperature is expressed in energy units.

We shall assume that the ionization in the channel can be computed by Sach's formula. This problem is considered in detail in reference 6.

The internal energy of the gas in the channel is expressed in the form

$$\varepsilon = \frac{3}{2}\frac{p}{\rho} + \frac{I}{m_a} = \frac{p}{\rho}\left[\frac{3}{2} + \frac{I}{(Z+1)T}\right], \qquad (2.3)$$

where I is the total energy of ionization plus the energy of dissociation, referred to a single atom. It is appropriate to apply Eq. (2.3) in the case of complete ionization, for example, for hydrogen, Z = 1, I = 15.74 ev. For incomplete ionization, the energy of ionization increases with increasing temperature. According to Sach's formula, the ratio l/T depends rather weakly on the density and temperature; therefore, for a not too wide an interval of change of these parameters, the expression in the square brackets can be considered to be approximately constant. In this case it is more suitable to take the expression for the energy in the form

$$\varepsilon = \frac{1}{\gamma - 1}\frac{p}{\rho}, \qquad (2.3a)$$

as was done by Drabkina.[2] Here γ is the effective ratio of specific heats. The value of γ is somewhat different for the gas in the channel and in the shock wave. According to reference 2, $\gamma = 1.25$ for hydrogen and 1.22 for air.

Transfer coefficients. The electrical conductivity σ and the thermal conductivity κ differ strongly for the ionized gas (see, for example, reference 7):

$$\sigma = \sigma_1(Z)T^{3/2} = 3\sigma'(Z)T^{3/2}/4e^2\sqrt{2\pi m}\lambda; \qquad (2.4)$$

$$\varkappa = \varkappa_1(Z)T^{5/2}. \qquad (2.5)$$

Here e and m are the charge and mass of the electron, $\lambda = \ln(3T^{3/2}/ze^3\sqrt{4\pi m_e})$, and $\sigma'(Z)$ is a dimensionless coefficient. For Z = 1, 2, 3, and 4 we have, respectively, $\sigma' = 1.95$, 1.135, 0.840, and 0.667. The value of $\kappa e^2/\sigma T$, according to the Wiedemann-Franz law, is of the order of unity. For Z = 1, 2, 3, and 4 this combination is equal to 1.62, 2.16, 2.40, and 2.60 respectively. The "Coulomb logarithm" λ is only slightly sensitive to the values of the quantities entering into it. For $\lambda = 5$, for example, we have $\sigma_1(1) = 3.4 \times 10^{-13}$ $\text{sec}^{-1}\text{ev}^{-3/2}$, and $\kappa_1(1) = 3.9 \times 10^{20}\,\text{cm}^{-1}\text{sec}^{-1}\text{ev}^{-5/2}$.

We note that the electrical conductivity of air increases with the temperature more slowly than $T^{3/2}$, because of the increase of Z as a consequence of ionization. At a temperature on the order of several electron volts, it changes approximately as $T^{1/2}$ and is equal to $2 \times 10^{14}\,\text{sec}^{-1}$ for $T \approx 3$ to 4 ev.

Radiation. A simple estimate, taking experimental data[3-5] into account, shows that if the radiation from the channel were black body radiation, it would carry several tenfold more energy than is actually released in the channel. In fact, the radiation is nonequilibrium and flows freely from the channel. For open radiation, div q = Q'_R, where Q'_R is the energy radiated per unit volume. The fundamental mechanism of open radiation is the retardation radiation

$$Q'_{\text{ret}} = 1.5\cdot 10^{-25} n_i n_e Z^2 T_{\text{ev}}^{1/2}\ (\text{erg-cm}^{-3}\text{sec}^{-1}) \qquad (2.6)$$

(see reference 8) and recombination radiation. For hydrogenlike atoms, the latter can be computed from the approximate formula

$$Q'_{\text{rec}} = 5\cdot 10^{-24} n_i n_e Z^4 T_{\text{ev}}^{-1/2}(\text{erg-cm}^{-3}\text{sec}^{-1}). \qquad (2.7)$$

Equation (2.7) was obtained by V. I. Kogan, using the cross section of recombination at the different levels given in reference 9.

For partially-ionized atoms of the different elements, which cannot be considered hydrogenlike, one must expect that the radiation of Z charged ions is greater than calculated by the "hydrogenlike" formulas, because of the incomplete screening. This circumstance can be taken into consideration if we use the effective change $Z_{eff} = Z + \Delta$ in place of the actual charge of the ion Z. According to Unsöld,[10] we can take as a sort of mean value (as a rough approximation, with great uncertainty) $\Delta = 1.5$.

Radiation in the discrete spectrum as a result of resonance absorption is forbidden for many lines and is close to the equilibrium (Planckian) in intensity. The presence of such radiation increases the thermal conductivity of the plasma. In the present research, however, we shall not consider this radiation thermal conductivity, since its calculation is a complicated independent problem which requires a detailed knowledge of the spectrum and the line widths.

Skin effect and the magnetic field. The penetration depth δ of the field after a time t can be estimated by the formula $\delta^2 \sim c^2t/2\pi\sigma$. According to Abramson and Gegechkori,[3,4] $\sigma = 2 \times 10^{14}\,\mathrm{sec}^{-1}$. At the instant $t = 10^{-6}$ sec, the radius of the channel becomes $a \sim 1$ mm, which yields $\delta^2/a^2 \sim 10^2$. Thus we can consider the electric field to be constant over the cross section and use the expression $j = \sigma E$ for the current density.

We now estimate the role of magnetic forces, for which we compare the magnetic pressure $H^2/8\pi$ with the gas-kinetic pressure. The latter has the same order of magnitude as the kinetic energy per unit volume. If an appreciable amount of the liberated Joule heat remains in the form of the kinetic energy of particles in the channel, then the pressure can be estimated as

$$p \sim \frac{1}{\pi a^2}\frac{J^2 t}{\pi a^2 \sigma} \sim \frac{H^2}{8\pi}\frac{\delta^2}{a^2}. \qquad (2.8)$$

It is then evident that we can regard the magnetic forces as inconsequential when we can neglect the skin effect.

The magnetic field begins to have a strong effect on the kinetics of the electrons (on the electrical conductivity and especially on the thermal conductivity) when the frequency of their rotation in the magnetic field $\omega = eH/mc$ is comparable with the collision frequency $1/\tau$. For typical values of the magnetic field and density in the channel, Mandel'shtam and his coworkers obtained in their experiments $\omega\tau \sim 0.1$.

We shall neglect both the magnetic forces and the effect of the magnetic field on the kinetics of the electrons.

The shell of the channel. Ionization jump. Since we are dealing with entirely different simplications in the channel region and the shock-wave region, it is necessary to establish the condition of joining the solutions on the boundary between the two regions. Physically, the joining takes place in the transition region, in the shell of the channel, where a transition occurs from a strongly ionized gas to a weakly ionized one, and where an intense ionization process is taking place. We shall not investigate the behavior of the quantities in the transition layer, but shall consider them (approximately) as discontinuities, as is usually done for discontinuities in hydrodynamics. We shall assume here that the transition region is not very wide.

We shall denote by $\dot{a}$ the velocity of motion of the discontinuity, and use the index 1 for quantities on the outside and the index 2 for quantities on the inside (the channel side). The laws of conservation of mass and momentum take the form

$$\rho_1(v_1 - \dot{a}) = \rho_2(v_2 - \dot{a}) \equiv g; \qquad (2.9a)$$

$$p_1 + \rho_1(v_1 - \dot{a})^2 = p_2 + \rho_2(v_2 - \dot{a})^2. \qquad (2.9b)$$

The density in the channel is very low, $\rho_2 \ll \rho_1$; therefore, the first condition yields $v_1 = \dot{a}$. The pressure jump is expressed in the form $\Delta p = p_1 - p_2 = g^2(\rho_2^{-1} - \rho_1^{-1})$. Considering $\rho_2 \ll \rho_1$, $g \sim \rho_2\dot{a}$, $p \sim \rho_1 a^2$, we find that the pressure jump is small:

$$\Delta p/p \sim \rho_2/\rho_1 \ll 1. \qquad (2.10)$$

We shall assume that the pressure does not undergo a jump. Then, neglecting the heat flow on the side of the cold dense gas, we obtain the condition

$$g(\varepsilon_2 + p/\rho_2) + q_2 = 0. \qquad (2.11)$$

from the conservation of energy.

3. QUASI-SELF-SIMILAR SOLUTION

As is well known (see reference 11), the motion described by two dimensional parameters is self-similar. In our case, the motion as a whole depends on a large number of parameters; however, we can find an approximate solution which is self-similar separately in the region of discharge and in the region of the shock wave. Curves representing the dependence of different quantities on the radius remain, in each of the regions, the same with passage of time, but their scales change in each region according to its own law.

Shock wave. The motion of the gas outside the channel is completely determined if the time dependence of the radius of the channel is given. The boundary of this channel plays the role of a piston which displaces the gas. If this dependence has a simple power form

$$a(t) = At^k \qquad (3.1)$$

and if the pressure in the wave is so large that the pressure of the undisturbed gas can be neglected, then the motion in the region of the shock is determined by the two dimensional parameters A, ρ_0 and is self-similar.

In place of the variables t, r in Eqs. (2.1), we introduce the variables t, $x = r/a_c(t)$, where a_c is the radius of the wave front. We introduce also the new dependent variables:

$$\rho'(x) = \rho/\rho_0, \qquad v'(x) = v/\dot{a}_c,$$
$$p'(x) = p/\rho_0\dot{a}_c^2. \tag{3.2}$$

Neglecting the heat released and transferred, we can rewrite Eq. (2.1) in the form

$$(v'-x)\frac{d\rho'}{dx} + \rho'\frac{dxv'}{xdx} = 0;$$
$$\left(1-\frac{1}{k}\right)v' + (v'-x)\frac{dv'}{dx} + \frac{1}{\rho'}\frac{dp'}{dx} = 0; \tag{3.3}$$
$$2\left(1-\frac{1}{k}\right)p' + (v'-x)\frac{dp'}{dx} + \gamma p'\frac{dxv'}{xdx} = 0.$$

The boundary conditions in a strong shock wave for x = 1 have the form

$$\rho' = (\gamma+1)(\gamma-1)^{-1}, \quad v' = 2(\gamma-1)^{-1},$$
$$p' = 2(\gamma-1)^{-1}. \tag{3.4}$$

Equations (3.3) with boundary conditions (3.4) were integrated numerically on an electronic computing machine for the values k = 1, $^3/_4$, $^3/_5$ and $\gamma = {^5/_3}$, $^7/_5$, $^9/_7$. The results of the integration are shown in Fig. 1. The position of the piston is determined by the point where $v' = x$. The pressure at the piston p_k can be obtained from the velocity of the piston

$$p_k = K_p\rho_0\dot{a}^2, \tag{3.5}$$

where the coefficient of resistance K_p will be considered as approximately equal to 0.9 (see Fig. 1, $K_p = p'(s)a^2/a_c^2$).

The Channel. Let us consider the case in which

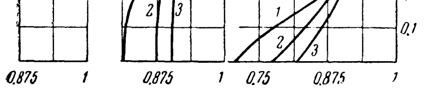

FIG. 1. Distribution of the velocity v′ (dashed lines), pressure p′ and density ρ′ behind the front of the shock wave as a function of $x = r/a_c$ for various values of k. The curves 1 correspond to γ = 5/3, 2 to γ = 7/5, and 3 to γ = 9/7.

we can neglect radiation. Then

$$q = -\varkappa dT/dr. \tag{3.6}$$

We shall assume that the temperature in the channel is appreciably higher than is necessary for complete ionization. Then, on the boundary of the channel, where the ionization is beginning to fall off, the temperature will be much less than at the center, and we can set (approximately) T = 0 for r = a.

We transform from the variables t, r to the variables t, $s = r^2/a^2(t)$, and also introduce the new dependent variables

$$\vartheta(s) = \frac{T}{T_0}, \quad u = \frac{1}{\vartheta}\frac{r}{2a}\left(\frac{r}{a} - \frac{v}{\dot{a}}\right),$$
$$y = \frac{r}{2a}\left\{\frac{q}{pa} + \frac{5}{2}\left(\frac{v}{a} - \frac{r}{a}\right)\right\}. \tag{3.7}$$

Here T_0 is the temperature on the axis. We shall regard the pressure as constant over the cross section of the channel. Equations (2.1a), (2.1c), (3.6) take the form

$$\frac{du}{ds} = \frac{1-(1/2k)}{\vartheta}, \quad \frac{dy}{ds} = \frac{\alpha\beta}{4}\vartheta^{5/2} - \left(2 - \frac{3}{4k}\right),$$
$$\frac{d\vartheta}{ds} = -\frac{y + {^5/_2}u\vartheta}{\alpha s\vartheta^{5/2}}, \tag{3.8}$$

where

$$\alpha = \varkappa(T_0)T_0/pa\dot{a}, \tag{3.9}$$
$$\beta = \sigma(T_0)E^2a^2/\varkappa(T_0)T_0. \tag{3.10}$$

The condition (2.11) becomes $y(1)/u(1) = I/(Z+1)T_0$. It is then evident that for self-similarity the temperature ought not to depend on the time. The same applies to the quantities α, β. Making use of (3.5), (3.10), we obtain the result that $a \sim t^{3/4}$, $E \sim t^{-3/4}$. The electric field is connected with the current by the relation

$$J = \langle\sigma\rangle \pi a^2 E, \tag{3.11}$$

where the angular brackets denote an averaging over the cross section of the channel. Thus, the self-similar mode is obtained only in the case of a definite law of current increase: $J \sim t^{3/4}$. Actually, the current changes sinusoidally, but for the first quarter period of the sine wave we can use approximately the results obtained from the self-similar mode.

Equations (3.8) have been integrated numerically for $k = {^3/_4}$ with the boundary conditions

$$\text{for } s = 0 \quad u = 0, \quad y = 0, \quad \vartheta = 1,$$
$$\text{for } s = 1 \quad y/u = I/(Z+1)T_0, \quad \vartheta = 0\,(\vartheta \ll 1).$$

In the actual integration, the parameter α was assigned and β was so chosen that the quantity $\vartheta(1)$ was as small as possible. The exact value of $\vartheta(1)$ is rather unimportant, since the curves hardly depend on it over the major part of the interval. The characteristic curves are shown in

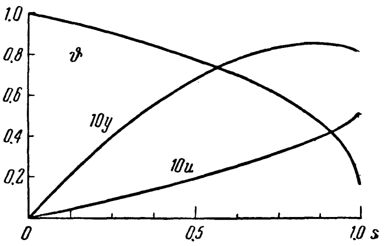

FIG. 2

Fig. 2. The values of $I/(Z+1)T_0 = 5.95$, 2.0, 1.56, 0.9, and 0.25 correspond to $\alpha = 16$, 8, 6.90, 5.33, and 4 and $\beta = 1.48$, 1.56, 1.60, 1.68, and 1.80. The coefficients $K_\sigma = \langle T^{3/2}/T^{3/2} \rangle$ and $K_\rho = \langle T_0/T \rangle$ change in this case from 0.661 to 0.632 and from 1.49 to 1.69, respectively. We shall henceforth take approximately $\beta = 1.6$, $K_\sigma = 0.655$, and $K_\rho = 1.55$. Knowing the dependence of α and β on the temperature, it is possible to find all the parameters of the channel. However, it is more suitable to use Eq. (4.4), in which we must replace σ by $\langle\sigma\rangle = \sigma_1 \langle T^{3/2} \rangle$. In accord with (2.3), it is necessary to replace in Eq. (4.4′) $(\gamma - 1)^{-1}$ by $3/2 + K_\rho I/(Z+1)T_0$. For the temperatures of interest, the single-term approximation $\xi \approx 3\sqrt{I/(1+Z)T_0}$ is valid with sufficient accuracy. Using Eqs. (3.10), (3.11), (4.4) and the values of the coefficients that have been obtained, we get the radius of the channel, the temperature, and the electric field. For hydrogen, we have:

$$a = 1.53\rho_0^{-5/28} (Jt^{-3/4})^{2/7} t^{3/4}; \tag{3.12}$$

$$T_k = 3.5\rho_0^{1/14} (Jt^{-3/4})^{2/7}; \tag{3.13}$$

$$E = 50\rho_0^{1/4} t^{-3/4}. \tag{3.14}$$

For the temperature in the channel, the condition $T_k = (\langle T^{3/2} \rangle)^{2/3}$ is assumed. Here we have expressed T_k in ev, a in mm, E in v/cm, J in kiloamperes, t in μsec, while the density unit is 0.9×10^{-4} g/cm^3. In reference 4, for a 15-kv discharge, a 2μhy self inductance and a 0.25μf capacitance, the measured values for the radius of the channel (for hydrogen at atmospheric pressure, $\rho_0 = 1$) were 1.00, 1.55 and 2.60 mm for 0.3, 0.5 and 1.0μsec, respectively. The corresponding values computed according to (3.12) are 1.00, 1.50, and 2.45. The agreement is rather good. Experimental data for the quantities in (3.13) and (3.14) are unfortunately lacking. The existence of such data would have made it possible to verify Eq. (2.5) for the thermal conductivity of the plasma, since the radiation does not play an important role in the given case.

4. HOMOGENEOUS MODEL OF A CHANNEL WITH A DENSE SHELL

If the removal of heat from the channel is brought about by transparent radiation, while the thermal conductivity can be neglected, then we can demonstrate a simple self-similar solution for the region of the channel: the pressure, temperature and density are constant over the cross section, while the velocity is proportional to the radius. The entire temperature drop is concentrated in the shell. The radiation is absorbed there, and the ionization of the gas entering the channel takes place in that region. If we consider the shell to be thin, we can obtain a set of equations for the basic parameters of the channel. In the general case, use can be made of these equations as a mathematical model describing, however roughly, the basic processes in the channel. In this case we can also take the thermal conductivity into consideration (approximately).

The equations for the energy balance in the channel and the shell have the form

$$\frac{dW}{dt} + p\frac{d\pi a^2}{dt} = Q_J; \tag{4.1}$$

$$\left(\varepsilon + \frac{p}{\rho}\right)\frac{dM}{dt} = Q_T + Q_R, \tag{4.2}$$

where M and W are the mass and energy of the gas in the channel. Equation (4.1) is obtained by integration of (2.1c) over the cross section of the channel (including the shell) without any assumption on the form of the distribution of the quantities over the cross section. For the homogeneous model, we set $W = M\epsilon$, $M = \pi a^2 \rho$. Equation (4.2) is obtained from (2.11). The expressions for the release of Joule heat Q_J and for the heat loss by radiation Q_R and by thermal conduction Q_T can be written in the form

$$Q_J = J^2/\pi a^2 \sigma, \quad Q_R = \pi a^2 Q_R' (p, T),$$
$$Q_T = 1.3 \cdot 2\pi\varkappa T. \tag{4.3}$$

In order of magnitude, $Q_T \sim \kappa(T/a)2\pi a$, while the coefficient in (4.3) is chosen in correspondence with the results of the previous section Eq. (3.10) wherein $T_k = (\langle T^{3/2} \rangle)^{2/3}$ was assumed for the characteristic temperature in the channel. Approximately, it can be obtained from (3.5) for a weak shock, or it can be considered equal to the pressure of the undisturbed gas, when the wave becomes weak and undergoes a transition to the acoustic type.

Making use of (3.5) and (2.3), we can rewrite (4.1) in the form

$$\rho_0 2\pi^2 a^3 \dot{a}^3 \xi = J^2 / \sigma, \tag{4.4}$$

$$\xi = K_p\left[1 + (\gamma - 1)^{-1} 2^{-1} \dot{a}^{-2} \frac{d^2a^2}{dt^2}\right] = K_p [1 + (\gamma - 1)^{-1} (2 - k^{-1})]. \tag{4.4'}$$

Here $k = \dot{a}t/a$. Comparing (4.1) and (4.2), we obtain

$$Q_T + Q_R = \eta Q_J, \tag{4.5}$$

where η is a coefficient on the order of unity. If, for example, we can neglect the change in temperature with time, then

$$\eta = \gamma\left[1 + (\gamma - 1)\, 2\, \dot{a}^2 \left(\frac{d^2a^2}{dt^2}\right)^{-1}\right]^{-1}. \tag{4.5'}$$

For a weak shock, when the pressure in the channel can be considered equal to the pressure of the undisturbed gas p_0, we get from (2.3) instead of (4.4)

$$p_0 2\pi^2 a^3 \dot{a} \gamma / (\gamma - 1) = J^2 / \sigma. \tag{4.6}$$

Equation (4.5) retains its form, but the coefficient η will be different. For example, if we neglect the change in temperature with time, then we have simply $\eta = 1$ in place of (4.5). Equations (4.4), (4.5), together with (4.3) and (3.5), (3.11) allow us to find all the parameters of the channel.

Let us consider the channel in air. The conductivity $\sigma(T)$ for air in the temperature range of interest to us changes comparatively slowly (see Sec. 3) and, by (2.4), can be taken to be approximately $\sigma = 2 \times 10^{14}\ \text{sec}^{-1}$. This is supported by the experimental data. If, making use of references 3 and 4, we take the electrical conductivity into account, then it is shown that within wide limits of change of the parameters of the discharge, σ does not depart appreciably from this value. Assuming $K_p = 0.9$, $\gamma = 1.2$ and $J \sim t$, we get $\xi = 4.5$. For these values of σ and ξ, we get for the channel radius (from Eq. (4.4)

$$a = 0.93\, \rho_0^{-1/6} J^{1/3} t^{1/2}. \tag{4.7}$$

Here a is in mm, J in kiloamperes, t in μsec, and we take as the density unit the density of the air at atmospheric pressure, $1.29 \times 10^{-3}\ \text{g/cm}^3$.

The experimental values of the radius[4] for a discharge voltage of 15 kv and capacitance $C = 0.15\ \mu\text{f}$, at 0.3, 0.5 and 1.0 μsec, are the following: for a coil inductance of $L = 2\ \mu\text{hy}$ (which corresponds to $\dot{J} = V/L = 7.5 \times 10^9$ amp/sec): 0.65, 0.95, and 1.55 mm, respectively; for $L = 12$ ($\dot{J} = 1.25 \times 10^9$), 0.33, 0.50 and 0.80 mm, respectively; for $L = 64$ ($\dot{J} = 2.4 \times 10^8$), 0.18, 0.25 and 0.40 respectively. The corresponding values computed from (4.7) are 0.67, 1.0 and 1.62 (for $L = 2$); 0.35, 0.57 and 0.99 (for $L = 12$), 0.21, 0.32, and 0.58 (for $L = 64$). The agreement is excellent. A certain saturation at larger self inductances and values of the time is explained by the fact that (4.4) does not take the initial pressure into account. If this were done in (4.1), for example, by means of the interpolation $p = K_p \rho_0 a^2 + p_0$, then the agreement with experiment would be improved.

The spark discharge in air has also been investigated experimentally by Norinder and Karsten.[12] The values of the radius computed by (4.7) agree satisfactorily with their experimental data.

The temperature in the channel can be calculated by (4.5) and (4.3). However, this computation is difficult in practice because of the absence of reliable data on the radiation of air. We shall only put down some estimates. The coefficient η is on the order of unity. For the same discharge which was considered previously,[3] at $t = 1\ \mu$sec, the Joule heat (for $L = 2$, 12 and 64 μhy) is $Q_J = 1.7 \times 10^{13}$, 3×10^{12}, 4.2×10^{11} erg/cm-sec, respectively. For $L = 64$, using (4.5), we obtain $T = 3.7$ ev, while all the heat is transferred by the electronic conductivity; we can neglect radiation ($Q_R \sim 10^{10}$ erg/cm-sec). For $L = 12$, the thermal conductivity and radiation have the same order of magnitude, but for $L = 2$, the heat is primarily conveyed by radiation. In the second case, the radiation is much greater than in the first, because of the high density of the plasma, but also because of the large value of the cross section of the channel. Taking $T = 4$ ev, we get, making use of (2.2) and (3.5), $n_i = 3.3 \times 10^{17}$, in the first case and $n_i = 9 \times 10^{17}$ in the second. These quantities greatly exceed the experimental value of 10^{17} obtained by Dolgov and Mandel'shtam.[5] According to their experimental data, $T \approx 4$ ev and $Z \approx 2$. Substituting these values in (2.5) and (4.3) we get $Q_T = 0.6 \times 10^{12}$ erg/cm-sec. We estimate the radiation crudely by using (2.7) with an effective charge equal to $Z + 1.5 = 3.5$. This gives $Q_R = 1.6 \times 10^{12}$ for $L = 12$ and $Q_R = 4.4 \times 10^{13}$ for $L = 2$. These results correspond in order of magnitude to the experimental values of Q_J; however, the accuracy of the estimates is not very great because of the very approximate method employed in considering the radiation of air. Therefore, the role of other possible mechanisms of heat transfer, for example, radiant thermal conductivity, is not completely clear.

In conclusion, let us consider the limits of applicability of the theory developed above.

The lower limit is determined by the fact that for appreciable ionization, the temperature in the channel ought to be larger than (approximately) one electron volt. For this, the current ought to be not too small and should increase after a rather short time. The corresponding estimate can be obtained from (3.13) for hydrogen and from (4.3) and (4.4) for air. Neglecting the weak dependence on the density, and disregarding radiation, we obtain a condition for both cases, which is very rough:

$$J \gg 10^{-2} t^{3/4}. \tag{4.8}$$

The upper limit is determined by the requirement of smallness of the magnetic pressure $H^2/8\pi = J^2/2\pi a^2 c^2$ in comparison with the gas-dynamic pressure. Using (3.5) and (4.4), we get

$$H^2 / 8\pi p = (J / J_0)^{2/3}, \; J_0 = (2^{1/2} K_p^{3/2} / \xi\pi^{1/2}) (c^3 \rho_0^{1/2} / \sigma), \tag{4.9}$$

where J_0 is the current at which the magnetic forces begin to be appreciable. For hydrogen, setting $\sigma = \sigma_1 T^{3/2}$, $\xi = (53/T)^{1/2}$, and using (2.4), we get

$$J_0 (H_2) = 50 \rho_0^{2/5} j^{-1/5}. \tag{4.10a}$$

For air, substituting a fixed value of the conductivity $\sigma = 2 \times 10^{14}$, and $\xi = 4.5$, we get

$$J_0 \text{ (air)} = 250 \rho_0^{1/2}. \tag{4.10b}$$

The current is expressed in kiloamperes, the time in μsec, and the density in units of 0.9×10^{-4} for hydrogen and 1.29×10^{-3} for air, both in g/cm^3.

Both criteria are well satisfied for typical cases of lightning in the atmosphere. For example, let the current of the lightning be 30 kiloamperes and the time of current flow 200μsec; then we get $0.55 \ll 30 \ll 250$. The form of the lightning current is not linear, so that the coefficient in (4.7) must be changed, but if (4.7) is used for a rough estimate, then we get, in the case considered, for the radius of the lightning channel $a \approx 4$ cm.

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