COLLECTIVE EXCITATION OF ODD NONSPEHERICAL NUCLEI

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A theory is developed for the energy states of odd nuclei corresponding to collective nucleon motion in which the axial symmetry of the nucleus is preserved. It is shown that the energy spectrum of the nucleus can be determined by only two parameters. Conditions under which the energy spectrum splits up into a system of rotational-vibrational bands are determined.

IN the articles of Davydov and Filippov[1] and Davydov and Chaban[2] a theory of collective excitations of states of even-even axially-symmetric nuclei was developed without assuming that the rotational energy is small in comparison with the energy of the surface vibrations. It was shown that the energy of excited states in which the axial symmetry of the nucleus is not destroyed, can be represented by a function which depends on only two parameters; conditions under which collective excitations split up into a system of rotational-vibrational bands were found. In the present work the theory is extended to the case of axially-symmetric odd nuclei, having a spin equal to or larger than $^3/_2$ in the ground state.

1. COLLECTIVE EXCITATIONS OF ODD NUCLEI WHICH PRESERVE THE AXIAL SYMMETRY OF THE NUCLEUS

According to the unified model of the nucleus,[3] in the case of strong coupling the classical energy of collective motion of the nucleus is obtained after averaging the energies of the interaction of external nucleons with the nuclear surface over the states of motion of the external nucleons. In the case that interests us — motion which preserves the axial symmetry of the nucleus ($\gamma = 0$, j_{i3} are good quantum numbers) – this energy can be written in the form

$$E = T + V(\beta), \tag{1.1}$$

$$T = {}^1/_2 B\dot{\beta}^2 + \hbar^2 \{J(J+1) - K^2\} / 6B\beta^2, \tag{1.2}$$

$$V(\beta) = {}^1/_2 C\beta^2 + A\beta + \hbar^2 D / 6B\beta^2, \tag{1.3}$$

where C and B characterize the properties of the nucleus, A and D depend on the number of external nucleons and on the state of their motion, $K = J_3 = \sum j_{i3}$ is the spin of the nucleus in the ground state, J is the total angular momentum of the nucleus, $\beta \geq 0$ determines the deviation of the nucleus from spherical symmetry.

Equation (1.1) corresponds to the Schrödinger equation

$$\{\hat{T} + V(\beta) - E\}\Psi = 0, \tag{1.4}$$

$$\hat{T} = -\frac{\hbar^2}{2B\beta^2}\left\{\frac{\partial}{\partial\beta}\left(\beta^2\frac{\partial}{\partial\beta}\right) + \frac{1}{3\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{3\sin^2\theta}\left[\frac{\partial^2}{\partial\varphi^2} + 2\cos\theta\frac{\partial^2}{\partial\varphi\,\partial\psi} + \frac{\partial^2}{\partial\psi^2}\right] - \frac{1}{3}\frac{\partial^2}{\partial\psi^2}\right\}. \tag{1.5}$$

The solution of Eq. (1.4) can be obtained in the form

$$\Psi = \sum_J \frac{1}{\beta} u_{JK}(\beta) D^J_{MK}(\varphi, \theta, \psi), \tag{1.6}$$

where $J = K, K+1, \ldots$; D^J_{MK} are the generalized spherical functions which are the irreducible representations of the rotation group.

Substituting Eq. (1.6) into (1.4) we see that the functions u_{JK} should satisfy the boundary condition

$$u_{JK}(0) = 0 \tag{1.7}$$

and the equation

$$\left\{-\frac{\hbar^2}{2B}\frac{d^2}{d\beta^2} + V_K(\beta) + \frac{\hbar^2[J(J+1) - K(K+1)]}{6B\beta^2} - E\right\}u_{JK} = 0, \tag{1.8}$$

where

$$V_K = {}^1/_2 C\beta^2 + A\beta + \hbar^2(D+K)/6B\beta^2 \approx V_K(\beta_K) + {}^1/_2 C_K(\beta - \beta_K)^2 \tag{1.9}$$

The quantities β_K and C_K which enter into Eq. (1.9) can be expressed in terms of A, B, C, D and K by means of the equations

$$\beta_K = -A/C + \hbar^2(D+K)/3B\beta_K^3,$$
$$C_K = C + \hbar^2(D+K)/B.$$

However, it is more convenient to consider them as some parameters characterizing the nucleus in the ground state $(J = K)$. Then Eq. (1.8) can be put in the form

$$\left[-\frac{\hbar^2}{2B}\frac{d^2}{d\beta^2} + W_{JK}(\beta) - \varepsilon\right]u_{JK} = 0, \qquad (1.8a)$$

where $\epsilon = E - V_K(\beta_K)$,

$$W_{JK}(\beta) = \frac{C_K}{2}(\beta - \beta_K)^2 + \frac{\hbar^2[J(J+1) - K(K+1)]}{6B\beta^2} \approx W_{JK}(\beta_{JK}) + \frac{1}{2}C_{JK}(\beta - \beta_{JK})^2. \qquad (1.10)$$

Here

$$\beta_{JK} = \beta_K + \hbar^2[J(J+1) - K(K+1)]/3BC_K\beta_{JK}^3, \qquad (1.11)$$

$$C_{JK} = C_K + (\hbar^2/B\beta_{JK}^4)[J(J+1) - K(K+1)], \qquad (1.12)$$

$$W_{JK}(\beta_{JK}) = {}^1/_2 C_K(\beta_{JK} - \beta_K)^2 + \hbar^2[J(J+1) - K(K+1)]/6B\beta_{JK}^2. \qquad (1.13)$$

After introduction of the dimensionless parameters*

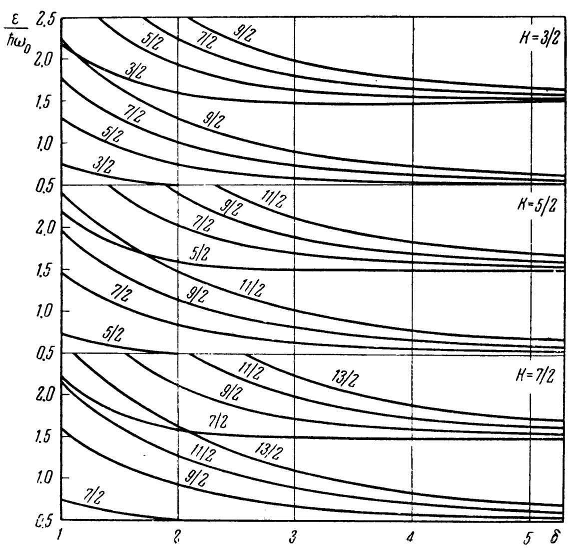
$$\delta = \beta_K(BC_K/\hbar^2)^{1/4}, \quad \xi = \beta_{JK}/\beta_K \geqslant 1, \quad \omega_0 = \sqrt{C_K/B},$$

Equations (1.11) to (1.13), respectively, take the form

$$\xi^3(\xi - 1) = [J(J+1) - K(K+1)]/3\delta^4, \qquad (1.11a)$$

$$C_{JK} = C_K(1 + [J(J+1) - K(K+1)]/\delta^4\xi^4), \qquad (1.12a)$$

$$W_{JK}(\beta_{JK})/\hbar\omega_0 = {}^1/_2\delta^2(\xi - 1)^2 + [J(J+1) - K(K+1)]/6\delta^2\xi^2. \qquad (1.13a)$$



Substituting Eq. (1.10) into (1.8a) we find

$$\left\{-\frac{\hbar^2}{2B}\frac{d^2}{d\beta^2} + \frac{1}{2}C_{JK}(\beta - \beta_{JK})^2 - [\varepsilon - W_{JK}(\beta_{JK})]\right\}u_{JK} = 0. \qquad (1.14)$$

In order to determine the eigenfunctions and eigenvalues of Eq. (1.14) we introduce the new variable

$$\zeta = \xi\delta_1(\beta - \beta_{JK})/\beta_{JK}, \quad \delta_1^4 = \delta^4 C_{JK}/C_K,$$

which varies in the interval $\xi\delta_1 \leq \zeta > \infty$, and the new function $v(\zeta)$ using the relation

$$u_{JK}(\beta) = v(\zeta)\exp(-\zeta^2/2).$$

Then the function $v(\zeta)$ will satisfy the equation

$$v''(\zeta) - 2\zeta v'(\zeta) + 2\nu v(\zeta) = 0,$$

$$\nu = \frac{\varepsilon - W_{JK}(\beta_{JK})}{\hbar\omega_{JK}} - \frac{1}{2}, \qquad (1.15)$$

$$\omega_{JK} = \left(\frac{C_{JK}}{B}\right)^{1/2} = \omega_0\left[1 + \frac{J(J+1) - K(K+1)}{\delta^4\xi^4}\right]^{1/2}$$

and the boundary condition

$$v(-\xi\delta_1) = 0, \quad \exp(-\zeta^2/2)v(\zeta) \to 0 \quad \text{for } \zeta \to \infty. \qquad (1.16)$$

The solution of Eq. (1.15) satisfying Eq. (1.16) can be put in the form

$$v_\nu(\zeta) = aH_\nu(\zeta), \qquad (1.17)$$

where

$$H_\nu(\zeta) \equiv [2\Gamma(-\nu)]^{-1}\sum_{K=0}^{\infty}\frac{(-1)^K}{K!}\Gamma\left(\frac{K-\nu}{2}\right)(2\zeta)^K$$

are Hermitian functions of the first kind. Here the energy of collective motion of the nucleus will be determined by the formula

$$\frac{\varepsilon}{\hbar\omega_0} = (\nu + {}^1/_2)\left[1 + \frac{J(J+1) - K(K+1)}{\delta^4\xi^4}\right]^{1/2} + \frac{\delta^2(\xi - 1)^2}{2} + \frac{J(J+1) - K(K+1)}{6\delta^2\xi^2}. \qquad (1.18)$$

The quantum number ν (which is, in the general case, not integral) is the kernel of the transcendental equation

$$H_\nu(-\delta_1\xi) = 0. \qquad (1.19)$$

The quantities ξ and δ_1, for given J and K, are determined using Eqs. (1.11a) and (1.12a) through the parameter δ. For $\delta > 2$, $\delta_1 \approx \delta$.

2. SPECTRUM OF COLLECTIVE EXCITATIONS OF ODD NONSPHERICAL NUCLEI

In the preceding section it was shown that the energies of collective excitations (for which the axial symmetry is preserved) of odd nonspherical

*From Eq. (1.8a) follows that for J = K the square of the amplitude of zero-point vibrations $\overline{\beta_{0s}^2} = \hbar/2\sqrt{BC_K}$. Thus, the parameter $\delta = \beta_K(2\overline{\beta_{00}^2})^{-1/2}$ is proportional to the ratio of the value of β_K, which defines the equilibrium form of the nucleus in the ground state, to the amplitude of zero-point vibrations of the nuclear surface.

Nucleus	J		Energy level, kev		$\hbar\omega_0$, kev	δ
	Theoret.	Exptl.	Theoret.	Exptl.		
Tb^{159}	3/2	3/2	0	0		
[4]	5/2	5/2	57.6	57		
	7/2	7/2	130	136	353	2.43
	3/2	3/2	364	364		
	5/2	—	440	—		
Gd^{157}	3/2	3/2	0	0		
[5]	5/2	5/2	55	55	732	3.5
	7/2	7/2	130	131		
	3/2	—	732	—		
Eu^{153}	5/2	5/2	0	0		
[4,5]	7/2	7/2	84	84		
	9/2	9/2	184	194	700	3.31
	11/2	—	300	—		
	5/2	5/2	700	700		
	7/2	—	805	—		
Re^{187}	5/2	5/2	0	0		
[4,5]	7/2	7/2	134.6	134.6		
	9/2	9/2	294	300		
	11/2	—	473	—	910	2.99
	5/2	—	910	910		
	7/2	—	1075	—		
U^{233}	5/2	5/2	0	0		
[4]	7/2	7/2	40.1	40.1		
	9/2	9/2	92	92.8	476	3.988
	11/2	—	151	—		
	5/2	—	476	476		
	7/2	—	528	—		
Np^{237}	5/2(−)	5/2(—)	0	0		
[4,6,7]	7/2(−)	7/2(—)	33.1	33.1		
	9/2(−)	9/2(—)	78	76.1	433	3.99
	5/2(−)	—	433	433		
	7/2(−)	—	476	—		
	5/2(+)	5/2(+)	59.8	59.8		
	7/2(+)	7/2(+)	103.2	103.2		
	9/2(+)	9/2(+)	157.6	157.2		
	11/2(+)	11/2(+)	220.2	224	433	3.60
	13/2(+)	13/2(+)	289	303		
	5/2(+)	—	492.8	—		
Th^{229}	5/2	5/2	0	0		
[4,6]	7/2	7/2	43.2	43.2		
	9/2	9/2	98.2	100		
	11/2	11/2	163.2	164	555	4.0
	5/2	—	555	—		
Ho^{165}	7/2	7/2	0	0		
[4]	9/2	9/2	94	94		
	11/2	11/2	217	216		
	13/2	—	356	360	989	4.06
	7/2	—	989	989		
	9/2	—	1108	1100		
U^{235}	7/2	7/2	0	0		
[4]	9/2	9/2	46.5	46.5		
	11/2	11/2	105	104.3		
	13/2	—	168	172	379	3.63
	7/2	—	379	379		
	9/2	—	436	430		
Lu^{175}	7/2	7/2	0	0		
[4,5,8]	9/2	9/2	113	113		
	11/2	11/2	248	250		
	7/2	—	990	—	990	3.80
	5/2	5/2	342	342		
	7/2	(5/2, 7/2)	431	431		
	9/2	—	540	—		
	5/2	—	1332	—		

nuclei can be represented by the formula (1.18) depending only on the two parameters ω_0 and δ. The values $\epsilon/\hbar\omega_0$ are given on the figure as a function of the parameter δ for $K = 3/2$, $5/2$ and $7/2$, respectively. From the figure it follows that for some values of δ the spectrum of collective excitations of the nucleus splits up into a system of rotational-vibrational bands.

In the table a comparison is given between the theoretical values of the excitation energies of the first and second rotational-vibrational bands of excited states of odd nuclei and experimental data. There the values of the parameters $\hbar\omega_0$ and δ, used in the calculation of theoretical values, are also given.

Comparing the spectrum of collective excitations of odd nuclei with the spectrum of collective excitation of even-even nuclei, it is possible to draw the following conclusions: (1) The break-up of collective excitations into a system of rotational-vibrational bands in odd nuclei sets in for lower values of δ than in even-even nuclei; (2) The values of the parameter ω_0, which can be called the frequency of vibration of the nuclear surface, in the ground state is smaller in odd nuclei than in even-even nuclei having the same value of the parameter δ.

For $\delta > 3$ the quantity ν takes on values near to integral ones 0, 1, 2, ...; further, according to Eq. (1.11a) one can approximately set

$$\xi = 1 + g\,[J(J+1) - K(K+1)]/3\delta^4.$$

Then Eq. (1.18) can be replaced by the approximate equality

$$\varepsilon/\hbar\omega_0 = (\nu + 1/2) + [J(J+1) - K(K+1)]/6\delta^2 - a\,[J(J+1) - K(K+1)]^2/\delta^6. \quad (2.1)$$

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BEHAVIOR OF THE DISTRIBUTION FUNCTION OF A MANY-PARTICLE SYSTEM NEAR THE FERMI SURFACE

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The form of the distribution function of a system of electrons is studied in the Hartree approximation near the Fermi surface, for the case of a weakly inhomogeneous distribution. It is shown that in this region the inhomogeneity has a particularly strong effect, so that the correct expression for the distribution function, as given in this paper, is decidedly different in this region from the expression usually employed (that calculated from the Thomas-Fermi model). It is pointed out that the latter expression is completely unsuitable for use in problems in which the neighborhood of the Fermi surface plays an important part.

As is well known, the distribution function (the density matrix in a mixed representation) is the most important quantity characterizing a many-particle system. By means of it one can calculate without difficulty quite a number of physical quantities for the system in question.