solutions of another kind. It is easily seen that the solution (16) cannot be reduced to Schwarzschild's solution, which follows from the general theory of relativity.

*There is an evident illusoriness in the currently widely discussed "test" of the general theory of relativity through measurement of the perihelion shifts of artificial satellites.[5] Such a test can provide no basis for a choice between the general theory of relativity and Birkhoff's theory.

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ON THE THEORY OF THE POSITIVE COLUMN IN AN ELECTRONEGATIVE GAS

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1. In a positive column where negative ions are produced spatially and disappear at the wall, their relative concentration, which was obtained in reference 1, satisfies the condition $\kappa > D_e/2D_p - 1$,* where D_e and D_p are the diffusion coefficients of electrons and positive ions. This condition is required for the flow of negative ions to the wall, where they recombine. The wall is a surface sink for the negative ions produced within the volume.

2. When decay of the negative ions through collisions with neutral atoms[2] is included, the situation becomes somewhat more complicated, but we still have a linear problem which can be completely solved. For ambipolar diffusion, subject to the assumptions $D_e \gg D_p$, $D_p \approx D_n$, $b_e \gg b_p$ ahd $b_p \approx b_n$, we obtain an equation† for the concentration of negative ions in the column:

$$\frac{2\varkappa(1+\varkappa)D_p/D_e-\varkappa}{1+2\varkappa}=\frac{\beta-\gamma\varkappa}{Z-(\beta-\gamma\varkappa)},$$

where γ is the rate of decay of negative ions per ion and the rest of the notation is that used in reference 1. This is a cubic equation in κ which can be solved as follows:

(a) $\kappa > D_e/2D_p - 1$. With this concentration more negative ions are created per unit volume than decay, so that the column is a spatial source of negative ions. The negative ions produced in the column diffuse to the wall, which serves as a surface sink.

(b) $\kappa = D_e/2D_p - 1$. In this case the negative ions created by electrons adhering to neutral atoms equals those vanishing through decay. There is no effective resulting creation or disappearance of negative ions in the column. Their radial flow is zero and the total number of negative ions in the column is determined only by processes in the space.

(c) $\kappa < D_e/2D_p - 1$. Here the number of negative ions disappearing from the column exceeds the number produced, so that the column is a spatial sink for negative ions. Their radial flow is directed toward the axis and a stationary state is possible only in the presence of a surface source at the wall.‡

Thus in a positive column, where the disappearance of negative ions obeys a linear law, small values of κ are possible when there is near the surface of the wall** a layer that produces a flow of negative ions into the column, where they disappear through decay as a result of collisions with neutral particles. Our conclusion that a surface source exists agrees with Günterschulze's hypothesis[4] of a layer of negative ions at the wall.

3. When we take into account the spatial recombination of positive and negative ions, we can in general distinguish two regions of the column, an inner region where recombination predominates over creation, and an outer region where creation is stronger than recombination. We are here concerned with effective spatial sources and sinks.†† Since the equations of balance are nonlinear the problem can be solved more or less simply only for regions close to the axis of the discharge.[1]

*In reference 3 it was assumed that $\varkappa \ll 1$ in the absence of spatial disappearance of negative ions. This is unacceptable to us, since $\varkappa$ is a solution of the system and is fully determined by the kinetics of the column. The analysis carried out in reference 1 and in the present note shows that $\varkappa$ can be small only when a spatial loss occurs.

†The solution is obtained as in reference 1.

‡The strength of the surface source per unit length of the column is given by the integral $2\pi\int_0^R(\beta-\gamma\varkappa)N_e r\,dr$, where N_e is the electron density.

**Reference 3 considers negative ion concentrations which fulfill the condition $b_p \varkappa / b_e \ll 1$. Then, as is easily seen from the expression for the flow given in reference 2, there must be a source of negative ions at the wall. We therefore consider the boundary condition $N_n(R) = 0$ not to be stringent enough.

††The inclusion of surface sources of negative ions will only shift the boundary between the regions toward the wall.

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SURFACE WAVES ON THE BOUNDARY OF A GYROTROPIC MEDIUM

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1. We consider the surface waves (s.w.) $\exp[i(hz - \omega t) + \gamma x]$ propagating along the interface $x = 0$ of two semi-infinite media. Medium 1 $(x > 0)$ is isotropic $(\epsilon = \epsilon_0,\ \mu = \mu_0)$. Medium 2 $(x < 0)$ is gyrotropic with a dielectric constant ϵ and magnetic permeability μ_{ik}:

$$\mu_{xx} = \mu_{zz} = \mu_1;\quad \mu_{yy} = \mu_3;\quad \mu_{xz} = -\mu_{zx} = i\mu_2, \tag{1}$$

that is, the direction of the external magnetizing field is such that $H_0 \parallel OY$. We shall consider H-mode surface waves $(H_z \neq 0)$ in a medium with tensor μ_{ik} (ferrites). All of our results will also be valid for media with tensor ϵ_{ik} (plasma, Hall effect etc.) when E, H, ϵ, μ_{ik}, ω are replaced by H, E, μ, ϵ_{ik}, $-\omega$.

2. From the continuity of E_y and H_z at $x = 0$ (H_z is given in terms of E_y in reference 2) we obtain an equation for $u = -hc/\omega$, the retardation coefficient of the wave ($u = c/v_\phi$, where v_ϕ is the phase velocity):

$$\mu_0 (u^2 - \varepsilon\mu_\perp)^{1/2} + \mu_\perp (u^2 - \varepsilon_0\mu_0)^{1/2} = \mu_0 \Gamma u, \tag{2}$$

$$\Gamma = \mu_2/\mu_1.$$

Equation 2 was analyzed graphically. Some of the results follow. For $\Gamma > 0$ and $\mu_\perp > 0$, Eq. (2) has a real root if

$$\text{for } \varepsilon\mu_\perp > \varepsilon_0\mu_0:\ \mu_\perp + \mu_0 > \mu_0\Gamma > \mu_0 (1 - \varepsilon_0\mu_0/\varepsilon\mu_\perp)^{1/2},$$
$$\text{for } \varepsilon\mu_\perp < \varepsilon_0\mu_0:\ \mu_\perp + \mu_0 > \mu_0\Gamma > \mu_0 (1 - \varepsilon_0\mu_\perp/\varepsilon\mu_0)^{1/2}. \tag{3}$$

This is the only root; the wave thus propagates in only one direction $(h < 0)$. For $\epsilon_0\mu_0 \neq \epsilon\mu_\perp$ slight gyrotropy $(\Gamma \ll 1)$ cannot invalidate the law for an isotropic boundary. Just as in the isotropic case,[1] a s.w. does not propagate for $\epsilon > 0$, $\mu > 0$. But with $\epsilon\mu_\perp$ close to $\epsilon_0\mu_0$ a unidirectional wave (only in the z direction) is possible even for slight gyrotropy (theoretically also for paramagnetics).

For $\mu_\perp < 0$, $\Gamma > 0$, and $|\mu_\perp| > \mu_0$ the condition for propagation of the direct wave $(h > 0)$ is $\mu_0\Gamma < |\mu_0 + \mu_\perp|$, while for the reverse wave $(h < 0)$, $\Gamma < (1 + |\epsilon\mu_\perp|/\epsilon_0\mu_0)^{1/2}$. Thus, depending on the values of ϵ and μ_{ik}, both waves are propagated or one alone, or, finally, s.w. are impossible.

3. We now consider the more complicated case of s.w. in a gyrotropic plate 3 $(0 < x < d,\ \mu = \mu_{ik})$ between isotropic media 1 $(\epsilon = \epsilon_0;\ \mu = \mu_0)$ and 2 $(\epsilon = \tilde{\epsilon};\ \mu = \tilde{\mu})$. Let d be large; for the boundary $x = d$ we set up an equation similar to (2), different from (2) only in the sign of the right-hand side, i.e., the boundary $x = d$ guides s.w. in a direction opposite to that on the boundary $x = 0$. Accordingly the field of the direct wave is concentrated at one boundary and the field of the reverse wave at the opposite boundary: for $\epsilon_0 = \tilde{\epsilon}$ and $\mu_0 = \tilde{\mu}$:

$$E_y = A_1 e^{\gamma_3 x} + A_2 e^{-\gamma_3 x},\quad A_{1,2} = \gamma_3\mu_0 \pm \gamma_1\mu_\perp \mp \mu_0\Gamma h.$$

When the boundary which conducts energy in the undesired direction is covered with an absorbing film we obtain a unidirectional system.

For $\gamma_3 d \leq 1$ we must take into account the interaction of the boundaries and investigate the characteristic equation. In the general case $(\epsilon_0 \neq \tilde{\epsilon},\ \mu_0 \neq \tilde{\mu})$ this equation is

$$h^2\Gamma^2 + h\Gamma(\gamma_1 P_1 - \gamma_2 P_2) - \gamma_1\gamma_2 P_1 P_2 - \gamma_3^2 = \gamma_3(\gamma_1 P_1 + \gamma_2 P_2)\coth \gamma_3 d, \tag{4}$$

$$P_1 = \frac{\mu_\perp}{\mu_0};\quad P_2 = \frac{\mu_\perp}{\tilde{\mu}};\quad \gamma_1^2 = h^2 - \frac{\omega^2}{c^2}\varepsilon_0\mu_0;$$

$$\gamma_2^2 = h^2 - \frac{\omega^2}{c^2}\tilde{\varepsilon}\tilde{\mu};\quad \gamma_3^2 = h^2 - \frac{\omega^2}{c^2}\varepsilon\mu_\perp.$$

This equation contains a term which is linear in h, so that the direct and reverse waves differ not only with respect to the field distribution but also with respect to the phase velocity and critical velocity. For $\epsilon\mu_\perp > \epsilon_0\mu_0 > \tilde{\epsilon}\tilde{\mu}$

$$\omega_{cr} = \frac{c}{d}\,\frac{\alpha P_1 + \beta P_2}{(\alpha P_1 + \Gamma)(\beta P_2 - \Gamma)}, \tag{5}$$

$$\alpha = \left(1 - \frac{\varepsilon_0\mu_0}{\varepsilon\mu_\perp}\right)^{1/2};\quad \beta = \left(1 - \frac{\tilde{\varepsilon}\tilde{\mu}}{\varepsilon\mu_\perp}\right)^{1/2}.$$