changes very slightly for small changes in the size of β.

The cases we are considering are of interest because the magnitudes of the lifetimes are determined by corrections in the 7-th order of perturbation theory, since the degree of K-forbiddenness is 7. Therefore any deficiencies of the scheme will strongly affect the result. The calculated values were found to be 30 to 40 times those observed in experiment. It should be remarked that, in addition to the perturbations considered here, there may also be perturbations due to the presence of the paired nucleon. Even though we cannot, because of possible deficiencies of the scheme, definitely assert that these perturbations give a contribution amounting to approximately $\frac{1}{4}$ of the contribution of the perturbations considered here (as would follow from our result), it is clear that these perturbations will be small. The main contribution will come from the perturbations which we have considered.

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*The levels are designated according to reference 5.

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PHONON INTERACTIONS OF ELECTRONS IN POLAR CRYSTALS

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THE interaction energy of two electrons in polar crystals can be split into parts. The first part is the direct Coulomb interaction of the electrons, including the polarization of the electron shells of the atoms. This polarization is evaluated by introducing the dielectric constant $\epsilon_\infty = n^2$, where n is the index of refraction of light. The second part is the interaction through the phonon field. An evaluation of the phonon interaction is also important for a system consisting of an electron and a hole, for instance, an exciton. This problem was considered by a number of authors: in the strong coupling approximation by Dykman and Pekar,[1] and in the intermediate-coupling approximation by Meyer,[2] Haken,[3] and others. The authors of the cited papers used different variational principles to find the energy spectrum of the exciton.

In the present communication the phonon interaction potential of the electrons is derived in the intermediate-coupling approximation, with allowance for their relative momentum.

The operator of the interaction energy of an electron with the phonon field can be written in the form[4,5] (we put henceforth $\hbar = 1$):

$$\sum_k V_k a_k e^{ikr} + V_k^* a_k^+ e^{-ikr}, \qquad (1)$$

$$V_k = -(i\omega/k)(\tfrac{1}{2}m\omega)^{1/4}(4\pi\alpha/V)^{1/2},$$

$$\alpha = \frac{e^2}{2}\left(\frac{2m}{\omega}\right)^{1/2}\left(\frac{1}{n^2} - \frac{1}{\varepsilon}\right). \qquad (2)$$

The quantity α plays the role of the coupling constant, m is the effective mass of the electron, ω the limiting frequency of the longitudinal optical vibrations, and the a_k are the second-quantization operators. In the center-of-mass system, the energy operator of the two-electron system under consideration is of the form

$$\hat{H} = -\frac{1}{2M}\nabla_k^2 - \frac{1}{2\mu}\nabla_r^2 + \frac{e^2}{n^2 r} + \sum_k \omega a_k^+ a_k$$

$$+ \sum_k 2\cos\frac{kr}{2}(V_k a_k e^{ikR} + \text{c. c.}), \qquad (3)$$

where

$$R = \tfrac{1}{2}(r_1 + r_2), \quad r = r_1 - r_2, \quad M = 2m.$$

The center-of-mass coordinates can be eliminated from (3) by means of the canonical transformation S;

$$S = \exp\left\{ i\left(P - \sum_k k a_k^+ a_k\right)\right\} R; \tag{4}$$

P is the total momentum of the system. After the transformation we have

$$S^{-1}HS = -\frac{1}{2\mu}\nabla_r^2 + \frac{e^2}{n^2 r} + \frac{P^2}{2M} + \sum_k \left(\omega + \frac{k^2}{2M} - \frac{(pk)}{M}\right) a_k^+ a_k$$

$$+ \sum_k 2\cos\frac{kr}{2}\cdot(V_k a_k + V_k^* a_k^+) + \frac{1}{2M}\sum_{kk'}(kk')\, a_k^+ a_{k'}^+ a_k a_{k'}. \tag{5}$$

In the following we consider, for the sake of simplicity, the case of zero total momentum. We single out in the energy operator (5) the following three terms:

$$H_0 = -\frac{1}{2\mu}\nabla_r^2 + \sum_k \left(\omega + \frac{k^2}{2M}\right) a_k^+ a_k$$

$$+ \sum_k V_k a_k e^{ikr/2} + V_k^* a_k^+ e^{-ikr/2}. \tag{6}$$

The operator H_0 describes the "free" motion of a polaron of effective mass μ in the center-of-mass system of the two particles. The remaining terms in (5) distort the motion of the free polaron and describe the influence of the second particle on the motion of the first one, thus playing the part of a potential energy operator. If we denote these terms by H_1, we can find the correction to the energy approximately from the relation $\varphi_q(r) = \langle\Omega_q, H_1\Omega_q\rangle$ where Ω_q is the eigenfunctional of the operator H_0, the approximate form of which was found in the paper by Lee, Low, and Pines.[5] Using the explicit form of these functionals we get, apart from an additive term which does not depend on r, the following expression for $\varphi_q(r)$:

$$\varphi_q(r) = -\frac{e^2}{r}\left(\frac{1}{r^2} - \frac{1}{\varepsilon}\right)\frac{a^2 - 2b^2}{a^2}$$

$$+ \frac{e^2}{r}\left(\frac{1}{n^2} - \frac{1}{\varepsilon}\right)\exp\left\{-(a^2 - b^2)^{1/2} r\right\} \tag{7}$$

$$\times a^{-2}\left[(a^2 - 2b^2)\cos br - 2(a^2 - b^2)^{1/2} b\sin br\right],$$

where η is a parameter defined in reference 5, $a^2 = M\omega$, and $2b = q(1 - \eta)$.

One must note that if the total relative momentum q is zero the expression for $\varphi_q(r)$ in the

limits of applicability of the intermediate coupling approximation is not approximate, but exact. The phonon interaction itself goes leads to the attraction of two electrons, but the magnitude of the attraction is insufficient for the formation of a "bipolaron," since the sum of the Coulomb potential $e^2/n^2 r$ and the phonon potential $\varphi_q(r)$ is always positive. As $r \to 0$ the potential $\varphi_q(r)$ stays finite, and as $r \to \infty$ it has a Coulomb character. If $r \to \infty$, the interaction of the two electrons is described by the usual Coulomb law, $e^2/\varepsilon r$.

The source of the deviation of the phonon interaction from the Coulomb interaction consists in taking into account the recoil during the emission of phonons. Owing to recoil, the electrons experience fluctuating displacements, which leads to an additional interaction energy.

In a similar manner one can show that the interaction of an electron with a hole in an exciton is described by the potential[3]

$$-(e^2/\varepsilon r) - e^2(1/n^2 - 1/\varepsilon)e^{-ar}/r.$$

The deviation of the potential from the Coulomb potential is important at distances of the order $(M\omega)^{-1/2}$ which amounts to 10 to 15 A for Cu_2O. The ground state and partly the next energy levels of the exciton are shifted downwards.

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