and for $r \ll v/\omega_k$ we have

$$F_x^k = \pi^2 I^2 ne^2 \beta / mc^3 \omega_k'. \quad (8)$$

If a current moves onto a plasma or a metal, such an expansion is impossible, since we must assume

$$\varepsilon = 1 - \omega_0^2/(\omega^2 - i\gamma\omega); \quad \gamma = \omega_0^2/4\pi\sigma;$$

$$\omega_0^2 = 4\pi ne^2/m; \quad \mu = 1, \quad (9)$$

where σ is the electrical conductivity for $\omega = 0$. In that case it is necessary to evaluate expression (3) more accurately, retaining the radical under the integral sign. We shall give the results for particular cases. If $r \gg c/\omega_0$ and $\beta \gg (\gamma/\omega_0) \times (c/\omega_0 r)$, we have*

$$F_x = I^2/c^2 r\sqrt{1-\beta^2}. \quad (10)$$

If the conditions $r \gg c/\omega_0$ and $\beta \ll (\gamma/\omega_0) \times (c/\omega_0 r')$ are satisfied, we get

$$F_x = \frac{2\pi I^2\beta}{c^3\sqrt{1-\beta^2}}\sigma \ln \frac{1.356\,c}{8\pi\sigma\beta r}. \quad (11)$$

For $r \ll c/\omega_0$ and $\beta \geq \gamma/2\omega_0$ the evaluation of the integral (3) gives

$$F_x = \frac{I^2\omega_0}{3c^3\sqrt{1-\beta^2}}\{\eta^2 K(\sqrt{1-\eta^2/4})$$

$$+ 2(2-\eta^2)E(\sqrt{1-\eta^2/4}) + \eta(\eta^2-3)\}, \quad (12)$$

where K and E are the complete elliptic integrals. In the particular case $\beta \gg \gamma/2\omega_0$, we expand (12) in powers of $\eta = \gamma/\omega_0\beta$,

$$F_x = (4I^2/3c^2\sqrt{1-\beta^2})\,\omega_0/c. \quad (13)$$

If $r \ll c/\omega_0$ and $\beta \leq \gamma/2\omega_0$, we get the following result

$$F_x = \frac{2I^2\omega_0}{c^3\sqrt{1-\beta^2}}\left\{-\frac{1}{6}\eta + \frac{2}{3\sqrt{|z_1|}}F(\varphi, k)\right.$$

$$\left. - \frac{1}{3}\frac{(2-\eta^2)|z_2|}{\sqrt{|z_1|}}\left[\frac{\sqrt{k'^2+|z_2|\,k'^{-2}}}{\sqrt{|z_2|(1+|z_2|)}} - k'^{-2}E(\varphi, k)\right]\right\}, \quad (14)$$

where $E(\varphi, k)$ and $F(\varphi, k)$ are incomplete elliptic integrals, and

$$k^2 = (z_1 - z_2)/z_1; \quad k'^2 = 1 - k^2;$$

$$\tan^2\varphi = 1/|z_2|; \quad \eta = \gamma/\omega_0\beta; \quad (15)$$

$$z_1 = 1 - \frac{\eta^2}{2} - \frac{\eta^2}{2}\sqrt{1-4/\eta^2};$$

$$z_2 = 1 - \frac{\eta^2}{2} + \frac{\eta^2}{2}\sqrt{1-4/\eta^2}. \quad (16)$$

The expansion of (14) for $\beta \ll \gamma/\omega_0$ gives

$$F_x = \frac{20\pi I^2\beta\sigma}{3c^3\sqrt{1-\beta^2}}\ln\frac{1.492\,\omega_0}{2\pi\sigma\beta}. \quad (17)$$

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*The second condition is in fact equivalent to $r \gg \delta$, where δ is the skin depth for a frequency v/r.

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ENERGY DEPENDENCE OF THE REACTION CROSS SECTIONS FOR SLOW NEUTRONS

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It follows from very general assumptions that the reaction cross section for low-energy neutrons is proportional to $E^{-1/2}$ (cf., e.g., reference 1):

$$\sigma_r = (\sigma_r E^{1/2})_0 E^{-1/2}, \quad (1)$$

where the index 0 denotes evaluation at the neutron energy $E = 0$. Expression (1) is essentially the first term in the series

$$\sigma_r = (\sigma_r E^{1/2})_0 (E^{-1/2} - \alpha + \gamma E^{1/2} + \cdots). \quad (2)$$

The aim of the present paper is to show that the assumptions leading to the 1/v law also determine the quantity α in (2). The effective reaction cross section can be expressed through the logarithmic derivative of the wave function of the incoming particle at the nuclear boundary (f_0). In the notations of Blatt and Weisskopf[1] the reaction cross section for s neutrons incident on a nucleus with spin zero is equal to

$$\sigma_r = \frac{-4\pi R \operatorname{Im} f_0}{(\operatorname{Re} f_0)^2 + (\operatorname{Im} f_0 - kR)^2} \cdot \frac{1}{k}. \qquad (3)$$

Expanding f_0 in a power series in k, we obtain only even powers of k:

$$f_0 = (\operatorname{Re} f_0)_0 (1 + ak^2 + \cdots) + i (\operatorname{Im} f_0)_0 (1 + bk^2 + \cdots). \qquad (4)$$

Qualitatively this follows from the fact that f_0 is determined by the neutron state inside the nucleus (for $r \le R$), which can approximately be characterized by the wave number $K = (K_0^2 + k^2)^{1/2}$, where $K_0^2 \gg k^2$. If the effect of the nucleus on the neutron can be described by an operator V that satisfies the condition

$$\int_0^\infty [\psi(r) V \varphi(r) - \varphi(r) V \psi(r)]\, dr = 0$$

[e.g., a complex potential $V = U(r) + iW(r)$], then one can prove (4) rigorously, following, e.g., Bethe.[2] Substituting (4) into (3) and using

$$k^2 = 2mE\hbar^{-2} (A/(A+1))^2, \qquad (5)$$

where E is the neutron energy in the laboratory system, m is the mass of the neutron, and A the mass number of the target nucleus, we obtain

$$(\sigma_r E^{1/2})_0 / \sigma_r E^{1/2} = 1 + \alpha E^{1/2} + \beta E + \cdots, \qquad (6)$$

where

$$\alpha = \alpha_0 = \frac{m}{\pi \hbar^2} \left(\frac{A}{A+1}\right)^2 (\sigma_r E^{1/2})_0. \qquad (7)$$

Expressions (6) and (2) are equivalent. For a nucleus with spin $i \neq 0$, the expansions (2) and (6) remain unchanged, but instead of (7) the relation between α and α_0 is

$$\alpha = \alpha_0 [x_-^2 / g_- + (1 - x_-)^2 / (1 - g_-)], \qquad (8)$$

where $g_- = i/(2i+1)$ is the statistical weight of the reaction channel with spin $J = i - 1/2$, and x_- is the relative contribution of this channel to the thermal cross section. The value of α goes through a minimum $\alpha_{min} = \alpha_0$ at $x_- = g_-$. Expressions (6) to (8) have been previously obtained[3] from the Breit-Wigner formula for an isolated level. Actually, as is clear from the foregoing considerations, the validity of these relations is not restricted to the range of applicability of the single-term Breit-Wigner formula, nor to the applicability of the concepts of the compound nucleus.

If the reaction induced by a slow neutron has only one open channel for a given channel spin then, using the reciprocity theorem, one can obtain from (6) an expression for the cross section of the reverse reaction close to its threshold:

$$(\sigma_{rev} E_n^{-1/2})_0 / \sigma_{rev} E_n^{-1/2} = 1 + \alpha \frac{A+1}{A} E_n^{1/2} + \beta_1 E_n + \cdots, \qquad (9)$$

where E_n is the kinetic energy of the emitted neutron in the center-of-mass system, and α is given by (7) and (8) if the statistical weights of the entrance and exit channels are identical.

The term $\alpha E^{1/2}$ in (6) can be noticed in experiment if the thermal reaction cross section is very large, but the coefficient β is small. This last condition is fulfilled if there are no narrow resonance levels for small neutron energies. In reference 3 the α term appears in the expression for the energy dependence of the reaction cross sections for the processes $He^3(n,p)$ and $B^{10}(n,\alpha)$. A value $\alpha = 4.1 \times 10^{-2}$ kev$^{-1/2}$ was found for the first reaction. Comparing (7) and (8), it appears that for low energies the reaction goes essentially through the channel $J = 0$. The cross section for the reaction $Li^7(p,n)$, measured in reference 4, agrees near the threshold with (9) if $\alpha \approx 0.21$. From this and from the value for $(\sigma E^{1/2})_0$ there follow two possibilities for the spin of the channel: (1) $x_- = 0$, $x_+ = 1$ and (2) $x_- = 0.75$, $x_+ = 0.25$. The mere presence of the α term in the expression for the reaction cross section does not yet tell anything about the resonance levels of the compound nucleus. However, the fact that the reaction has a very large cross section and goes essentially through one of two possible channels, as in the reaction $He^3(n,p)$, supports the argument in favor of the presence of the level.

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