value of φ obtained from (3) differs on the average from the exact solution obtained numerically in reference 9 by less than 0.5%, and never differs by more than 1%. In the same region, φ' differs by less than 2.5% on the average and by not more than 5% from the exact value of φ'.

In conclusion it should be noted that any attempt to derive an extremely accurate approximation of the Thomas-Fermi function at small and at large distances from the nucleus is devoid of meaning, for there Eq. (1) does not correctly indicate the potential in the atom.

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PRODUCTION OF A STAR AND A FAST PROTON OR ANTIPROTON

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THE production of a star by a γ-quantum through an intermediate π-meson pair was investigated in reference 1. In the present work we consider an analogous process: a high-energy γ-quantum produces a proton-antiproton pair one particle of which is absorbed in the nucleus, producing a star. The other particle carries away energy of the order of the energy of the star. The study is carried out for the ultra-relativistic region where only small angles between the momentum of the γ-quantum and that of the emitted proton (or antiproton) are of importance. The strong interaction between the proton (and antiproton) and the nucleus is accounted for by using the optical model. The nucleus is regarded as an ideal black body of a given radius, as far as the proton and antiproton are concerned.

We shall assume that the behavior of nucleons is described by Dirac's equation. The anomalous magnetic moment of the nucleon is not of great importance for such high energies. Let us assume, for example, that the proton is absorbed and the antiproton is emitted to an infinite distance. Dirac's equation for such a process can be written as follows: ($\hbar = c = 1$)

$$(\gamma\nabla - \gamma_4 E_1 + m)\,\psi_{p_1}(\mathbf{r}) = \frac{ie}{\sqrt{2\omega}}(\boldsymbol{\gamma}\mathbf{e})\,e^{i\mathbf{kr}}\psi_{p_2}^{(-)}(\mathbf{r}). \qquad (1)$$

where $\mathbf{p}, E_1$ ($\mathbf{p}_2, E_2$) are the momentum and energy of the proton (or antiproton). We shall denote the momentum of a γ-quantum of frequency ω by $\mathbf{k}$, and its polarization vector by $\mathbf{e}$. The wave function of the antiproton, which is a free particle in its final state, is a superposition of a plane wave and of a wave diffracted on the black nucleus.[2]

We find the wave function ψ_{p_1} of the proton by means of the Green's function[3] of Eq. (1), under the condition that the antiproton is at infinity. We obtain the cross section for the process by calculating the total flux of protons incident upon the nucleus:

$$d\sigma = \int j(\mathbf{s}_1)\,d\mathbf{s}_1\,|F|^2\,dp_2\,d\mathbf{k}_2/(2\pi)^3;$$

$$j(\mathbf{s}_1) = \left(\psi_{\mathbf{p}_1}(\mathbf{s}_1)\,\frac{i\boldsymbol{\gamma}\mathbf{p}_1}{p_1}\,\psi_{\mathbf{p}_1}(\mathbf{s}_1)\right),$$

where $\mathbf{k}_2$ is the transverse momentum of the emitted antiproton and F is in the nature of a nucleon form-factor.[4] Integration over $\mathbf{s}_1$ ($\mathbf{s}_2$) is carried out along a circle with radius R, perpendicular to $\mathbf{p}_1$ ($\mathbf{p}_2$) and passing through the center of the nucleus.

We obtain the following expression for the differential cross-section of the process, averaged over possible polarization of the γ-quantum:

$$d\sigma(E_2, \boldsymbol{\eta}) = \frac{e^2}{\omega^3}\frac{|F|^2 R}{(2\pi)^3 m}\{[E_2^2 + (\omega - E_2)^2]\,K(\varepsilon) + E_2(\omega - E_2)\,E(\varepsilon)\}\,\frac{dE_2\,d\boldsymbol{\eta}}{(1+\eta^2)^{3/2}}; \qquad (2)$$

$$\boldsymbol{\eta} = \mathbf{k}_2/m, \quad \varepsilon^2 = \eta^2/(1+\eta^2), \quad d\boldsymbol{\eta} = \eta\,d\eta\,d\varphi_\eta,$$

where $K(\epsilon)$ and $E(\epsilon)$ are complete elliptic integrals of the first and second kind respectively. It can be seen from Eq. (2) that $\eta_{eff} \sim 1$, i.e. $\vartheta_{eff} \sim m/E_2$. The form-factor F can be determined from comparison with experiment. At high energies, the differential cross section for small angles attains large values.

Integration over E_2 and η can be carried out only for F = 1. We have then

$$\sigma = (e^2/24\pi^2)(R/m)\,\Phi(\eta_{max}),$$

$$\Phi(\eta_{max}) = \int\limits_0^{\eta_{max}} \left\{4K\left(\frac{\eta}{\sqrt{1+\eta^2}}\right) + E\left(\frac{\eta}{\sqrt{1+\eta^2}}\right)\right\} \frac{\eta d\eta}{(1+\eta^2)^{3/2}}. \quad (3)$$

If we put $\eta_{max} = \infty$ (in general, η_{max} should be of the order of unity) we have $\Phi(\infty) = 9\pi^2/8$, $\sigma \sim 10^{-28}\,cm^2$. As in the scalar case,[1] the total cross section is independent of the γ-quantum energy and is proportional to R/m and not to R^2, since in the effective region for the process, that ahead of the nucleus, the ψ-function of the emitted particle has a shadow and the whole process is determined by the penumbra region.

Equation (2) has been obtained under the assumption that the nuclear radii (R_1 with respect to protons and R_2 with respect to antiprotons) are equal, $R_1 = R_2 = R$. The cross section for the process is then independent of which particle, the proton or the antiproton, is free in the final state. If $R_1 \neq R_2$, and $\Delta R \gg 1/m$, then the cross section for the process with emission of the more strongly interacting particle is, in the given approximation, exponentially small ($\sim \exp(-\alpha\Delta R)$, $\alpha \sim m$). The cross section for the process with emission of the less strongly interacting particle (the proton) can be obtained from Eq. (2) by replacing R with $2R_1$ for $R_1 > R_2$ (or with $2R_2$ for $R_2 < R_1$).

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OVERHAUSER EFFECT IN NONMETALS

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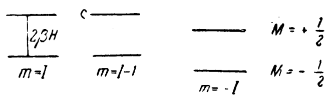
RECENTLY a number of papers[1-4] have appeared in which different modifications of the Overhauser method are proposed to obtain nuclear polarization in nonmetals. In particular, these papers discuss the polarization of the nuclei of paramagnetic atoms in salts and of the nuclei of donor and acceptor impurities in silicon and germanium. The problem is mainly to obtain a nonequilibrium nuclear polarization thanks to the fast passage effect. In the present paper we shall be concerned with obtaining a stationary nuclear polarization.

Let us consider a system consisting of a nucleus of spin I and an electron located in a magnetic field of strength H. The system will have 2(2I + 1) levels corresponding to two values of the electron spin component (M), and 2I + 1 values of the nuclear spin component (m). Assuming the external field to be sufficiently strong (Zeeman energy of the electron considerably larger than the spin-spin interaction energy of the electron with the nucleus) we get 2I + 1 transitions in the paramagnetic-resonance spectrum (selection rules: $\Delta M = \pm 1$, $\Delta m = 0$). To evaluate the population of the levels, we neglect the spin-spin interaction energy and the Zeeman energy of the nuclear spin, and we obtain 2I + 1 pairs of levels with an energy level difference in each pair equal to $2\beta H$ (see figure).



We consider the most important case, where we can neglect for the nuclear spin all interactions except the contact interaction, which is proportional to $(\mathbf{S}\cdot\mathbf{I})\,\delta(\mathbf{r})$. In that case we have, for the relaxation processes involving the nuclear spin, the selection rule $\Delta(M + m) = 0$.

Let complete saturation be reached (that is, let the saturation parameter be equal to unity) for all 2I + 1 paramagnetic resonance levels. We get then Overhauser's known result, i.e., the degree of polarization is equal to

$$f = B_I(2I\delta), \quad \delta = \beta H/kT, \quad (1)$$