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14

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POSSIBILITY OF DETERMINING THE INTERACTION CONSTANTS FROM EXPERIMENTS ON $K_{\mu 3}$ DECAYS

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Expressions are derived for the spectra and polarization of μ mesons created in $K_{\mu 3}$ decays for fixed pion energies and arbitrary complex interaction functions. It is shown that, accurate to within an unimportant phase shift, it is possible to determine all interaction functions from measurements of the spectra and three polarizations, with the exception of the second vector function. The latter cannot be separated in the above experiments from the first-vector and scalar interaction functions. The presence of tensor interaction can be ascertained from measurements of the spectrum and polarizations of μ mesons at an energy close to maximum.

EXPRESSIONS for the spectrum of μ mesons, as well as for their polarization, were obtained by various authors.[1-11] However, no calculations were made for the case of arbitrary complex constants,* nor were they analyzed from the point of view of the possibility of determining all the constants experimentally. If the interaction is assumed to be local, the most general expression for the transition matrix element is of the form

$$\langle\bar{\psi}_\mu \Big\{(g_S + i\gamma_5 g_S') + (g_V + i\gamma_5 g_V')\gamma_4 + {}^1/_2 M^{-1}(g_T + i\gamma_5 g_T') \times (\gamma_4 \hat{p}_\pi - \hat{p}_\pi \gamma_4)\Big\}\psi_\nu\rangle (2M^{3/2}E_\pi^{1/2})^{-1}. \qquad (1)$$

Here M is the mass of the K meson, and p_π

*In fact, the interaction constants may depend on the pion energy, and shall henceforth be referred to as "functions."

and E_π are the 4-momentum and energy of the pion respectively.

Only one vector variant is taken into account in the matrix element, because the second vector variant can be reduced to the first vector and scalar variants, as was noted by Pais and Treiman.[11] It must be indicated that this reduction is possible only if the electromagnetic interaction is neglected. The expression for the spectrum is written in the following manner:

$$\delta W_1(E_\pi,E_\mu) = (32\pi^3M^3)^{-1}dE_\pi dE_\mu\{A_0 + A_1E_\mu + A_2E_\mu^2\}. \quad (2)$$

Here A_i, like B_i, C_i, and D_i below, are functions of the pion energy and of the interaction function. The expressions for A_i, B_i, C_i and D_i are given in the Appendix.

For the degree of longitudinal polarization, multiplied by the probability of decay with given meson energies (or, in other words, for the differences of the spectra of μ-mesons polarized with and against the momentum at fixed pion energy), we obtain the following expression

$$\delta W_2(E_\pi,E_\mu) \quad (3)$$
$$= (32\pi^3M^3)^{-1}dE_\pi dE_\mu\{B_0 + B_1E_\mu + B_2E_\mu^2 + B_3E_\mu^3\}.$$

In the case of transverse polarization in the plane of the decay (along the direction of $\mathbf{J}_2$, which makes a sharp angle with the pion momentum and a right angle with the muon momentum) the analogous expression is

$$\delta W_3(E_\pi,E_\mu)$$
$$= -|p_\pi||\sin\theta|(32\pi^3M^3)^{-1}dE_\pi dE_\mu\{C_0 + C_1E_\mu + C_2E_\mu^2\}.$$

Here θ is the angle between the pion and muon momenta.

For the difference in the spectra of muons polarized in the direction $\mathbf{J}_3 = (\mathbf{p}_\pi \times \mathbf{p}_\mu)/|(\mathbf{p}_\pi \times \mathbf{p}_\mu)|$, perpendicular to the plane of decay (which is forbidden by the law of conservation of combined parity), we obtain, for a fixed pion energy, the following expression

$$\delta W_4(E_\pi,E_\mu) = p_\mu p_\pi|\sin\theta|\{D_0 + D_1E_\mu\}dE_\pi dE_\mu(32\pi^3M^3)^{-1} \quad (4)$$

The expressions for δW_2 and δW_3 contain six combinations of interaction functions. These expressions are polynomials of third and second degree respectively. It is possible therefore to set up seven equations from the experimental data. An investigation of the determinant of the system has shown that at least six of the seven equations are linearly independent.

From the spectral data it is possible to obtain three more equations and the polarization perpendicular to the plane of the decay yield two more. A total of eleven equations can thus be obtained. Hence, with accuracy to a non-essential phase shift, it is possible to obtain all the interaction functions. Naturally, the second vector variant cannot differ in this case from S or V_1. However, with this reservation, a complete experiment is possible.

It is of interest, however, to obtain certain information on the interaction functions from data on only part of the experiments. Such information can be obtained from data on the spectra of the longitudinally-polarized muons at fixed energy E_π. If we regroup the terms in expression (3), the result is

$$\delta W_2(E_\pi,E_\mu) = dE_\pi dE_\mu(32\pi^3M^3)^{-1}\{b_T\Phi_T + b_V\Phi_V + b_{VT}\Phi_{VT} + b_{ST}\Phi_{ST} + b_S\Phi_S + b_{SV}\Phi_{SV}\}. \quad (5)$$

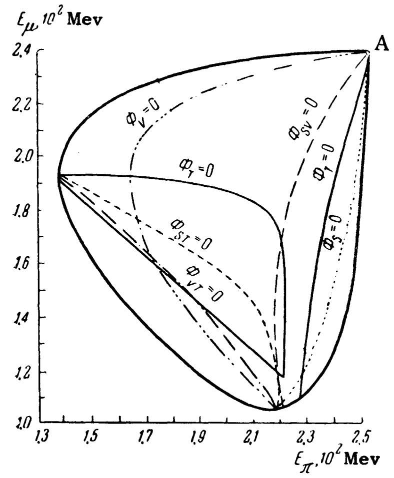
Here

$$b_T = -2\mathrm{Re}(g'_Tg^*_T); \qquad b_V = -2\mathrm{Re}(g'_Vg^*_V);$$
$$b_S = -2\mathrm{Re}(g'_Sg^*_S); \qquad b_{VT} = -2\mathrm{Re}(g_Vg'^*_T - g'_Vg^*_T);$$
$$b_{ST} = -2\mathrm{Re}(g_Sg'^*_T + g'_Sg^*_T); \quad b_{SV} = -2\mathrm{Re}(g_Sg'^*_V - g'_Sg^*_V).$$

The values of the functions Φ_i are given in the Appendix.

The diagram shows the region of variation of the pion and muon energies, allowed by the laws of momentum and energy conservation. This region is plotted for K^0 decays, i.e., the following mass values are assumed: $M_{K^0} = 495$ Mev, $m_{\pi^\pm} = 139.6$ Mev, $m_\mu = 105.7$ Mev, $c^2 = 1$.

The diagram shows, inside the allowed zone, the curves on which the functions Φ_i vanish. The closer a point is to the curve, the less the probability contribution due the term corresponding to the curve. These curves intersect at several points. If a probability other than zero is observed at such points, it means that there is at least one non-vanishing variant, whose line does not pass through the given point. Of particular in-



Zero lines of pure and interference terms.

terest is point A, corresponding to maximum muon energy. It is not necessary to know the pion energy to carry out measurements at this point, and the experiment can thus be facilitated somewhat. If it is found that the probability density for the emission of longitudinally polarized mesons differs from zero near the point, this means that at least one of the S or T variants is present, provided the measurement is carried out in a region on the order of 3 to 5 Mev from the limiting energy. The line of the zeros of the first vector variant will approach quite gently the boundary of the region in the boundary energy. If the above energy density is seen to be unequal to zero in a region on the order of 0.5 Mev, this means the existence of the tensor variant, for in this case the interference term ST would differ from zero. Naturally, if the π-meson energy is fixed, no such accuracy is necessary.

It must be emphasized here again that in our notation the second vector variant $<\bar{\psi}_\mu p_\pi(g_\mu - i\gamma_5 g'_{V_2})\psi_\nu>$ is reduced to the first vector and scalar variants. Therefore the presence of a scalar variant can be interpreted also as the presence of the second vector variant. In connection with this, it is particularly important to establish the presence of a tensor variant.

The remaining points require simultaneous measurement of the pion energy.

We call attention to the fact that $\delta W_3/|\sin\theta|$ is a second-order polynomial in E_m if, and only if, the interference term VT differs from zero, i.e., if the tensor and vector variants exist simultaneously. If there is no such term, then $\delta W_3/|\sin\theta|$ depends linearly on the muon energy or becomes constant.

The expressions for the spectrum and for the longitudinal polarization of the pions were integrated by various authors (the spectra by Furuchin et al.[2] and Matinina,[7,8] and the spectra and polarizations by Okun'[1]) under the assumption that the interaction functions are independent of the pion energy. From the expressions obtained by Okun' it is easily seen that if the interaction constants are considered independent of the pion energy, it is possible to determine experimentally, from measurements of the spectrum and of the longitudinal polarization, all twelve combinations of the interaction constant.

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APPENDIX

We give below the values of the functions A_i, B_i, C_i, and D_i:

$$A_0 = a_T\left[\left(ME_\pi - \frac{\Delta}{2}\right)(2ME_\pi - \Delta - E_\pi^2 + m_\pi^2) - m_\pi^2 m_\mu^2 + m_\mu^2 E_\pi^2\right]M^{-2} + a_{VT}\left(\frac{\Delta}{2} - ME_\pi - m_\pi^2 + E_\pi^2\right)m_\mu M^{-1}$$
$$+ a_V\left(m_\mu^2 + ME_\pi - \frac{\Delta}{2}\right) + a_{SV} m_\mu (M - E_\pi) + a_{ST}\left(ME_\pi - \frac{\Delta}{2}\right)(E_\pi - M)M^{-1} + a_S\left(\frac{\Delta}{2} - ME_\pi - m_\mu^2\right);$$
$$A_1 = a_T 2(2ME_\pi - \Delta)(M - E_\pi)M^{-2} + a_{VT}(E_\pi - M) + a_V 2(M - E_\pi) + a_{ST}(2E_\pi M - m_\pi^2 - M^2)M^{-1} + a_{SV}(-m_\mu);$$
$$A_2 = a_T 2M^{-2}(m_\pi^2 - 2ME_\pi + 2M^2) - 2a_V.$$

Here

$$\Delta = M^2 + m_\pi^2 + m_\mu^2;\ a_T = |g_T|^2 + |g'_T|^2;\ a_V = |g_V|^2 + |g'_V|^2;\ a_S = |g_S|^2 + |g'_S|^2;\ a_{VT} = 2\mathrm{Re}(g_V g_T^* - g'_V g_T'^*);$$
$$a_{ST} = 2Re(g_T g_S^* + g'_T g_S'^*);\ a_{SV} = 2Re(g_S g_V^* - g'_S g_V'^*).$$

$$B_0 = b_S m_\mu^2(M - E_\pi) + b_{SV}\left(\frac{\Delta}{2} - ME_\pi - m_\mu^2\right)m_\mu + b_{ST}\left[m_\mu^2\left(\frac{\Delta}{2} - m_\pi^2\right) + (E_\pi - M)E_\pi m_\mu^2\right]M^{-1} + b_V m_\mu^2(M - E_\pi)$$
$$+ b_{VT} M^{-1} m_\mu\left(ME_\pi - \frac{\Delta}{2}\right)(M - E_\pi) + b_T(M - E_\pi)(E_\pi^2 - m_\pi^2)m_\mu^2 M^{-2};$$
$$B_1 = b_S\left(ME_\pi - \frac{\Delta}{2}\right) + b_{SV} m_\mu(E_\pi - M) + b_{ST} M^{-1}\left(m_\mu^2 + \frac{\Delta}{2} - ME_\pi\right)(E_\pi - M) + b_V\left(\frac{\Delta}{2} - ME_\pi - 2m_\mu^2\right)$$
$$+ b_{VT} M^{-1}\left[\frac{\Delta}{2}m_\mu + E_\pi^2 m_\mu + Mm_\mu(M - 3E_\pi)\right] + b_T M^{-2}\left[2m_\mu^2 m_\pi^2 - 2E_\pi^2 m_\mu^2 + \left(\frac{\Delta}{2} - ME_\pi\right)(2ME_\pi - \Delta - E_\pi^2 + m_\pi^2)\right];$$
$$B_2 = b_{SV} m_\mu + b_{ST}(m_\pi^2 - 2ME_\pi + M^2)M^{-1} + b_V 2(E_\pi - M) + b_{VT} m_\mu(E_\pi - M)M^{-1} + b_T 4M^{-1}\left(\frac{\Delta}{2} - E_\pi\Delta/2M - ME_\pi + E_\pi^2\right);$$
$$B_3 = 2b_V + b_T(4E_\pi M - 2m_\pi^2 - 2M^2)M^{-2};\quad C_0 = -b_S m_\mu + b_{ST} m_\mu(E_\pi - M)M^{-1} + b_V m_\mu - b_{VT} m_\mu^2 M^{-1}$$
$$+ b_T m_\mu\left[2\left(ME_\pi - \frac{\Delta}{2}\right) - E_\pi^2 + m_\pi^2\right]M^{-2};\quad C_1 = b_{SV} + b_{ST}\frac{m_\mu}{M} + b_{VT}(E_\pi - M)M^{-1} + b_T 2m_\mu(M - E_\pi);$$
$$C_2 = b_{VT} 2M^{-3}.\quad D_0 = M^{-1}\left(d_{SV} + \frac{m_\mu}{M}d_{ST} + \frac{M - E_\pi}{M}d_{VT}\right);\quad D_1 = -2d_{VT}M^{-2}.$$

Here

$$d_{SV} = -2\mathrm{Im}\,(g_S g_V^* - g_S' g_V'^*);\quad d_{ST} = 2\mathrm{Im}\,(g_T g_S^* + g_T' g_S'^*);\quad d_{VT} = -2\mathrm{Im}\,(g_V g_T^* - g_V' g_T'^*).$$

$$\Phi_S = E_\mu\left(ME_\pi - \frac{\Delta}{2}\right) + m_\mu^2 (M - E_\pi);\quad \Phi_{SV} = E_\mu^2 m_\mu + E_\mu (E_\pi - M)\, m_\mu + m_\mu \left(\frac{\Delta}{2} - ME_\pi - m_\mu^2\right);$$

$$\Phi_{ST} = E_\mu^2 \left(\frac{m_\pi^2}{M} - 2E_\pi + M\right) + E_\mu \left\{- m_\mu^2 + \frac{E_\pi m_\mu^2}{M} + (E_\pi - M)\left(\frac{\Delta}{2M} - E_\pi\right)\right\} + M^{-1} \left\{m_\mu^2 \left(\frac{\Delta}{2} - m_\pi^2\right) + (E_\pi - M)\, E_\pi m_\mu^2 \right\};$$

$$\Phi_V = 2E_\mu^3 + E_\mu^2 (-2M + 2E_\pi) + E_\mu \left\{\left(\frac{\Delta}{2} - ME_\pi\right) - 2m_\mu^2\right\} + m_\mu^2 (M - E_\pi);$$

$$\Phi_{VT} = E_\mu^2 M^{-1} m_\mu (E_\pi - M) + E_\mu M^{-1} \left\{\frac{\Delta}{2} m_\mu + E_\pi^2 m_\mu + M m_\mu (M - 3E_\pi)\right\} + M^{-1} \left\{m_\mu\left(ME_\pi - \frac{\Delta}{2}\right) (M - E_\pi)\right\};$$

$$\Phi_T = M^{-2}E_\mu^3 \{4E_\pi M - 2m_\pi^2 - 2M^2\} + E_\mu^2 M^{-1} 4\left(\frac{\Delta}{2} - \frac{E_\pi\Delta}{2M} - ME_\pi + E_\pi^2\right) + E_\mu M^{-2} \left\{2m_\pi^2 m_\mu^2 - 2E_\pi^2 m_\mu^2 \right.$$
$$\left. + \left(\frac{\Delta}{2} - ME_\pi\right)(2ME_\pi - \Delta - E_\pi^2 + m_\pi^2)\right\} + (M - E_\pi)(E_\pi^2 - m_\pi^2)\, m_\mu^2 M^{-2}.$$

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15

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THEORY OF EXCITATION OF HYDROMAGNETIC WAVES

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Excitation of hydromagnetic and magnetoacoustic waves by external currents is investigated. Damping of the waves as a result of conductivity and viscosity is taken into account. The intensity of excitation by currents is compared with the intensity of excitation by mechanical means.

1. As is well known, propagation of hydromagnetic and magnetoacoustic waves is possible in a conducting liquid located in an external magnetic field.[1] In the experiments of Lundquist,[2] hydromagnetic waves were excited in liquid mercury by mechanical means, with the use of a rotating disk equipped with blades. Excitation of hydromagnetic waves is also possible by means of external variable currents. It is therefore of interest to determine the intensity of the excitation of hydromagnetic waves by this method and to compare this intensity with that of the excitation of hydromagnetic waves by mechanical means. The present paper is devoted to a consideration of this problem.