$$\frac{\partial n}{\partial t} + \operatorname{div}\frac{\mathbf{j}}{e} - (N_d - n)(\tilde{\beta} + \beta_b + n\beta + \beta_{ext}) + (N_a + n)\, n\, (\tilde{\beta}' + \beta_b' + n\beta') = 0, \quad (2.32)$$

$$\frac{\partial \varepsilon_e}{\partial t} + \operatorname{div}\mathbf{W}_e - \mathbf{E}\cdot\mathbf{j} + An\frac{T_e - T}{T}\left(\frac{T_e}{T}\right)^{1/2} - (N_d - n)[kT\tilde{\beta} + kT\beta_b - n\varepsilon_0\beta + (\varepsilon_{ext} - \varepsilon_0)\beta_{ext}] \quad (2.33)$$
$$+ (N_a + n)\, n\, [kT_e\tilde{\beta}' + kT_e\beta_b' - n_0\beta'] = 0.$$

By solving this system of two equations together with the transport equations one can uniquely determine T_e and the non-equilibrium values of n for various processes.

[1] V. P. Shabanskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 657 (1956), Soviet Phys. JETP **4**, 497 (1957).

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PHOTOPRODUCTION OF ELECTRON AND μ - MESON PAIRS ON NUCLEONS

I. T. DIATLOV

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Production of electron or μ-meson pairs on nucleons by high-energy gamma-quanta is examined. It is shown under what conditions the cross sections for these processes can be expressed in terms of the electromagnetic form factors of the free nucleon.

1. An investigation of the cross sections of the radiation processes involving production of electrons and μ mesons from nucleons at large angles, makes it possible to judge the electromagnetic properties of the nucleon (form factors)[1] or, if these are known from other experiments, to determine the limits of validity of modern quantum electrodynamics.

It was indicated in reference 2 that the electromagnetic form factor of the free nucleon can be written in the form ($\hbar = c = 1$):

$$\Gamma_\mu(q^2) = a(q^2)\gamma_\mu + i\frac{b(q^2)}{2M}\frac{1}{2}(\gamma_\mu\hat{q} - \hat{q}\gamma_\mu),$$

where q is the momentum transferred by the electromagnetic field to the nucleon ($q^2 = \mathbf{q}^2 - q_0^2$; $\hat{q} = q_\nu\gamma_\nu$), M is the nucleon mass, and $a(q^2)$ and $b(q^2)$ are real functions. When q^2 goes to zero, $a(q^2)$ goes to 1 or 0 for protons and neutrons respectively, and $b(q^2)$ goes to the anomalous magnetic moment μ_0 (in nuclear magnetons). Substantial deviations from these limiting values of $a(q^2)$ and $b(q^2)$ are expected when $q \gtrsim \mu$, where μ is the mass of the pion. We shall therefore be interested in recoils $q \gtrsim \mu$, i.e., as will be seen later on, in sufficiently large angles. Furthermore, to determine the form factor $b(q^2)$ it is necessary to consider large recoils, since $b(q^2)$ enters into the formula together with the factor $q(M)(1)$.

2. We consider the production of electron or muon pairs on nucleons by gamma quanta. Graphs corresponding to this process are divided into two groups. In the first group (1a) only one photon line goes to the nucleon line. Along this photon line the nucleon acquires a recoil momentum $q = k - p_+ - p_-$ (p_+ and p_- are the momenta of the pair components and k is the momentum of the incident quantum). These are graphs having the electromagnetic vertex part of the free nucleon, and can be expressed in terms of functions a and b of formula (1) by inserting Γ_μ from formula (1) when writing the matrix elements corresponding to graphs (1a). In the second group, two photon lines go to the nucleon portion of the graph (Figs. 1b and 1c indicate the general form of such a graph and two simplest graphs for a nucleon interacting only with the electromagnetic field). It is impossible to account for the meson interactions for these by introducing simple form factors of type (1).

As to the matrix elements of such graphs, it can be assumed that they are of the same order of magnitude (or less) as the matrix elements of the simplest graphs (1c), corresponding to the

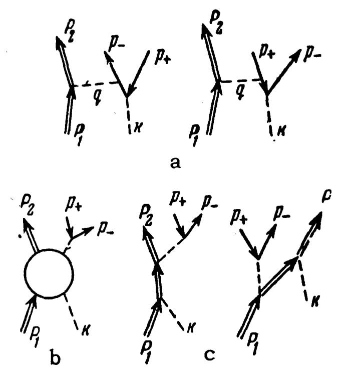

FIG. 1

process that takes place on a point nucleon interacting only with the electromagnetic field. The interaction with meson fields leads to a smearing of the electromagnetic charge of the nucleon which, apparently, reduces the matrix elements compared with the corresponding ones for the point nucleon. These considerations can be confirmed by direct examination of several very simple meson graphs. Consequently, under these conditions, when it is possible to neglect graphs 1c compared with 1a, the cross section of the process can be expressed in terms of the quantities $a(q^2)$ and $b(q^2)$ and this takes care of the influence of the meson fields.

The matrix elements corresponding to graphs 1a and 1c are:

$$M_a = -e^3(2\pi)^4\overline{U}(P_2)\Gamma_\mu U(P_1)\overline{u}(p_-)\left\{\frac{\gamma_\mu}{q^2}\frac{i(-\hat{p}_+ + \hat{k}) - m}{-2p_+\cdot k}\frac{\hat{e}}{\sqrt{2\omega}} + \frac{\hat{e}}{\sqrt{2\omega}}\frac{i(\hat{p}_- - \hat{k}) - m}{-2p_-\cdot k}\frac{\gamma_\mu}{q^2}\right\}v(p_+); \tag{2}$$

$$M_c = -e^3(2\pi)^4\overline{U}(P_2)\left\{\gamma_\mu\frac{i(\hat{P}_1 + \hat{k}) - M}{2P_1\cdot k}\frac{\hat{e}}{\sqrt{2\omega}} + \frac{\hat{e}}{\sqrt{2\omega}}\frac{i(\hat{P}_2 - \hat{k}) - M}{-2P_2\cdot k}\gamma_\mu\right\}U(P_1)\overline{u}(p_-)\frac{\gamma_\mu}{q_1^2}v(p_+), \tag{3}$$

$$pk = \mathbf{p}\cdot\boldsymbol{\omega} - \varepsilon\omega; \quad P_2 = P_1 + q; \quad q_1 = p_+ + p_-;$$

P_1 and P_2 are the momenta of the initial and final nucleons, m is the mass of the light particles, and $\hat{e} = e_\mu\gamma_\mu$ where e_μ is the polarization vector of the gamma quantum.

In order of magnitude

$$M_a \sim \frac{e^3(2\pi)^4}{\sqrt{\omega}}\frac{\mathbf{e}\cdot\mathbf{p}}{p\cdot k}\left(1 - \frac{\mathbf{q}\cdot\mathbf{p}}{Ep}\right)\frac{1}{q^2}; \quad M_c \sim \frac{e^3(2\pi)^4}{\sqrt{\omega}}\frac{\mathbf{e}\cdot\mathbf{p}}{P\cdot k}\left(1 - \frac{\mathbf{q}\cdot\mathbf{p}}{Ep}\right)\frac{1}{q_1^2}; \tag{4}$$

p is the momentum of any one of the light particles (we assume that their momenta are of the same order), P is the momentum of the nucleon, and E is its energy.

From the Weizsäker[3] analysis of radiation processes it is clear that the principal contribution to the total cross section will be made at large energies by M_a, since the principal contribution to the double cross section is introduced by the angles $\vartheta \lesssim m/p$, and at such angles $M_a \gg M_b$. We are interested, however, in large Q ($q \gtrsim \mu > m$), which corresponds to large angles ($\vartheta \geq \mu/p$). It is therefore necessary to examine the ratio of M_a to M_c at such angles.

From (4) we see that M_a consists of two terms whose ratio is proportional to $\sim|\mathbf{q}|/E$. Neglecting the terms $\sim|\mathbf{q}|/E$ means considering only small recoils $|\mathbf{q}| \ll M$, neglecting the energies transferred to the nucleon, and neglecting the term containing $b(q^2)$ in the expression (1). In order to be able to discard graphs 1c without making such approximations, it is necessary to satisfy the condition

$$\frac{\mathbf{q}\cdot\mathbf{p}}{Ep}\frac{P\cdot k}{p\cdot k}\frac{q_1^2}{q^2} \gg 1. \tag{5}$$

Recoils $|\mathbf{q}| \geq \mu$ mean that the momenta of the light particles should be greater than μ (ω, $|\mathbf{p}_+|$, $|\mathbf{p}_-| > \mu > m$). Then, when $\vartheta \ll 1$ (it will be shown below that only this case is of importance), we have

$$p\cdot k \sim \frac{\omega}{p}[m^2 + (p\vartheta)^2], \quad \mathbf{q}\cdot\mathbf{p} \sim p(E - M) + O(p^2\vartheta^2),$$
$$q^2 \sim O(p^2\vartheta^2), \quad q_1^2 \sim 2m^2 + O(p^2\vartheta^2), \tag{6}$$
$$P\cdot k \sim M\omega + O(p^2\vartheta^2).$$

From the energy-conservation law (the nucleon was at rest before collision) we have

$$E - M \approx 0(p^2\vartheta^2/M). \tag{7}$$

Insertion of (6) and (7) into (5) gives the conditions under which it is possible to consider only graphs 1a

$$\vartheta^2 \ll M/p, \qquad p \gg M. \tag{8}$$

The first of conditions (8) imposes limitations on the energies acquired by the recoil nucleon during the process. It follows from (7) and (8) that the recoil energies are considerably smaller than the energies of the light particles ($\epsilon \cong p$): $E \ll p$. If the second condition of (8) is not satisfied ($p \lesssim M$), then $M_a \gg M_c$ as before, but the terms with $|\mathbf{q}|/E$ in M_a are of the same order as in M_c, and consequently all these, including those containing $b(q^2)$, should be discarded. We then obtain for the quanta with energy $\omega \lesssim M$ and angles $\vartheta^2 \ll M/\omega \sim 1$ the ordinary Bethe-Heitler formula multiplied by $a^2(\mathbf{q}^2)$ ($q^2 = \mathbf{q}^2$ for $|\mathbf{q}| \ll M$, and the recoil energy of the nucleon is neglected).

3. If conditions (8) are satisfied we obtain, by inserting expression (1) in lieu of Γ_μ into Eq. (3), the probability of pair production on a nucleon at rest.

$$dW = (2\pi)^{-4} |M_a|^2 (2\pi)^{-9} d^3p_+ d^3p_- \delta(M + \omega - \varepsilon_+ - \varepsilon_- - [M^2 + (\boldsymbol{\omega} - \mathbf{p}_+ - \mathbf{p}_-)^2]^{1/2}). \quad (9)$$

Integrating (9) over the absolute value of the momentum p_- and then averaging over the polarization of the gamma-quantum and summing over the spins of the pair components, we obtain ($\alpha = 1/137$):

$$d\sigma = \frac{\alpha^3}{(2\pi)^2} \frac{p_+ p_- dp_+}{\omega^3} do_+ do_- \frac{1}{q^4} \Big\{ a^2(q^2) \Big[f_{\mathrm{B-H}} + \frac{q^2}{4M^2} f_1 \Big] + 2a(q^2) b(q^2) \frac{q^2}{4M^2} f + b^2(q^2) \frac{q^2}{4M^2} \Big[f_{\mathrm{B-H}} + \frac{q^2}{4M^2} f_1 - f \Big] \Big\}, \quad (10)$$

where

$$f_{\mathrm{B-H}} = \frac{16 p_+^2 p_-^2 (p_+\vartheta_+)^2}{[m^2 + (p_+\vartheta_+)^2]^2} + \frac{16 p_+^2 p_-^2 (p_-\vartheta_-)^2}{[m^2 + (p_-\vartheta_-)^2]^2} - \frac{16 p_+^2 p_-^2 (p_+^2 + p_-^2)(\vartheta_+ \vartheta_-)}{[m^2 + (p_+\vartheta_+)^2][m^2 + (p_-\vartheta_-)^2]} - 8\omega^2 \frac{[(p_+\vartheta_+)^2 + (p_-\vartheta_-)^2] p_+ p_-}{[m^2 + (p_+\vartheta_+)^2][m^2 + (p_-\vartheta_-)^2]}; \quad (11)$$

$$f = -8\omega^2 [(p_+\vartheta_+)^2 + (p_-\vartheta_-)^2] / [m^2 + (p_+\vartheta_+)^2][m^2 + (p_-\vartheta_-)^2];$$

$$f_1 = 8m^2 (p_+ - p_-)^2 \omega^2 / [m^2 + (p_+\vartheta_+)^2][m^2 + (p_-\vartheta_-)^2];$$

$$q^2 = (p_+ \boldsymbol{\vartheta}_+ + p_- \boldsymbol{\vartheta}_-)^2 = 2M(\omega - p_+ - p_-); \quad (12)$$

$\boldsymbol{\vartheta}_+$ and $\boldsymbol{\vartheta}_-$ are the angles between $\boldsymbol{\omega}$ and $\mathbf{p}_+$ and $\mathbf{p}_-$ respectively ($\boldsymbol{\omega} \cdot \boldsymbol{\vartheta}_\pm = 0$). The modulus of $\mathbf{p}_-$ should be expressed in terms of the remaining variables from the conservation laws. $f_{\mathrm{B-H}}$ is the usual angular distribution of pairs at relativistic energies (Bethe-Heitler). The remaining terms of (10) represent either corrections to account for the recoil or pair production due to the magnetic moment. All these tend to zero as $M \to \infty$. Formulas (10) and (11) are correct for all angles $\vartheta^2 \ll M/p$, i.e., $q^2 \ll Mp$. Consequently, there exists a sufficiently broad region in which these formulas are of definite interest from the point of view of investigating the limits of validity of quantum electrodynamics and a study of the nucleon form factors

$$\mu^2 \lesssim q^2 \ll Mp, \quad \mu^2/p^2 \lesssim \vartheta^2 \ll M/p. \quad (13)$$

If small $q \sim \mu$ are considered it is possible to neglect recoil in (10). We then obtain the usual Bethe-Heitler formula for the angular distribution at higher energies, multiplied by $a^2(q^2)$:

$$d\sigma = a^2(q^2)\, d\sigma_{\mathrm{B-H}}. \quad (14)$$

For lower energies, $p \lesssim M$, formula (14), as already indicated, remains valid in the angle interval $\vartheta \ll 1$.

However, if we do not neglect recoil, then formula (10) holds for angles $\vartheta \sim \mu/p$, and the terms with f_1 can be neglected for angles $\vartheta \gg \mu/p$ in production of a pair of muons, and for all angles (13) in production of a pair of electrons, since then

$$|f_1| / |f_{\mathrm{B-H}}| \sim 2m^2/(p\vartheta)^2 \ll 1$$

($|f_1|/|f_{\mathrm{B-H}}| \ll 1$ also for $|p_+ - p_-| \ll p_\pm$). In all the above cases formula (10) becomes

$$d\sigma = \frac{\alpha^3}{(2\pi)^2} \frac{p_+ p_- dp_+}{\omega^3} do_+ do_- \cdot q^{-4} \Big\{ a^2(q^2) f_{\mathrm{B-H}} + 2a(q^2) b(q^2) \frac{q^2}{4M^2} f + b^2(q^2) \frac{q^2}{4M^2} [f_{\mathrm{B-H}} - f] \Big\}. \quad (15)$$

By determining the angular distribution at various energies it is possible, in principle, using formulas (10) to (15), to obtain the values of the form factors[2] or, if these are known, to check how substantially the electromagnetic interaction varies at high energies.

4. If we integrate the δ-function in (9) over the angle between the projections of the vectors $\mathbf{p}_+$ and $\mathbf{p}_-$ on a plane perpendicular to the direction $\boldsymbol{\omega}$ ($p_+\boldsymbol{\vartheta}_+$ and $p_-\boldsymbol{\vartheta}_-$), and then integrate over the angles ϑ_+ and ϑ_- to a certain $\vartheta_{\max}$ ($\vartheta_{\max} \ll M/p$) at constant p_+ and p_-, i.e., at constant q^2 (12), we obtain the energy distribution of the pair particles that travel in the cone $\vartheta_\pm \lesssim \vartheta_{\max}$.

The integration should be over the region bounded by inequalities

$$|q^2 - x^2 - y^2| \leqslant 2xy; \quad x \leqslant \Lambda_+, \quad y \leqslant \Lambda_-, \quad (16)$$

$$x = p_+\vartheta_+; \quad y = p_-\vartheta_-; \quad \Lambda_+ = p_+\vartheta_{\max}; \quad \Lambda_- = p_-\vartheta_{\max}.$$

Integration over region (16) can be performed rigorously only if $|p_+ - p_-| \geq \sqrt{q^2/\vartheta_{\max}}$ or approximately if $\Lambda_+^2 \gg q^2$ ($p_+ \leq p_-$ in the formulas that follow).

For the case $\Lambda_+^2 \gg q^2$ we get

$$d\sigma = \alpha^3 \frac{dp_+ dp_-}{\omega^3} \frac{2M}{q^4} \Big\{ a^2(q^2) \Big[\Phi_{\mathrm{B-H}} + \frac{q^2}{4M^2} \Phi_1 \Big] + 2a(q^2) b(q^2) \frac{q^2}{4M^2} \Phi + b^2(q^2) \frac{q^2}{4M^2} \Big[\Phi_{\mathrm{B-H}} + \frac{q^2}{4M^2} \Phi_1 - \Phi \Big] \Big\}; \quad (17)$$

$$\Phi_{\mathrm{B-H}} = (p_+^2 + p_-^2) \frac{q^2}{\sqrt{q^4 + 4q^2m^2}} L - 2p_+ p_- \Big[1 + \frac{2m^2}{\sqrt{q^4 + 4q^2m^2}} L \Big];$$

$$\Phi = -2\omega^2 \Big\{ 2 \ln \frac{p_+ \vartheta_{\max}}{m} + \frac{m^2}{\sqrt{q^4 + 4q^2m^2}} L \Big\}; \quad (18)$$

$$\Phi_1 = -\frac{(p_+ - p_-)^2}{p_+ p_-} \omega^2 \frac{m^2}{\sqrt{q^4 + 4q^2m^2}} L;$$

$$L = \ln \frac{[\sqrt{q^4 + 4q^2m^2} - q^2]\, m^2}{\sqrt{q^4 + 4q^2m^2}\, (q^2 + m^2) + q^2 (q^2 + 3m^2)}.$$

For $q^2 \ll M^2$ ($q^2 \lesssim \mu^2$) we neglect all the terms in (17) except for the first one, containing Φ_{B-H}. For $q^2 \gg m^2$ it is possible to neglect Φ_1 entirely, the remaining functions assume the simple form

$$\Phi_{B-H} = (p_+^2 + p_-^2)\, q^2 L \,/\, \sqrt{q^4 + 4q^2m^2} - 2p_+p_-;$$
$$\Phi = -4\omega^2 \ln (p_+\vartheta_{max}/m). \quad (19)$$

We leave L in its previous form, in view of the large coefficients in the terms with m^2.

Formula (17) can still be integrated over dp_+ for $p_+ + p_- = \text{const} \approx \omega$ ($q^2 = \text{const}$) and $2Mdp_- = -dq^2$. Then, for $q^2 \gg m^2$:

$$d\sigma = -\alpha^3 \frac{dq^2}{3q^4}\Big\{a^2(q^2)\Big[\frac{q^2}{\sqrt{q^4+4q^2m^2}}L - \frac{1}{2}\Big] - 3ab\frac{q^2}{M^2}\Big[\ln\frac{\omega\vartheta_{max}}{2m} - 1\Big] + b^2\frac{q^2}{4M^2}\Big[\frac{q^2L}{\sqrt{q^4+4q^2m^2}} + 6\ln\frac{\omega\vartheta_{max}}{2m} - \frac{13}{2}\Big]\Big\}. \quad (20)$$

Analogous results can also be obtained for bremsstrahlung. For this purpose it is necessary to replace in the matrix elements (2)

$$\mathbf{p}_+, \varepsilon_+ \to -\mathbf{p}_1, -\varepsilon_1, \quad \mathbf{p}_-, \varepsilon_- \to \mathbf{p}_2, \varepsilon_2; \quad \boldsymbol{\omega}, \omega \to -\boldsymbol{\omega}, -\omega$$

and in formulas (9) to take $d^3p_2d^3\omega/(2\pi)^6$ instead of the statistical factor $d^3p_+d^3p_-/(2\pi)^6$. The results can be obtained from formulas (10) to (15) by substituting

$$\frac{p_+p_-dp_+}{\omega^3}do_+do_- \to \frac{p_2}{p_1}\frac{d\omega}{\omega}do_{\mathbf{p}_2}do_\omega;$$

$$\vartheta_+ \to \vartheta; \quad \vartheta_- \to \vartheta - \vartheta_2, \quad p_+ \to -p_1$$

(ϑ_2 is the angle between $\mathbf{p}_2$ and $\mathbf{p}_1$).

In conclusion, the author expresses sincere gratitude to I. M. Shmushkevich for suggesting the topic and for valuable advice.

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THE POLARIZATION OF THE INTERNAL-CONVERSION ELECTRONS EMITTED AFTER β-DECAY

V. B. BERESTETSKII and A. P. RUDIK

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A discussion is given of the correlation of the polarization of internal-conversion electrons with the direction of emission of the electrons in the preceding β-decay. If one neglects the Coulomb field of the nucleus, then in the case of a magnetic multipole the polarization is longitudinal and does not depend on the energy. In the case of an electric multipole both longitudinal and transverse polarizations occur, with dependence on the energy.

1. Owing to the nonconservation of parity in β-decay the daughter nucleus is polarized in the direction of the emitted β-decay electron (the parent nucleus is supposed unpolarized, and the direction of emission of the neutrino is not observed). Therefore if an internal-conversion process occurs after the β-decay, the conversion electrons must possess a preferred polarization.* This effect can be used both in studying β-decay and also in studying the properties of nuclear levels, since (as will be shown below) the character of the polarization

*Our attention was called to the existence of such an effect by A. I. Alikhanov and V. A. Liubimov.