DETERMINATION OF THE ENERGY OF FAST PARTICLES FROM THE ANGULAR DISTRIBUTION OF THEIR REACTION PRODUCTS

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An analysis is given of the error in determining the energy E of fast colliding particles from the angular distribution of the produced particles. It turns out that to determine the energy by this method one should take account of the connection between the total number of observed star tracks and the energy E. The dependence predicted by the Landau theory is used in the present paper. An approximate distribution of E as a function of the angles and the number of tracks observed is obtained.

1. A method widely used at present for determining the energy of colliding particles is based on the analysis of the angular distribution of the particles produced in the collision process and on the simplest relations of relativistic kinematics. In the course of this analysis two assumptions are usually made: (1) the velocity of the secondaries is close to light velocity, and (2) in the center-of-mass system, the outgoing particles emerge on the average symmetrically with respect to the plane perpendicular to the line of motion.

The first assumption is well satisfied for sufficiently high energies of the primaries ($\gtrsim 10^{12}$ ev), with which we shall deal from now on (cf., for example, the direct measurements of energy of primaries in reference 1).

The second assumption, which is strictly true for nucleon-nucleon collisions, requires additional justification in the most commonly occurring case of collision of nucleons with heavy nuclei. By using the hydrodynamical theory of multiple production proposed by Landau,[2] one can evaluate the degree of asymmetry, if one invokes the additional assumption that the "tube" model[3,4] is valid. According to the results of Amai et al.[5] using this model, the asymmetry in the c.m. system is small, and we shall neglect it in what follows.

Under these assumptions, the energy E of the primary particles, expressed in units of Mc^2 (where M is the nucleon mass), is determined by the formulas

$$-\ln\gamma = \frac{1}{n}\sum_{i=1}^{n}\ln\tan\vartheta_i, \qquad (1)$$

$$E = 2\mu\gamma^2, \qquad (2)$$

where θ_i is the angle in the laboratory system between the direction of motion of the i-th secondary and the direction of the primary; n is the number of tracks of charged secondaries from which the value of E is computed, and μ is the mass of a "tube."

The following important question concerns the size of the possible error in this method. This question was posed in the paper of Castagnoli et al.[6] who estimated the errors taking into account only fluctuations in the value of the energy as a function of the angular distribution. However such an approach is inadequate since it does not take into account several factors, each of which can even change the errors by an order of magnitude. Among these factors are the effect of the energy spectrum of the primaries,* the distribution of "tube" lengths and the relation between the number of observed particles and the energy E. Neglect of the last factor means essentially that the calculations of Castagnoli et al. refer to an artificial case where the dispersion of the distribution of the total number of particles as a function of the energy E is infinite. Furthermore, computations carried out by us show that using the actual dependence can shift the most probable value of E by an order of magnitude from the value obtained when this relation is not taken into account.

In the present paper an attempt is made to treat the problem of the possible errors of the method of determining the energy E from the angular distribution of the secondaries.

The analysis will be based on the hydrodynam-

*The possible importance of the spectrum was first pointed out by N. L. Grigorov.

ical theory of multiple production, extended to the case of the collision of a nucleon with heavy nuclei by using the "tube" model.

2. Let us first start with the case where we know that nucleon-nucleon collisions occur.*

We consider the two stochastic variables,

$$-\eta = \frac{1}{N}\sum_{i=1}^{N} \ln \tan \vartheta_i$$

and the total number of charged particles, N.

According to Landau's theory, the expectation value of N is

$$MN = {}^4/_3 (E/2)^{1/4} = N_0.$$

The (conditional) probability density of the quantity η under the condition that the energy is E and the number of observed particles is n† is given by the relation (cf. reference 6)

$$p\left(\eta/E,n\right) = \left(\frac{2\pi\sigma^2(E)}{n}\right)^{-1/2} \exp\left\{-\left(\eta - \frac{1}{2}\ln\frac{E}{2}\right)\frac{n}{2\sigma^2(E)}\right\}, \quad (3)$$

where

$$\sigma^2(E) = {}^1/_2 \ln(E/2).$$

Omitting unimportant normalization factors from now on, we can write the probability $p(N\,|\,E)$ for fixed E in the form

$$p\left(\frac{N}{E}\right) \infty N_0^{-1/2} \exp\left\{-\frac{(N-N_0)^2}{\alpha N_0}\right\}, \quad (4)$$

where α is a constant which we set equal to unity‡ (cf. Appendix). In order to find the desired probability density $p(E\,|\,\eta, N)$ for the energy of the primary to be in the interval E to E + dE under the condition that definite values of η and N were observed, we must use Bayes' formula

$$p(E|\eta, N) \sim p(E)\,p(\eta, N|E) = p(E)\,p(\eta|N, E)\;p(N|E), \quad (5)$$

where $p(E)$ is the spectrum of incident particles, which was set equal to $E^{-2.7}$ in the numerical calculations.**

Figures 1 and 2 show examples of the variation of $p(E\,|\,\eta, N)$ for nucleon-nucleon collisions, for different values of N and of E_{eff}, which is defined from the relation $\eta = \frac{1}{2}\ln(E_{eff}/2)$.

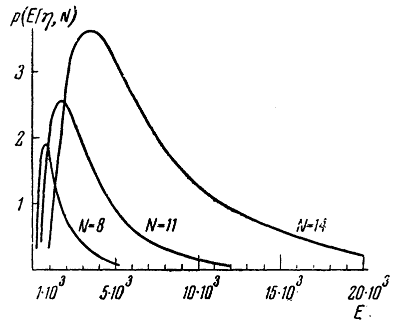

FIG. 1. η = 4.3; $E_{eff} = 10^4$.

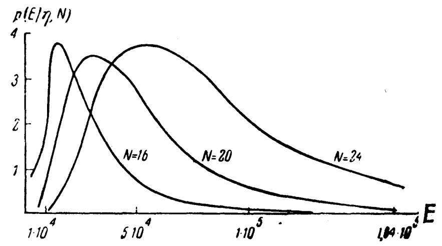

FIG. 2. η = 5.4; $E_{eff} = 10^5$.

3. Let us consider next the collision of a nucleon with a heavy nucleus. In using the tube model we must also take account of the distribution of paths in the nuclear matter and the change (compared to the case of nucleon-nucleon collision) which this distribution produces in the dependence of the mean number of particles N and the angular distribution on the value of E.

Expressing the tube length l in terms of number of nucleons, we have approximately[3,4]:

$$\overline{N} = N_0 l^{3/4}; \quad \sigma^2(E) = \frac{1}{2}\ln\left[E/2\left(\frac{l+1}{2}\right)^3\right].$$

Remembering that the probability $p(l)$ of collision with a tube of length l has the form $p(l) \sim l$, $1 \le l \le l_{max}$, and again using Bayes' formula, we get:

$$p\left(E\,|\,\eta, N, l\right) = \frac{p(\eta, N, l\,|\,E)\,p(E)}{\int_0^\infty p(\eta, N, l\,|\,E)\,p(E)\,dE} \quad (6)$$

$$= \frac{p(\eta\,|\,N, l, E),\ p(N\,|\,l, E)\,p(E)}{\int_0^\infty p(\eta\,|\,N, l, E)\,p(N\,|\,l, E)\,p(E)\,dE}$$

where

$$p(\eta\,|\,N, l, E) = \left(2\pi\frac{\sigma^2(E)}{N}\right)^{-1/2}\exp\left\{-\left(\eta - \ln\sqrt{\frac{E}{2l}}\right)^2\frac{N}{2\sigma^2(E)}\right\},$$

$$p(N\,|\,l, E) = (2\pi\overline{N})^{-1/2}\exp\{-(N-\overline{N})^2/2\overline{N}\}.$$

Since

$$p(E, l\,|\,\eta, N) = \frac{p(\eta\,|\,N, l, E)\,p(N\,|\,l, E)\,p(E)\,p(l)}{\int_0^\infty p(\eta\,|\,N, l, E)\,p(N\,|\,l, E)\,p(E)\,dE}, \quad (7)$$

*Such a case was realized experimentally by using an emulsion stack sandwiched with light material (brass).[7]

†As a special case, we may have n = N.

‡The final result depends very strongly on the numerical value of α. Within the framework of the Landau theory, we know only that its order of magnitude is unity.

**The shape of the spectrum in the high-energy region is not known; however, as auxiliary computations showed, a small change of the exponent (say to 2.5) practically does not change the final result.

we have finally:

$$p(E\,|\,\eta,\,N) = \int_0^{l_{\max}} p(E,\,l\,|\,\eta,\,N)\,dl \qquad (8)$$

$$\sim \int_0^{l_{\max}} \frac{p(\eta\,|\,N,\,l,\,E)\,p(N\,|\,l,\,E)\,p(E)\,l}{\int_0^{\infty} p(\eta\,|\,N,\,l,\,E)\,p(N\,|\,l,\,E)\,p(E)\,dE}\,dl.$$

Figure 3 shows the function $p(E\,|\,\eta, N)$ for $N = 14$; $\eta = 4.3$; $l_{\max} = 5$ (the energy $E_{\rm eff}$ determined from the angular distribution is equal to 10^4).

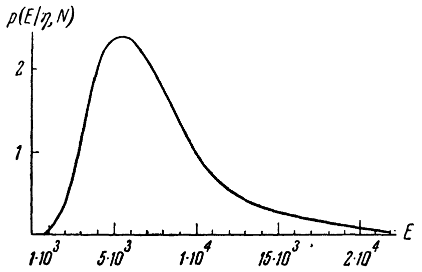

FIG. 3

4. On the basis of the computations and the graphs presented here we can draw the following conclusions.

1. In determining E it is necessary, in addition to its relation to the angular distribution (through the quantity η), to take account of its dependence on the total number N of observed tracks.

2. For given values of η and N, the distribution of possible values of E is characterized by considerable dispersion. The size of the dispersion as well as the location of the maximum of the distribution depend essentially on the relation between η and N.

3. The probability distribution is also affected by the parameters of the collision model (for example, by the value of the dispersion α).

In conclusion the authors express their indebtedness to G. B. Zhdanov for a fruitful discussion of the questions treated in this paper, and to Z. S. Maksimova and R. M. Povarova for carrying out the numberical computations.

APPENDIX

If the probability distribution $p(N_t)$ of the total number of particles is known, the probability that N of the particles are charged is given by

$$p(N) = \sum_{N_t = N}^{\infty} p(N\,|\,N_t)\,p(N_t),$$

where

$$p(N\,/\,N_t) = \frac{N_t!}{N!\,(N_t - N)!}\,q^N (1-q)^{N_t - N},$$

and q is the probability of production of a charged particle, which we may set equal to $2/3$. As usual, for sufficiently large N and N_t, these distributions can be represented by a Gaussian.

If the total number of particles is distributed according to the law

$$p\left(N_t\,|\,E\right) = (2\pi\sigma^2(E))^{-1/2} \exp\left\{-\frac{(N_t - \overline{N}_t)^2}{2\sigma^2(E)}\right\},$$

where $\sigma^2(E) = DN_t$ is the dispersion of N_t, and $DN_t = \alpha' N_t = \alpha' N_t$ (α' = const), the dispersion $D(N)$ of the quantity N is given by the relation

$$D(N) = \overline{N}\,[1 + (\alpha' - 1)\,q] = \alpha N.$$

According to reference 8, $\alpha' \sim 1$; then $\alpha \sim 1$. Unfortunately at present, within the framework of the hypotheses which are the basis of Landau's theory, one can only determine the order of magnitude of the lower limit on the value of α. Quantum fluctuations and peripheral collisions, which were not included in Ref. 8, should apparently increase the value of α.

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23