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THEORY OF THE ANISOTROPY OF THE WIDTH OF FERROMAGNETIC RESONANCE ABSORPTION LINE

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The dependence of the width of a resonance absorption line on the internal field is found from the Landau-Lifshitz equations. Specific examples of ferrites with single-axis and cubic symmetry are considered.

1. The width of a radio-frequency resonance absorption line, considering only spin-spin relaxation, can be described with the equation[1,2]

$$\dot{\mathbf{M}} = \gamma[\mathbf{M}\times\mathbf{H}] - \frac{1}{\tau}(\mathbf{M} - \chi_0\mathbf{H}), \qquad (1)$$

where χ_0 is the equilibrium susceptibility, and $M = \chi_0 H_M$. In the case of weak radio-frequency fields, when $|h| \ll H_M$, this equation leads to a Lorentzian line shape. When applied to ferromagnets, however, χ_0 is no longer constant. The magnitude of χ_0 can be deduced from Eq. (1), assuming a constant magnitude for the vector **M**. Then

$$\chi_0 = M^2/(\mathbf{M}\cdot\mathbf{H})$$

and

$$\dot{\mathbf{M}} = \gamma[\mathbf{M}\times\mathbf{H}] - \lambda M^{-2}[\mathbf{M}\times[\mathbf{M}\times\mathbf{H}]], \qquad (2)$$

where $\lambda = \chi_0/\tau$.

The Landau-Lifshitz equation (2), in which the magnitude of the magnetization vector **M** is constant, is conveniently expressed in polar coordinates, where the orientation of the vector **M** is given by its polar angle ϑ and its azimuthal angle φ. Introducing the radial, polar, and azimuthal components of the field, H_M, H_ϑ, and H_φ, Eq. (2) becomes

$$\dot{\vartheta} = -\gamma(H_\varphi - \alpha H_\vartheta), \quad \dot{\varphi}\sin\vartheta = \gamma(H_\vartheta + \alpha H_\varphi), \qquad (3)$$

where a dimensionless attenuation parameter $\nu = \lambda/\gamma M$ has been introduced.

Analogously, Eq. (1) becomes in spherical coordinates

$$\dot{M} = \frac{1}{\tau}(\chi_0 H_M - M), \qquad (4)$$

$$\dot{\vartheta} = \frac{\chi_0}{M\tau}H_\vartheta - \gamma H_\varphi, \quad \dot{\varphi}\sin\vartheta = \frac{\chi_0}{M\tau}H_\varphi + \gamma H_\vartheta. \qquad (5)$$

When M = const., with $\chi_0 = \alpha\gamma M\tau$, these equations reduce to Eqs. (3).

2. In a state of thermodynamic equilibrium the direction of the magnetization vector **M** in a ferromagnet coincides with the direction of the effective internal field $\mathbf{H}_M$, whose magnitude in turn can be found using the free energy F:

$$H_M = -F_M \equiv -\partial F/\partial M. \qquad (6)$$

The equilibrium orientation of the vector **M**, given by the angles ϑ_0 and φ_0, is found from the conditions

$$F_\vartheta \equiv \partial F/\partial\vartheta = 0, \quad F_\varphi \equiv \partial F/\partial\varphi = 0. \qquad (7)$$

If the free energy at this point is a minimum, the equilibrium is stable.

Now let us consider the non-equilibrium state, which arises when the effective field changes with time. In this event the orientation of the vector **M** will change, owing to the appearance of the components H_ϑ and H_φ in the non-equilibrium state. It is not difficult to show that

$$H_\vartheta = -F_\vartheta / M;\ H_\varphi = -F_\varphi / M \sin\vartheta_0. \tag{8}$$

If the deviation from equilibrium is small, i.e., for small values of

$$\Delta\vartheta(t) = \vartheta(t) - \vartheta_0;\ \Delta\varphi(t) = \varphi(t) - \varphi_0, \tag{9}$$

we can restrict ourselves to linear terms in the expansions of F_ϑ and F_φ,

$$F_\vartheta = F_{\vartheta\vartheta}\Delta\vartheta + F_{\vartheta\varphi}\Delta\varphi;\ F_\varphi = F_{\vartheta\varphi}\Delta\vartheta + F_{\varphi\varphi}\Delta\varphi, \tag{10}$$

where the second derivatives $F_{\vartheta\vartheta}$, $F_{\vartheta\varphi}$, and $F_{\varphi\varphi}$ are evaluated at equilibrium.

Now from Eq. (3), using Eqs. (8) to (10), we arrive at a system of linear equations, describing small damped oscillations of the magnetization vector about its equilibrium position:

$$\begin{aligned}\gamma^{-1}M\sin\vartheta_0\Delta\dot{\vartheta} &= \{F_{\varphi\vartheta} - \alpha F_{\vartheta\vartheta}\sin\vartheta_0\}\Delta\vartheta \\ &+ \{F_{\varphi\varphi} - \alpha F_{\vartheta\varphi}\sin\vartheta_0\}\Delta\varphi; \\ -\gamma^{-1}M\sin\vartheta_0\Delta\dot{\varphi} &= \{F_{\vartheta\vartheta} + \alpha F_{\vartheta\varphi}(\sin\vartheta_0)^{-1}\}\Delta\vartheta \\ &+ \{F_{\vartheta\varphi} + \alpha F_{\varphi\varphi}(\sin\vartheta_0)^{-1}\}\Delta\varphi.\end{aligned} \tag{11}$$

In the absence of attenuation terms ($\alpha = 0$), these reduce to the equations used successfully in previous work[3,4] to determine the spectrum of the natural frequencies of oscillation of the magnetization.

The system of homogeneous equations (11) possesses a periodic solution with frequency ω, if this frequency satisfies the secular equation

$$\omega^2 - i\omega\Delta\omega - \omega_0^2 = 0,$$

where ω_0 is the resonant frequency of oscillation,

$$\begin{aligned}\omega_0/\gamma &= (1+\alpha^2)^{1/2}H^* \\ &\equiv (1+\alpha^2)^{1/2}(M\sin\vartheta_0)^{-1}\{F_{\vartheta\vartheta}F_{\varphi\varphi} - F_{\vartheta\varphi}^2\}^{1/2},\end{aligned} \tag{12}$$

and $\Delta\omega$ is the breadth of the resonance absorption line.

$$\Delta\omega/\gamma \equiv \Delta H = (\alpha/M)\{F_{\vartheta\vartheta} + F_{\varphi\varphi}(\sin^2\vartheta_0)^{-1}\}. \tag{13}$$

To find the resonant frequency ω_0 and the line breadth ΔH, it is therefore necessary to have an exact expression for the free energy per unit volume of the crystal.

When the allowance for attenuation $\alpha \neq 0$, leads, owing to the smallness of the parameter α, to a negligible shift of the resonant frequency ω_0. This explains the good agreement between experiment and Artman's theoretical calculations[5] of the angular variation of the resonant field in ferrites with cubical symmetry of the crystal lattice.

In the case where the effective field H_M, given by Eq. (6), does not coincide at equilibrium with the direction of the external applied field, the width of an absorption line may become anisotropic relative to different crystallographic directions. In non-metallic ferromagnets this can depend on anisotropy of the shape of the sample as well as on the crystalline anisotropy.

3. As an example, let us consider the case of a uniaxial single-crystal ferrite. Let the polar axis of the coordinate system lie along the hexagonal axis of the crystal, which is the axis of least magnetization, and let the aximuth angle be measured from the direction $[10\bar{1}0]$. Then the angle-dependent part of the free energy per unit volume of an ellipsoidal specimen has the form

$$\begin{aligned}F &= K\sin^2\vartheta - MH(\sin\theta\cos\phi\sin\vartheta\cos\varphi \\ &+ \sin\theta\sin\phi\sin\vartheta\sin\varphi + \cos\theta\cos\vartheta) \\ &+ \tfrac{1}{2}M^2(N_x\sin^2\vartheta\cos^2\varphi + N_y\sin^2\vartheta\sin^2\varphi + N_z\cos^2\vartheta),\end{aligned} \tag{14}$$

where ϑ and φ are respectively the polar and azimuth angles, which determine the orientation of the external field. We limit ourselves here to anisotropy of the first order, and neglect magnetostriction effects.

The problem of determining the equilibrium orientation of the vector **M** for an arbitrary orientation of the external applied field (equivalent, in the absence of this field, to the problem of domain structure) is very complicated. The calculation can be performed comparatively simply only for the case where the external field lies in the base plane, i.e., where $\theta = \pi/2$.

In this event, if $\phi = \pi/2$, and

$$(N_y - N_x)M < H < \frac{2K}{M} + (N_y - N_z)M,$$

then

$$\begin{aligned}H^{*2} &= \left\{\left[\frac{2K}{M} + (N_y - N_z)M\right]^2 - H^2\right\}\frac{2K + (2N_y - N_x - N_z)M^2}{2K + (N_y - N_z)M^2}; \\ \frac{\Delta H}{\alpha} &= \frac{2K}{M} + (2N_y - N_x - N_z)M \\ &+ \frac{[2K + (N_y - N_z)M^2]^2 - H^2M^2}{M[2K + (N_y - N_z)M^2]}.\end{aligned} \tag{15}$$

If, on the other hand, $\phi = \pi/2$ and $H \geq 2K/M + (N_y - N_z)M$ then

$$\begin{aligned}H^{*2} &= H^2 - 2KH/M - (N_y - N_x)(HM - 2K) \\ &- M(N_y - N_z)[H - (N_y - N_x)M]; \\ \Delta H/\alpha &= 2H - 2K/M - (2N_y - N_x - N_z)M.\end{aligned} \tag{16}$$

Material	Direction of External field	H_{res}	ΔH	$\lvert\alpha\rvert$	$1/\tau$, sec^{-1}
Manganese ferrite with zinc impurities	[111]	3176	82	$12.5\cdot10^{-3}$	$7.0\cdot10^{8}$
	[110]	3225	70	$10.7\cdot10^{-3}$	$6.1\cdot10^{8}$
	[100]	3407	56	$8.4\cdot10^{-3}$	$5.1\cdot10^{8}$
Manganese ferrite	[111]	3214	66	$9.9\cdot10^{-3}$	$5.8\cdot10^{8}$
	[110]	3280	50	$7.5\cdot10^{-3}$	$4.3\cdot10^{8}$
	[100]	3442	46	$6.8\cdot10^{-3}$	$4.1\cdot10^{8}$

In the case where $\varphi = 0$ (field directed along the $[10\bar{1}0]$ axis) it is necessary only to interchange N_x and N_y everywhere. For an arbitrary ellipsoid, $N_x \neq N_y$, and in general the line width will be anisotropic with different orientations of the external field with respect to the base plane.

For a spheroidal sample ($N_x = N_y = N$, $N_z = 4\pi - 2N$), there is no anisotropy of the line width within the base plane, since for a given orientation of the field within this plane we have

$$\frac{\Delta H}{\alpha} = \frac{2\,[2K/M - (4\pi - 3N)\,M]^2 - H^2}{2K/M - (4\pi - 3N)\,M},$$

$$\text{for } H < 2K/M - (4\pi - 3N)\,M,$$

$$\frac{\Delta H}{\alpha} = 2H - \frac{2K}{M} + (4\pi - 3N)\,M, \tag{17}$$

$$\text{for } H \geqslant 2K/M - (4\pi - 3N)\,M.$$

If attenuation is neglected ($\alpha = 0$), we arrive at the expression for the resonant frequency given in reference 4.

4. As a second example, consider the case of a single crystal with cubic symmetry. This case is of especial interest, since the majority of single-crystal ferrites have this symmetry. For the coordinate axes x, y, and z, we select the axes [010], [001], and [100], respectively. The chosen polar axis is [100], which for the monocrystals with negative anisotropy of interest here is the axis of difficult magnetization. The azimuth angle is measured from the [010] axis. We limit ourselves henceforth to a spherical specimen ($N_x = N_y = N_z = 4\pi/3$), for which we assume that the applied external field H lies in the plane [110].

If, as before, we confine our attention only to anisotropy of the first order (which is possible except at excessively low temperatures) and neglect effects of magnetostriction, then the angle-dependent part of the free energy per unit volume has the form

$$F = \tfrac{1}{4}K\,[\sin^2 2\vartheta + \sin^4\vartheta\sin^2 2\varphi] \tag{18}$$

$$- MH\,[\cos\theta\cos\vartheta + \sin\theta\sin\vartheta\sin(\varphi + \pi/4)],$$

where θ is the polar angle that determines the orientation of **H** in the (110) plane.

It is not difficult to show that if the external applied field **H** is sufficiently large, and lies along one of the principal crystallographic directions belonging to the plane (110), then **M** is parallel to **H** at equilibrium.

Calculating the second derivatives of the free energy at equilibrium from Eq. (18), and substituting these in Eqs. (12) and (13), we find the resonant frequencies and line breadths for different directions of the external field.

If $\theta = 54°44'$, i.e., if the field **H** lies along the [111] axis, which, for a crystal with negative anisotropy, is the axis of easy magnetization, then

$$\Delta H = 2\alpha H\,(1 + 4\,|K|/3MH). \tag{19}$$

If $\theta = \pi/2$, i.e., the field **H** lies along the [110] axis, then

$$\Delta H = 2\alpha H\,(1 + |K|/2MH) \tag{20}$$

Finally, if $\theta = 0$, where the field lies along the axis [100] of difficult magnetization, then

$$\Delta H = 2\alpha H\,(1 - |K|/MH) \tag{21}$$

The corresponding formulas for the resonant frequencies, neglecting relaxation, have been computed by Artman.[5] We notice that for a single-crystal ferrite with positive anisotropy the signs in front of $|K|/MH$ in the expressions (19) to (21) are reversed.

It follows from the above definitions that the spin-spin relaxation time is related to α by the expression

$$\frac{1}{\tau} = \gamma M^{-2}\,(\mathbf{M}\cdot\mathbf{H})\,\alpha, \tag{22}$$

in which we can set $\mathbf{M}\cdot\mathbf{H} = MH$.

The angle dependence of the width of a resonance absorption line in a ferrite of cubic symmetry has been studied previously.[6,7] In the first of these references, a manganese ferrite with a small zinc impurity was investigated at 9100 Mcs. The effective anisotropic field K/M at room temperature amounted to -71 ± 1 oersteds. In the second reference a manganese ferrite of composition $Mn_{0.98}Fe_{1.86}O_4$ was studied at 9300 Mcs. The effective anisotropic field at room temperature was -79 ± 3 oersteds.

The experimental results for both ferrites and the calculated values of α and $1/\tau$ are given in the table.

It is evident from the table that α and $1/\tau$ are decreasing functions of the applied external field, with $1/\tau$ decreasing more slowly.

It is also not difficult to verify that, as a consequence of the smallness of the term $|K|/MH$, the variation of τ with different directions of the applied external field cannot be explained solely by the presence of the crystalline anisotropy of the sample. Calculation of second-order anisotropy cannot substantially alter this situation. Thus the relaxation time τ in Eq. (1), turns out to be a slowly rising function of the field strength when M = const. Therefore the Landau-Lifshitz equation (1) agrees best with experiment if the parameter λ is determined from the relation

$$\lambda = M^2/\tau(\mathbf{M}\cdot\mathbf{H}) \qquad (23)$$

Further experimental investigation of the angular variation of the breadth of resonance absorption lines in ferrites of different compositions at various microwave frequencies and temperatures is of interest for a more detailed explanation of the dependence of the relaxation time τ on the intensity of the applied magnetic field.

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ON THE $\pi \to e + \nu + \gamma$ DECAY

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The $\pi \to e + \nu + \gamma$ decay is investigated for vector and axial-vector interactions. An exact relation between the probability of the vector-type part of the decay $\pi \to e + \nu + \gamma$ and the probability of the decay $\pi^0 \to 2\gamma$ can be established by assuming that the direct interaction between π mesons and the electron-neutrino field, suggested by Gell-Mann and Feynman, exists in the vector-type theory. It is found that the axial-vector-type decay accounts for the main part of the total probability for the $\pi \to e + \nu + \gamma$ decay. The ratio of the total probability for the $\pi \to e + \nu + \gamma$ decay to the probability for the $\pi \to \mu + \nu$ decay is of order 5×10^{-6}. Expressions for the angular and energy distributions of the electrons and quanta are obtained.

GELL-MANN and Feynman[1] suggested a scheme for a universal weak interaction of the nucleons with the electron-neutrino field is of the vector- and axial-vector-type. The interaction Hamiltonian has the form

$$H_1 = (\bar{\psi}\gamma_\mu (G_V + \gamma_5 G_A)\tau^+\psi) J_\mu + \text{Herm. conj.}, \qquad (1)$$
$$J_\mu = (\bar{\psi}_e \gamma_\mu {}^1/_2 (1 + \gamma_5)\psi_\nu),$$

where

$$\psi = \begin{pmatrix}\psi_P \\ \psi_N\end{pmatrix}, \quad \tau^+ = \sqrt{2}\begin{pmatrix}01\\00\end{pmatrix} = \frac{1}{\sqrt{2}}(\tau_x + i\tau_y),$$
$$\gamma_5 = -\begin{pmatrix}01\\10\end{pmatrix}, \quad \gamma_\mu = \{\beta, \beta\alpha\},$$

and G_V and G_A are coupling constants. Gell-Mann and Feynman (see also reference 2) assume that there exists a direct interaction of the π mesons with the electron-neutrino field, which is described by the Hamiltonian