It is evident from the table that α and $1/\tau$ are decreasing functions of the applied external field, with $1/\tau$ decreasing more slowly.

It is also not difficult to verify that, as a consequence of the smallness of the term $|K|/MH$, the variation of τ with different directions of the applied external field cannot be explained solely by the presence of the crystalline anisotropy of the sample. Calculation of second-order anisotropy cannot substantially alter this situation. Thus the relaxation time τ in Eq. (1), turns out to be a slowly rising function of the field strength when M = const. Therefore the Landau-Lifshitz equation (1) agrees best with experiment if the parameter λ is determined from the relation

$$\lambda = M^2/\tau(\mathbf{M}\cdot\mathbf{H}) \tag{23}$$

Further experimental investigation of the angular variation of the breadth of resonance absorption lines in ferrites of different compositions at various microwave frequencies and temperatures is of interest for a more detailed explanation of the dependence of the relaxation time τ on the intensity of the applied magnetic field.

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ON THE $\pi \to e + \nu + \gamma$ *DECAY*

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The $\pi \to e + \nu + \gamma$ decay is investigated for vector and axial-vector interactions. An exact relation between the probability of the vector-type part of the decay $\pi \to e + \nu + \gamma$ and the probability of the decay $\pi^0 \to 2\gamma$ can be established by assuming that the direct interaction between π mesons and the electron-neutrino field, suggested by Gell-Mann and Feynman, exists in the vector-type theory. It is found that the axial-vector-type decay accounts for the main part of the total probability for the $\pi \to e + \nu + \gamma$ decay. The ratio of the total probability for the $\pi \to e + \nu + \gamma$ decay to the probability for the $\pi \to \mu + \nu$ decay is of order 5×10^{-6}. Expressions for the angular and energy distributions of the electrons and quanta are obtained.

GELL-MANN and Feynman[1] suggested a scheme for a universal weak interaction of the nucleons with the electron-neutrino field is of the vector- and axial-vector-type. The interaction Hamiltonian has the form

$$H_1 = (\bar{\psi}\gamma_\mu(G_V + \gamma_5 G_A)\tau^+\psi)J_\mu + \text{Herm. conj.}, \tag{1}$$
$$J_\mu = (\bar{\psi}_e\gamma_\mu{}^1/_2(1+\gamma_5)\psi_\nu),$$

where

$$\psi = \begin{pmatrix}\psi_P\\ \psi_N\end{pmatrix}, \quad \tau^+ = \sqrt{2}\begin{pmatrix}01\\00\end{pmatrix} = \frac{1}{\sqrt{2}}(\tau_x + i\tau_y),$$
$$\gamma_5 = -\begin{pmatrix}01\\10\end{pmatrix}, \quad \gamma_\mu = \{\beta, \beta\alpha\},$$

and G_V and G_A are coupling constants. Gell-Mann and Feynman (see also reference 2) assume that there exists a direct interaction of the π mesons with the electron-neutrino field, which is described by the Hamiltonian

$$H_2 = 2iG_V [\Phi^+ T^+ \nabla_\mu \Phi - (\nabla_\mu \Phi^+) T^+ \Phi] J_\mu + \text{Herm. conj.}, \quad (2)$$

where $\Phi = \{\varphi, \varphi^0, \varphi^+\}$ are the wave functions of the π mesons, and

$$T^+ = \frac{1}{\sqrt{2}}(T_x + iT_y) = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

is the isotopic spin operator for the π mesons. The vector part of the interaction (1) together with the interaction (2) is completely analogous to the interaction of π mesons with the electromagnetic field

$$H_{el} = e\left\{\bar{\psi}\gamma_\mu\left(\frac{1}{2}\tau_z + \frac{1}{2}\right)\psi + i[\Phi^+ T_z \nabla_\mu \Phi - (\nabla_\mu \Phi^+) T_z \Phi]\right\} A_\mu. \quad (3)$$

This allows us to establish an exact relation between the probability of the vector-type part of the $\pi^\pm \to e^\pm + \nu + \gamma$ decay and the probability of the decay of a neutral π meson into two γ quanta. Since the axial-vector-type part of the $\pi^\pm \to e^\pm + \nu + \gamma$ decay is mainly determined by terms which are simply related to the $\pi^\pm \to e^\pm + \nu$ decay, it is possible to estimate the probabilities for the $\pi \to e + \nu + \gamma$ decays in the A- and V-type theory more correctly than was done earlier by Treiman and Wyld.[3]

We write the matrix elements for the $\pi^0 \to 2\gamma$ decay (M_γ) and for the vector-type part of the $\pi \to e + \nu + \gamma$ decay (M_V) decay in the form

$$M_\gamma(k_1, k_2) = e^2 \varphi^0(q) A_\mu(k_1) A_\nu(k_2) U^\gamma_{\mu\nu}(k_1, k_2),$$
$$M_V(k_1, k_2) = eG_V \varphi^+(q) A_\mu(k_1) J_\nu(k_2) U^V_{\mu\nu}(k_1, k_2). \quad (4)$$

Here, for the $\pi^0 \to 2\gamma$ decay, k_1 and k_2 are the 4-momenta of the quanta, $\varphi^0(q)$ is the wave function of the neutral meson, and $q = k_1 + k_2$ is its momentum; for the $\pi \to e + \nu + \gamma$ decay, k_1 is the momentum of the quantum, $k_2 = p_e + p_\nu$ is the total momentum of electron-neutrino system, and $\varphi(q)$ is the wave function of the charged meson.

We shall show that, with the strong interactions treated exactly (but neglecting the radiative corrections connected with the electromagnetic and weak interactions),

$$U^\gamma_{\mu\nu}(k_1, k_2) = U^V_{\mu\nu}(k_1, k_2). \quad (5)$$

We shall assume that the momenta of the emitted γ quanta and the leptons ($k \sim \mu/2$) are small compared with the average momenta Λ of the

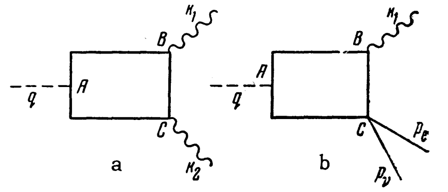

FIG. 1. (a) $\pi^0 \to 2\gamma$ decay; ---- meson line, ∿∿∿ photon line; (b) $\pi \to e + \nu + \gamma$ decay; —— electron and neutrino lines; p_e and p_ν are the 4-momenta of the electron and the neutrino, $p_e + p_\nu = k_2$.

virtual particles participating in the decay.*

We restrict ourselves first to the case when only π mesons and nucleons participate in the strong interactions. We consider an arbitrary Feynman diagram (Fig. 1) for the processes which we are concerned with. The quantities $U^\gamma_{\mu\nu}(k_1, k_2)$ and $U^V_{\mu\nu}(k_1, k_2)$ for this diagram are products of two factors: (1) the integral over the momenta of the virtual particles and the sum over the spin variables, and (2) the sum over the isotopic spin variables. We see from the comparison of the interaction Hamiltonians (1) and (2) with (3) that the spin-momentum part of the diagram is the same for $U^\gamma_{\mu\nu}$ and $U^V_{\mu\nu}$. Thus $U^\gamma_{\mu\nu}$ and $U^V_{\mu\nu}$ have the form

$$U^{\gamma,V}_{\mu\nu}(k_1, k_2) = J_{\mu\nu}(k_1, k_2) F^{\gamma,V}_{ABC};$$
$$F^{\gamma,V}_{ABC} = \mathrm{Sp}\{\tau_A \ldots \tau_B \ldots \tau_C \ldots\}, \quad (6)$$

where τ_A, τ_B, τ_C are isotopic spin operators; index A denotes the initial π meson, B and C denote the two γ quanta for the π^0 decay, and the γ quantum and the leptonic current in the decay $\pi \to e + \nu + \gamma$. In the case of the π^0 decay, τ_A is to be substituted by τ_z, and τ_B and τ_C are to be substituted by $(\tau_z + 1)/2$ for γ quanta emitted by nucleons, and by T_z for γ quanta emitted by mesons. In the case of the decay $\pi^- \to e^- + \nu + \gamma$, τ_A is replaced by $\tau = (\tau_x - i\tau_y)/\sqrt{2}$, τ_B by $(\tau_z + 1)/2$ (or T_z), and τ_C by τ^+ (or T^+).

The quantity $J_{\mu\nu}(k_1, k_2)$ should be a pseudotensor of second rank, i.e.,

$$J_{\mu\nu}(k_1, k_2) = A\varepsilon_{\mu\nu\lambda\sigma} k_{1\lambda} k_{2\sigma}, \quad (7)$$

where A is independent of k_1 and k_2 with an accuracy up to terms of order $(k/\Lambda)^2$, which we discard. Together with the diagram of Fig. 1 we

*It follows from the experimental lifetime of the π^0 meson that Λ cannot be essentially smaller than M, the mass of the nucleon, for the condition $\Lambda \ll M$ would introduce an additional factor $(\Lambda/M)^4$ into the estimate for the lifetime of the π^0 meson, in disagreement with experiment. (This conclusion is not derived from perturbation theory, since the succeeding terms in the expansion with respect to g^2 would be of order $g^2(\Lambda/M)^2 \lesssim 1$.)

now consider a diagram in which the vertices B and C are interchanged. It is easily seen that due to the symmetry of (7) under the interchange $\mu \to \nu$, $k_1 \to k_2$, the sum of these diagrams is equal to

$$U^{\gamma,V}_{\mu\nu}(k_1, k_2) + U^{\gamma,V}_{\nu\mu}(k_2, k_1) = J_{\mu\nu}(k_1, k_2)\,[F^{\gamma,V}_{ABC} + F^{\gamma,V}_{ACB}]. \qquad (8)$$

We investigate the vector structure of the expression $F_{ABC} + F_{ACB}$ in the isotopic spin space. We note here that the interaction with the electromagnetic field can formally be described (neglecting the radiative corrections with respect to e^2) by a sum of the interactions of the nucleons and the π mesons with an isotopic spin vector α_i^γ (components $\alpha_z^\gamma = 1/2$, $\alpha_x^\gamma = \alpha_y^\gamma = 0$), and an isotopic spin scalar $\beta = 1/2$. Then the Hamiltonian for the interaction with the electromagnetic field is written in this way:

$$H_{el} = e\{\bar{\psi}\gamma_\mu\tau_i\psi + i\,[\Phi^+ T_i \nabla_\mu \Phi - (\nabla_\mu \Phi^+)\, T_i \Phi]\}\, A_\mu \alpha_i^\gamma + e\bar{\psi}\gamma_\mu\psi A_\mu \beta \qquad (3')$$

which is formally invariant in isotopic spin space. The expression F_{ABC} also takes a form invariant in isotopic spin space:

$$F^\gamma_{ABC} = F_{ikl}\varphi_i^0\alpha_k^\gamma\alpha_l^\gamma + F'_{ik}\varphi_i^0\alpha_k^\gamma\beta + F''_{il}\varphi_i^0\alpha_l^\gamma\beta + F'''_i\varphi_i^0\beta^2, \qquad (9)$$

where φ_i^0 is the isotopic spin vector of the π^0 meson ($\varphi_1^0 = \varphi_2^0 = 0$, $\varphi_3^0 = 1$). The term F'_{ik} corresponds to the case when the quanta emitted at vertex B refer to α, and those emitted at C refer to β. the term F''_{il} corresponds to the opposite case. In analogy, the interaction with the leptonic field can be written in the form of an interaction with the isotopic spin vector α_i^V (components $\alpha_z^V = 0$, $\alpha_x^V = 1/\sqrt{2}$, $\alpha_y^V = i/\sqrt{2}$). Here the expressions for F^V_{ABC} and F^V_{ACB} take the form (φ_i^+ is the isotopic spin vector of the π^- meson; $\varphi_1^+ = 1/\sqrt{2}$, $\varphi_2^+ = -i/\sqrt{2}$, $\varphi_3^+ = 0$):

$$F^V_{ABC} = F_{ikl}\varphi_i\alpha_k^\gamma\alpha_l^V + F''_{il}\varphi_i\beta\alpha_l^V, \qquad (9')$$
$$F^V_{ACB} = F_{ikl}\varphi_i\alpha_k^V\alpha_l^\gamma + F'_{ik}\varphi_i\alpha_k^V\beta,$$

since the isotopic scalar β participates only in the emission of quanta (i.e. at the vertex B in the diagram of Fig. 1). It is easily seen that those terms in the expression $F^{\gamma,V}_{ABC} + F^{\gamma,V}_{ACB}$ in which both interactions in the vertices B and C involve the isotopic spin vector, reduce to zero. Indeed, in this case the quantity F_{ikl} in (9) and (9′) should be a pseudotensor of third rank in isotopic spin space. However, after the summation of the isotopic spin variables of the mesons and after taking the spur of the isotopic spin variables of the nucleons, the only expression left to represent $F_{ikl} + F_{ilk}$ is the unit tensor ϵ_{ikl}. But ϵ_{ikl} is antisymmetric in the indices k, l, while $F_{ikl} + F_{ilk}$ is symmetric in the same indices. Hence $F_{ikl} + F_{ilk} = 0$ (so that, for example, the γ quanta in the π^0 decay cannot be both emitted by virtual π mesons).

In the same way, all terms F_{ABC} reduce to zero in which both γ quantum emissions in the vertices B and C involve the isotopic spin scalar. Indeed, it is impossible to construct an isotopic spin vector F_i^γ, since there is no distinguished direction in isotopic spin space in expression (6). The only non-zero terms in (9) and (9′) are the terms F'_{ik} and F''_{il}. The quantities F'_{ik} and F''_{ik} are tensors of second rank in isotopic spin space, and they can therefore be expressed in terms of the unit tensor δ_{ik}, i.e. $F'_{ik} = C\delta_{ik}$, $F''_{ik} = D\delta_{ik}$, where C and D are certain numerical constants. Substituting these expressions for F'_{ik} and F''_{ik} in (9) and (9′), we find without difficulty that $F_{ABC} + F_{ACB} = F^V_{ABC} + F^V_{ACB} = 1/2\,(C + D)$, as can also be seen from (5).

The above proof is easily generalized for the case when hyperons and K mesons also participate in the strong interactions. In the interaction Hamiltonian describing the interaction of the hyperons and K mesons with the electromagnetic field we then have to replace the matrices $(\tau_z + 1)/2$ and T_z by $T_z + 1/2 + S/2$ and $T_z + S/2$, respectively (S is the strangeness), We also assume that the Hamiltonian for the interaction of the hyperons and K mesons with the leptonic field has the form (1), (2), where τ^+ and T^+ are replaced by the corresponding projection of the isotopic spin of the strange particle.

With the help of (4), (5), and (7) we can write the matrix elements M_γ and M_V in the form

$$M_\gamma(k_1, k_2) = ae^2\varphi^0(q)\,A_\mu(k_1)\,A_\nu(k_2)\,\varepsilon_{\mu\nu\lambda\sigma}k_{1\lambda}k_{2\sigma};$$
$$M_V(k_1, k_2) = aeG_V\varphi^+(q)\,A_\mu(k_1)\,J_\nu(k_2)\,\varepsilon_{\mu\nu\lambda\sigma}k_{1\lambda}k_{2\sigma} \qquad (10)$$

The constant a is to be determined from the experimental lifetime of the π^0 meson.

It is convenient to split up the matrix element $M_A(k_1, k_2)$ (corresponding to the axial-vector-type part of the $\pi \to e + \nu + \gamma$ decay) into two parts. The first part contains the diagram of Fig. 2, in which a γ quantum is emitted by an electron, as well as terms of zeroth order in k_1,

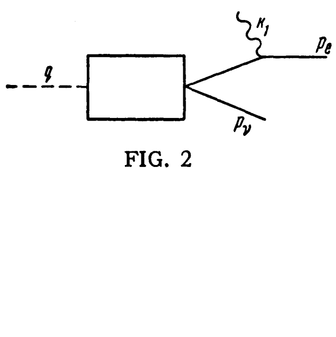

FIG. 2

k_2 from the expancion of diagram Fig. 1 in powers of k. The second part contains the terms of second order in k_1, k_2 in the expansion of diagram Fig. 1.

As is known,[3-5] it is not necessary to calculate the closed loops for the determination of the first part of $M_A(k_1, k_2)$. It can be obtained by introducing the phenomenological interaction

$$H_{\pi e\nu} = g_{\pi e\nu} \frac{\partial \varphi^+}{\partial x_\mu} \left(\bar{\psi}_e \gamma_\mu \frac{1}{2} (1 + \gamma_5) \psi_\nu\right) + \text{Herm. conj.,} \quad (11)$$

for the description of the decay $\pi \to e + \nu$, and by observing that on the strength of the gauge invariance $\partial\varphi^+/\partial x_\mu$ should be changed to $\partial\varphi^+/\partial x_\mu + ieA_\mu\varphi^+$. (The expression $ieg_{\pi e\nu}A_\mu\varphi^+J_\mu$ corresponds to the terms of zero order in k_1, k_2 in the expansion of diagram Fig. 1). Then[3-5] the Hamiltonian (11) is equivalent to the Hamiltonian:

$$H_{\pi e\nu} = m_e g_{\pi e\nu} \varphi^+ (\bar{\psi}_e \, {}^1\!/_2 (1 + \gamma_5) \psi_\nu) + \text{Herm. conj.,} \quad (12)$$

where the emission of the γ quantum is effected only through the bremsstrahlung of the electron. The radiative decay of the π meson with the Hamiltonian (12) was considered earlier.[4] Therefore we have to concern ourselves only with the part of $M_A(k_1, k_2)$ containing the terms quadratic in k_1, k_2. It is easily seen that the general expression for this matrix element must have the form:

$$M_A(k_1, k_2) = ibeG_A \varphi^+(q) \left[A_\mu(k_1) k_{1\nu} - A_\nu(k_1) k_{1\mu}\right] J_\mu(k_2) k_{2\nu}. \quad (13)$$

Strictly speaking, we cannot determine the constant b by theoretical considerations.* However, since it is determined in perturbation theory by integrals of the same type as those determining the constant a in (10), we may assume that b is of the same order as a. Direct calculation shows that the matrix element of Hamiltonian (12) does not interfere with the matrix elements (10) and (13), if we neglect higher powers of the ratio m_e/μ. It is therefore convenient to first compute $dW^{(1)}$, the probability of the transition $\pi \to e + \nu + \gamma$ on account of the terms $M_V + M_A$.

After the summation over the polarizations of the emitted quantum and over the spins of the electron, we obtain for the differential probability of the decay $dW^{(1)}$ the following expression:

$$dW^{(1)} = \frac{1}{\pi^2} \frac{G_V^2}{e^2} W_{\pi^0} \frac{k}{\mu^2} \left\{ (1 + \lambda^2) \left(1 - \frac{(\mathbf{k}\cdot\mathbf{n})(\mathbf{k}\cdot\boldsymbol{\nu})}{k^2}\right) + 2\lambda \frac{\mathbf{k}\cdot\boldsymbol{\nu} - \mathbf{k}\cdot\mathbf{n}}{k} \right\} \delta(E_e + E_\nu + k - \mu) \frac{d\mathbf{k}\, d\mathbf{p}_e}{(2\pi)^3}. \quad (14)$$

Here $\mathbf{p}_e = \mathbf{n}p_e$ and $\mathbf{k}$ are the momenta of the electron and the γ quantum, respectively, ν is a unit vector in the direction of the neutrino momentum, W_{π^0} is the probability for the decay of the π^0 meson into two γ quanta, μ is the mass of the π meson, $\lambda = bG_A/aG_V$. W_{π^0} is connected with the constant a in (10) through the relation:

$$W_{\pi^0} = \pi^2 \mu^3 a^2.$$

By integration we may obtain various partial distributions from (14). Thus the energy distribution of the electrons has the form $[p_e = (\mu/2)y]$

$$dW_e^{(1)} = \frac{1}{2^6\pi^3} \frac{G_V^2}{e^2} W_{\pi^0} \mu^4 y^2 \left\{ (1 + \lambda^2) \left[(1 - y)^2 + \frac{1}{6} y^2\right] + \lambda (1 - y) \left(1 - 2y + \frac{1}{2} y^2\right) \right\} dy, \quad (15)$$

and the energy distribution of the γ quanta $(k = (\mu/2)y)$ is:

$$dW_\gamma^{(1)} = \frac{1}{2^6\pi^3} \frac{G_V}{e^2} W_{\pi^0} \mu^4 \frac{4}{3} (1 + \lambda^2) x^3 (1 - x)\, dx. \quad (16)$$

The integral probability $W^{(1)}$ is equal to

$$W^{(1)} = \frac{1}{960\pi^3} \frac{G_V^2 \mu^4}{e^2} W_{\pi^0} (1 + \lambda^2). \quad (17)$$

We estimate the numerical value of $W^{(1)}$. The constant G_V^2 was determined in experiments[6] on the β decay of O^{14}: $G_V^2\mu^4 = 0.51 \times 10^{-13}$. We estimate the lifetime of the π^0 meson with Orear[7] to be $W_{\pi^0} \gtrsim 0.5 \times 10^{16}\ \text{sec}^{-1}$, and we find $W^{(1)} = 2.4\ \text{sec}^{-1}$ $(\lambda = 1)$. Thus the ratio of $W^{(1)}$ to the total probability for the $\pi \to \mu + \nu$ decay is

$$W^{(1)} / W_{\mu+\nu} \approx 6 \cdot 10^{-8}. \quad (18)$$

This estimate of the number of the $\pi \to e + \nu + \gamma$ decays is very close to the estimate of Treiman and Wyld (reference 3).*

Let us now consider $W^{(2)}$, which is the $\pi \to e + \nu + \gamma$ decay probability due to the terms in which a γ quantum is emitted by the electron. A corresponding calculation was made in reference 4† for the $\pi \to \mu + \nu + \gamma$ case. The results can be easily translated for the $\pi \to e + \nu + \gamma$ decay. We obtain for the differential probability the expression:

*The quantity b has the same phase as a, so that we may take them to be real. This follows from the invariance of the strong and electromagnetic interactions under time reversal (or charge conjugation). The proof may be carried through with the help of the phenomenological Hamiltonian corresponding to the matrix elements (10) and (13).

*An analogous calculation for the $\pi \to \mu + \nu + \gamma$ decay gave the negligibly small value $W^V_{\mu+\nu+\gamma} \sim 5 \times 10^{-10}\ W_{\mu+\nu}$ for the probability of a $\pi \to \mu + \nu + \gamma$ decay through the V variant.

†We note that formula (6) in reference 4, which gives the total probability for the $\pi \to \mu + \nu$ decay, contains the incorrect factor ½. Accordingly, the values in the tables of this reference should be multiplied by ½.

$$dW^{(2)} = \frac{1}{8\pi^3} \cdot \frac{e^2}{\mu^2} W_{e+\nu} \frac{1}{E_e E_\nu k} \Big\{ 2(E_e k - \mathbf{k}\cdot\mathbf{p}_e)(k\mu - kE_e + \mathbf{k}\cdot\mathbf{p}_e) + \mu^2 \Big[p_e^2 - \frac{(\mathbf{p}_e\cdot\mathbf{k})^2}{k^2} \Big] \Big\} \frac{d\mathbf{p}_e d\mathbf{k}}{(E_e k - \mathbf{k}\cdot\mathbf{p}_e)^2} \delta(E_e + E_\nu + k - \mu), \quad (19)$$

where $W_{e+\nu}$ is the probability for the $\pi \to e + \nu$ decay, equal to $1.3 \times 10^{-4} W_{\mu+\nu}$ in the Gell-Mann-Feynman scheme. The energy distribution of the electrons has the form

$$dW_e^{(2)} = \frac{1}{2\pi} e^2 W_{e+\nu} (1-y) \Big\{ \ln \Big[\Big(\frac{\mu}{m}\Big)^2 \frac{y^2}{1-y} \Big] + \frac{2y}{(1-y)^2} \Big[-2 + \ln \Big(\frac{\mu}{m} y\Big)^2 \Big] \Big\} dy. \quad (20)$$

The electron spectrum diverges near the upper limit ($y \to 1$), which corresponds to the emission of long-wave quanta. Therefore the total probability for a $\pi \to e + \nu + \gamma$ decay in which the electron is emitted with an energy below a certain maximum value y_{max} increases logarithmically with y_{max}. Because of this, and also because of the presence of the large term $\ln(\mu/m)^2$ in (20), the probability for the radiative decay with the quantum emitted by a virtual electron in comparatively large, and substantially surpasses the probability $W^{(1)}$.

It follows that we need consider only the term $W^{(2)}$ in the computation of the total probability for the $\pi \to e + \nu + \gamma$ decay. The results of the computation of this probability as a function of the maximal electron energy are tabulated in the table. (We use $W_{e+\nu} = 1.3 \times 10^{-4} W_{\mu+\nu}$).

y_{max}	0.3	0.5	0.7	0.9
$\frac{10^6 W_{e+\nu+\gamma}}{W_{\mu+\nu}}$	0.36	0.85	1.8	3.3

The table shows that the probability for the $\pi \to e + \nu + \gamma$ decay is greater by almost two orders of magnitude than the probability $W^{(1)}$ for the decay through the V variant.

Besides the energy distribution of the electrons, we are also interested in the angular distribution of the quanta over the angle θ between the directions of emission of the electron and of the quantum. This distribution is obtained from (19) by integrating over the energies of the electron and the quantum. For $\theta \gg m_e/\mu$ it is equal to

$$dW^{(2)}(\theta) = \frac{e^2}{4\pi} W_{e+\nu} \frac{\sin\theta d\theta}{\alpha^3} \times \Big\{ \alpha + (1-\alpha)\ln(1-\alpha) + 2\alpha^2(1-\alpha)\Big(\ln\frac{1}{1-y_{max}} - 1\Big)\Big\}, \quad (21)$$

where $\alpha = \sin^2\theta/2$ and y_{max} is the maximal energy (measured in $\mu/2$) up to which the electron spectrum is integrated. Expression (21) behaves like $d\theta/\theta$ for small θ. However, the divergence at $\theta \to 0$ in (21) is only seeming. It derives from the neglect of quantities $\sim m/\mu$ in the calculation of (21). For $\theta \sim m/\mu$ we really have an expression of order $1/(\theta^2 + 4(m^2/\mu^2))$ instead of $1/\theta^2$. The presence of this factor is perfectly reasonable: it is related to the fact that in the relativistic case the bremsstrahlung of the electron is concentrated in the narrow solid angle $\sim m/E$ around the direction of the momentum of the electron. This also accounts for the appearance of the large logarithmic term $\ln(\mu/m)^2$ in (20). Thus we have the most favorable conditions for the detection of the $\pi \to e + \nu + \gamma$ decay through the vector and axial-vector variant if the registering electron and quantum move in the same direction.

Of special interest is the experimental investigation of the vector-type part of the $\pi \to e + \nu + \gamma$ decay, since this would give the possibility to verify the hypothesis of Gell-Mann and Feynman about the existence of a direct interaction of the π mesons with the electron-neutrino field, as well as the theorem about the connection between the $\pi^0 \to 2\gamma$ decay and the vector-type part of the $\pi \to e + \nu + \gamma$ decay. It is possible in principle to single out the vector-type part of the $\pi \to e + \nu + \gamma$ decay from the background of the more probable axial-vector variant if one selects the electrons and quanta flying in opposite directions, since in the vector variant of the decay the greater part of the quanta is emitted into angles close to 180° relative to the direction of flight of the electron. In this case, as seen from (21), the probability for the axial-vector variant of such a decay, where the γ quantum is emitted by the electron, will be of order $10^{-7} W_{\mu+\nu} \sin\theta\, d\theta$. The probability for the vector variant of the decay will be of the same order. For a unique isolation of those terms in the probability of the transition which correspond to the vector variant (i.e., the determination of λ in (14)) one needs identical measurements for two different energies of the electron, which further complicates an already difficult task.

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Note added in proof (June 16, 1958): After the completion of this paper new data[8] appeared on the relation between the probabilities of the decays $\pi \to e + \nu$ and $\pi \to \mu + \nu$, from which it follows that $W_{e+\nu}/W_{\mu+\nu} < 10^{-5}$. This ratio $W_{e+\nu}/W_{\mu+\nu}$ contradicts the theory of Gell-Mann and Feynman, and indicates a significant suppression of the axial-vector variant of the decay $\pi \to e + \nu$. In this situation it is of even greater interest to verify the theory of Gell-Mann and Feynman for the vector variant, and to investigate experimentally the decay $\pi \rightarrow e + \nu + \gamma$. This decay should go mainly through the vector variant because of the suppression of the axial-vector variant, so that the probability of the $\pi \rightarrow e + \nu + \gamma$ decay will be determined by formulas (14) to (18).

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