COUPLED MAGNETOELASTIC WAVES IN FERROMAGNETIC MEDIA AND FERROACOUSTIC RESONANCE

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Submitted to JETP editor February 20, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 228-239 (July, 1958)

A phenomenological theory is given for coupled magnetoacoustic waves in ferromagnetic media and ferrites (the coupling between the elastic and magnetic waves is due to magnetostriction and spontaneous magnetization). The acoustic velocity in ferromagnetic media is determined and is found to be a function of the magnetization and longitudinal magnetic field. The acoustic absorption factor associated with the electrical conductivity and relaxation of the magnetization is determined. The possibility of resonant ultrasonic excitation of magnetic waves is indicated.

1. As is well known, any deviation of the magnetization from the equilibrium value (at a given temperature) in ferromagnetic media and ferrites is propagated in the form of waves, the dispersion properties of which are similar to those of spin waves.[1]

Because of magnetostriction and ponderomotive effects due to spontaneous magnetization, there is coupling in an elastically deformed ferromagnetic material between what we shall call "magnetic" waves and the elastic waves. In media of high conductivity, the coupled magnetoelastic waves produced in this way are similar to the magnetoelastic waves that propagate in metals in an external magnetic field and to the magnetohydrodynamic waves in liquid conductors. However, in contrast with magnetohydrodynamic waves, the magnetoelastic waves considered here can propagate in both ferromagnetic media and ferrites and, in addition, do not require an external magnetic field.

The coupling between magnetic and elastic waves offers the possibility of acoustic excitation of magnetic waves; moreover, the excitation should be especially intense in those cases in which the frequency and wave vector of the magnetic wave coincide with the frequency and wave vector of the elastic wave.

The interaction between magnetic and elastic waves leads to a dependence of the acoustic velocity in ferromagnetic media on the spontaneous magnetization and the external magnetic field. This interaction means additional acoustic absorption in ferromagnetic media; this absorption depends on the electrical conductivity of the medium and the magnetization relaxation mechanism. It is to be distinguished from another acoustic absorption mechanism, also characteristic of ferromagnetic media, in which the presence of an external acoustic field causes a deviation in the spin wave distribution function from the equilibrium value by virtue of the increased entropy.[2]

In the present paper we present a phenomenological theory of coupled magnetoelastic waves in ferromagnetic media and ferrites. Pure magnetic waves are considered first.

2. The free energy in a ferromagnetic medium can be given in the form:

$$\mathscr{H} = \int \left\{ \frac{1}{2} \alpha_{ik} \frac{\partial \mathbf{M}}{\partial x} \frac{\partial \mathbf{M}}{\partial x_k} + \beta(\mathbf{M}) + \frac{\mathbf{h}^2 + \mathbf{e}\cdot\mathbf{d}}{8\pi} - \mathbf{M}\cdot\mathbf{H}_0 \right\} dV. \tag{1}$$

Here the first term represents the exchange energy associated with inhomogeneity of the magnetization $\mathbf{M}$ (the α_{ik} are the exchange integrals*), the second term includes the exchange energy which depends on M and the anisotropy energy, while the third and fourth terms are the energy of the electromagnetic field and the energy of the magnetic moment in the external magnetic field $\mathbf{H}_0$ ($\mathbf{h}$ and $\mathbf{e}$ are the magnetic and electric fields respectively, $\mathbf{d}$ is the induction associated with the changing magnetization).

In the case of a uniaxial crystal the function $\beta(\mathbf{M})$ can be given by

$$\beta(\mathbf{M}) = \beta_1(M^2) + \beta_2(\mathbf{M}\cdot\mathbf{n}/M), \tag{2}$$

*In order-of-magnitude the quantities $\alpha_{ik} \sim \Theta_c a^2/\hbar g M_0$, where Θ_c is the Curie temperature, a is the lattice constant, and M_0 is the saturation magnetization.

where $\mathbf{n}$ is a unit vector in the direction of easiest magnetization.

The time derivative of the magnetization of the ferromagnetic medium is determined by the Landau-Lifshitz equation[3]

$$\partial \mathbf{M}/\partial t = g\,[\mathbf{M}\times\mathbf{H}^{(e)}] - \lambda M^{-2}\,[\mathbf{M}\times[\mathbf{M}\times\mathbf{H}^{(e)}]], \tag{3}$$

where $\mathbf{H}^{(e)}$ is the effective magnetic field, an expression for which is given below, λ is the damping factor and g is the gyromagnetic ratio.

We determine the time derivative of the energy in a volume V. Using Maxwell's equations we have:

$$\frac{d\mathscr{H}}{dt} = \int\limits_V \left\{-\frac{\partial \mathbf{M}}{\partial t}\left[\mathbf{H}_0 + \mathbf{h} - \frac{\partial\beta}{\partial \mathbf{M}} + \alpha_{ik}\frac{\partial^2\mathbf{M}}{\partial x_i \partial x_k}\right] - \mathbf{j}\cdot\mathbf{e}\right\} dV$$

$$+ \int\limits_S \left\{\frac{c}{4\pi}\,[\mathbf{h}\times\mathbf{e}]_k + \alpha_{ik}\frac{\partial \mathbf{M}}{\partial x_i}\frac{\partial \mathbf{M}}{\partial t}\right\} ds_k,$$

where $\mathbf{j}$ is the density of the conduction current and S is the surface which encloses the volume V.

If $\lambda = 0$ and $\sigma = 0$, from energy conservation considerations, the exchange integral must vanish. Whence, using Eq. (3) for $\partial\mathbf{M}/\partial t$ with $\lambda = 0$, it is easy to obtain the following expression for the effective magnetic field:

$$\mathbf{H}^{(e)} = \mathbf{H}_0 + \mathbf{h} - \partial\beta(\mathbf{M})/\partial\mathbf{M} + \alpha_{ik}\partial^2\mathbf{M}/\partial x_i \partial x_k. \tag{4}$$

For finite values of λ and σ the time derivative of the energy is

$$\frac{d\mathscr{H}}{dt} = -\int\limits_V \frac{j^2}{\sigma}\,dV - \int\limits_V \frac{\lambda}{M^2}\,[\mathbf{M}\times\mathbf{H}^{(e)}]^2\,dV \tag{5}$$

$$+ \int\limits_S \left\{\frac{c}{4\pi}\,[\mathbf{h}\times\mathbf{e}]_k + \alpha_{ik}\frac{\partial \mathbf{M}}{\partial x_i}\frac{\partial \mathbf{M}}{\partial t}\right\} ds_k,$$

where σ is the electrical conductivity of the media.

3. We now explain the dispersion properties of the magnetic waves. We use the symbol $\mathbf{M}_0$ to denote the equilibrium value of the magnetization per unit volume of the ferromagnetic medium at a given temperature and $\boldsymbol{\mu}(\mathbf{r}, t)$ to express the deviation of the magnetization from the equilibrium value at a point $\mathbf{r}$ and time t.

The quantity $\mathbf{M}_0$ can be determined from the minimum energy condition which yields

$$(\partial\beta/\partial\mathbf{M})_{\mathbf{M}=\mathbf{M}_0} = \mathbf{H}_0. \tag{6}$$

Assuming that $\mu \ll M_0$, from Eqs. (2), (4) and (6) we obtain the following expression for the effective magnetic field in the case of a uniaxial crystal with $\mathbf{H}_0 \parallel \mathbf{n}$:

$$\mathbf{H}^{(e)} = \mathbf{h} - H_0\boldsymbol{\mu}/M_0 - \beta\boldsymbol{\mu}_\perp - a\mathbf{n}(\mathbf{n}\cdot\boldsymbol{\mu}) + \alpha\Delta\boldsymbol{\mu},$$

where β is the anisotropy constant $\beta = -\beta_2'(1)/M_0^2$ and $a = 4M_0^2\beta_1''(M_0^2)$.

With $\lambda = 0$ the complete system of linearized equations is

$$\begin{aligned} \frac{\partial\boldsymbol{\mu}}{\partial t} &= gM_0\left[\mathbf{n}\times\left(\mathbf{h} - \frac{H_0}{M_0}\boldsymbol{\mu} - \beta\boldsymbol{\mu} + \alpha\Delta\boldsymbol{\mu}\right)\right], \\ \operatorname{curl}\mathbf{h} &= \frac{1}{c}\frac{\partial\mathbf{d}}{\partial t} + \frac{4\pi}{c}\mathbf{j}, \\ \operatorname{curl}\mathbf{e} &= -\frac{1}{c}\frac{\partial}{\partial t}(\mathbf{h} + 4\pi\boldsymbol{\mu}). \end{aligned} \tag{7}$$

We seek a solution for this system in the form of plane waves $e^{-i(\omega t - \mathbf{k}\cdot\mathbf{r})}$. First we consider the case $\sigma = 0$. Neglecting the displacement current, the relation between ω and k is of the form:

$$\omega = (\Omega\Omega_1)^{1/2}, \tag{8}$$

where

$$\begin{aligned} \Omega &= gM_0(\alpha k^2 + \beta + H_0/M_0 + 4\pi\sin^2\vartheta), \\ \Omega_1 &= gM_0(\alpha k^2 + \beta + H_0/M_0) \end{aligned} \tag{9}$$

and ϑ is the angle between $\mathbf{k}$ and $\mathbf{M}_0$.

This relation is the same as the well known relation between frequency and wave vector for Bloch spin waves.[4] It is valid when $\epsilon\omega \ll \sigma \ll c^2k^2/\omega$.

We now consider the magnetic and electromagnetic waves in ferrites, taking the displacement current into account. Writing $\mathbf{d} = \epsilon(\omega)\mathbf{e}$, where $\epsilon(\omega)$ is the dielectric constant, the following dispersion equation is obtained from Eq. (7):

$$\left\{1 + \frac{\tau\xi^2}{x^2 - \xi^2}\right\}\left\{1 + \frac{\tau\xi^2}{x^2 - \xi^2}\cos^2\vartheta\right\} - x^2 = 0, \tag{10}$$

where

$$\tau = \frac{4\pi g M_0}{4\pi g M_0 + \Omega_1}, \qquad \xi^2 = \frac{\tau^2}{\varepsilon}\left(\frac{ck}{4\pi g M_0}\right)^2, \qquad x = \tau\frac{\omega}{4\pi g M_0}.$$

If $\xi^2 \gg 1$, Eq. (10) leads to the solution:

$$\omega^2 = \frac{c^2k^2}{\varepsilon}\left\{1 \pm \frac{4\pi g M_0}{\omega}\cos\vartheta\right\}, \quad \omega^2 = \Omega\Omega_1. \tag{11}$$

The first of these equations determines the frequency of the electromagnetic waves which propagate in a medium of anisotropic magnetic susceptibility with the following values for the two waves (right-handed and left-handed circular polarization):

$$\begin{aligned} \mu_1(\omega) &= 1 - (4\pi g M_0/\omega)\cos\vartheta, \\ \mu_2(\omega) &= 1 + (4\pi g M_0/\omega)\cos\vartheta. \end{aligned} \tag{12}$$

The second solution coincides with (8).

If $\xi^2 \ll 1$, Eq. (10) has the following roots:

$$\omega = ck\sqrt{\frac{1-\tau}{\varepsilon}}, \quad \omega = ck\sqrt{\frac{1-\tau\cos^2\vartheta}{\varepsilon}}, \tag{13}$$

$$\omega = 4\pi g M_0/\tau = \Omega_1 + 4\pi g M_0 = g(B_0 + \beta M_0).$$

If the conductivity of the medium is high, ($\sigma \gg \epsilon\omega$, $\sigma \gg c^2k^2/\omega$) the frequency of the magnetic waves is

$$\omega = \Omega_1 + 4\pi g M_0. \tag{14}$$

4. We now determine the damping of the magnetic waves due to the finite conductivity and relaxation processes.

If the absorption is small, the damping factor can be defined as

$$\Gamma = -(1/\overline{\mathscr{H}})\overline{d\mathscr{H}/dt}, \tag{15}$$

where the bar denotes time averages of expressions defined in Eqs. (1) and (5); in place of the fields **e** and **h** we substitute their values for $\sigma = 0$, $\lambda = 0$ or $\sigma = \infty$ and $\lambda = 0$.

If $\epsilon\omega \ll \sigma \ll c^2k^2/\omega \sim c^2\hbar^2/\Theta_c a^2$, it can be shown that:

$$\Gamma = 16\pi^2 \frac{\sigma g M_0}{c^2k^2}(\Omega + \Omega_1 \cos^2\vartheta) + 2\frac{\lambda}{gM_0}(\Omega_1 + 2\pi g M_0 \sin^2\vartheta). \tag{16}$$

If $\sigma \gg 4\pi g M_0$ and $\sigma \gg c^2k^2/4\pi g M_0$

$$\Gamma = \frac{1}{\sigma}(ck)^2 \frac{gM_0}{\Omega_1 + 4\pi g M_0}(1 + \sin^2\vartheta) + 2\frac{\lambda}{gM_0}(\Omega_1 + 4\pi g M_0). \tag{17}$$

5. It can be easily shown that the absorption of the magnetic waves is small ($\Gamma \ll \omega$) if

$$\lambda \ll gM_0, \quad \sigma \ll c^2k^2/gM_0.$$

These inequalities are obviously the existence conditions for Bloch spin waves.

It follows from the second condition that

$$k \gg k_0,$$

where

$$k_0 = \begin{cases} \delta_0^{-1}, & l \ll \delta_0 \\ (\delta_0^2 l)^{-1/3}, & l \gg \delta_0, \end{cases} \tag{18}$$

l is the electron free path length and δ_0 is the depth of the skin layer corresponding to the frequency $\omega = gM_0$ ($\delta_0^2 = c^2/2\pi g M_0 \sigma_0$, σ_0 is the static conductivity; when $l \gg \delta_0$ the expression for k_0 corresponds to the anomalous skin effect).

Thus, there are no spin waves for wavelengths large compared with $\lambda_0 = 1/k_0$.

If $k \ll k_0$, the magnetic wave spectrum is determined by Eq. (14). In this case the frequency is a weak function of wave vector.

The dependence of spin-wave frequency on wave vector, as is well known, is associated with the dependence of the magnetization and other thermodynamic quantities on temperature. A Bloch law ($T^{3/2}$) for the magnetization corresponds to the spectrum $\omega = \Theta_c(ak)^2/\hbar$ which is obtained if we neglect the terms $\beta + H_0/M_0$ and $4\pi \sin^2\vartheta$ in Eq. (9), i.e., if the magnetic interaction and anisotropy are neglected. It can be shown that frequencies $\omega = \Theta_c(ak)^2/\hbar$ are excited at temperatures much higher than $4\pi g M_0\hbar$ ($\sim 1°K$).

At temperatures $T \le 4\pi g M_0 \hbar$ the magnetic interaction plays an important role in the spectrum and the frequency is determined by Eq. (8). In this case we have a T^2 relation for the magnetization instead of the $T^{3/2}$ relation.[5] Equation (8) applies only when $k \gg k_0$. Whence it may be concluded that the spectrum $\omega \sim k \sin\vartheta$ is excited at temperatures which satisfy the condition

$$gB_0\hbar\,(a^2\sigma\Theta_c/\hbar c^2)^{1/2} < T < gB_0\hbar.$$

In the temperature region $T \ll gB_0\hbar(a^2\sigma\Theta_c/\hbar c^2)^{1/2}$, in place of the spectrum given in (8) we must use that given in (14) which yields an exponential dependence for deviations of the magnetization from the saturation value.

6. We now investigate coupled magnetoacoustic waves in ferromagnetic media.

The equation of motion for the magnetization and the elasticity equation are:

$$\frac{\partial \mathbf{M}}{\partial t} + \frac{\partial}{\partial x_k}(\mathbf{M}\dot{u}_k) = g[\mathbf{M}\times\mathbf{H}^{(e)}] - \frac{\lambda}{M^2}[\mathbf{M}\times[\mathbf{M}\times\mathbf{H}^{(e)}]], \tag{19}$$
$$\rho\ddot{\mathbf{u}} = \mathbf{f},$$

where **u** is the elastic displacement, $\mathbf{H}^{(e)}$ is the effective magnetic field and **f** is the force which acts on a unit volume of the medium (expressions for these are given below).

The energy in the ferromagnetic medium can be given in the form:

$$\mathscr{H} = \int\Big\{\frac{1}{2}\alpha_{ik}\frac{\partial \mathbf{M}}{\partial x_i}\frac{\partial \mathbf{M}}{\partial x_k} + \frac{\mathbf{h}^2 + \mathbf{e}\cdot\mathbf{d}}{8\pi} + \beta(\mathbf{M}) - \mathbf{M}\cdot\mathbf{H}_0 + \frac{1}{2}\rho\dot{\mathbf{u}}^2 + \frac{1}{2}\lambda_{iklm}u_{ik}u_{lm} + F_{lm}(\mathbf{M})u_{lm}\Big\}dV. \tag{20}$$

This expression differs from that given in (1) in the presence of the three last terms; the first two represent elastic energy and the third the magnetostriction energy (λ_{iklm} is the elasticity tensor).

Using Maxwell's equations and (19), and assuming that the current density is

$$\mathbf{j} = \sigma\Big\{\mathbf{e} + \frac{1}{c}[\dot{\mathbf{u}}\times\mathbf{B}]\Big\}, \quad \mathbf{B} = \mathbf{H} + 4\pi\mathbf{M},$$

it is easy to show that

$$\frac{d\mathcal{H}}{dt}=\int\Big\{-\Big(g\,[\mathbf{M}\times\mathbf{H}^{(e)}]-\frac{\lambda}{M^2}[\mathbf{M}\times[\mathbf{M}\times\mathbf{H}^{(e)}]]\Big)\Big(\mathbf{H}_0+\mathbf{h}-\frac{\partial\beta}{\partial\mathbf{M}}+\alpha_{ik}\frac{\partial^2\mathbf{M}}{\partial x_i\partial x_k}-\mathbf{G}\Big)-\frac{1}{\sigma}j^2+\dot{u}_i\Big[f_i-\frac{\partial\sigma_{ik}}{\partial x_k}-\frac{1}{c}[\mathbf{j}\times\mathbf{B}]_i-\mathbf{M}\frac{\partial}{\partial x_i}\Big(\mathbf{H}_0+\mathbf{h}-\frac{\partial\beta}{\partial\mathbf{M}}+\alpha_{pr}\frac{\partial^2\mathbf{M}}{\partial x_p\partial x_r}-\mathbf{G}\Big)\Big]\Big\}dV$$
$$+\int\Big\{\frac{c}{4\pi}[\mathbf{h}\times\mathbf{e}]_k+\alpha_{ik}\frac{\partial\mathbf{M}}{\partial x_i}\frac{\partial\mathbf{M}}{\partial t}+\dot{u}_i\Big[\sigma_{ik}+\delta_{ik}\mathbf{M}\Big(\mathbf{H}_0+\mathbf{h}-\frac{\partial\beta}{\partial\mathbf{M}}+\alpha_{pr}\frac{\partial^2\mathbf{M}}{\partial x_p\partial x_r}-\mathbf{G}\Big)\Big]\Big\}ds_k,$$

where

$$\sigma_{ik}=\lambda_{iklm}u_{lm}+F_{ik}(\mathbf{M}),\quad \mathbf{G}=u_{lm}\partial F_{lm}(\mathbf{M})/\partial\mathbf{M}. \tag{21}$$

When $\sigma=\lambda=0$ the volume integral vanishes. Whence it is easy to show that the effective magnetic field $\mathbf{H}^{(e)}$ and the volume force $\mathbf{f}$ are

$$\mathbf{H}^{(e)}=\mathbf{H}_0+\mathbf{h}-\frac{\partial\beta}{\partial\mathbf{M}}+\alpha_{ik}\frac{\partial^2\mathbf{M}}{\partial x_i\partial x_k}-\mathbf{G},$$
$$f_i=\frac{\partial\sigma_{ik}}{\partial x_k}+\frac{1}{c}[\mathbf{j}\times\mathbf{B}]_i+\mathbf{M}\frac{\partial}{\partial x_i}\mathbf{H}^{(e)}. \tag{22}$$

Thus, for finite values of λ and σ the time derivative of the energy is given by

$$\frac{d\mathcal{H}}{dt}=-\int\frac{j^2}{\sigma}dV-\int_V\frac{\lambda}{M^2}[\mathbf{M}\times\mathbf{H}^{(e)}]^2dV+\int_S\Big\{\dot{u}_i(\sigma_{ik}+\delta_{ik}\mathbf{H}^{(e)}\mathbf{M})+\frac{c}{4\pi}[\mathbf{h}\times\mathbf{e}]_k+\alpha_{ik}\frac{\partial\mathbf{M}}{\partial x_i}\frac{\partial\mathbf{M}}{\partial t}\Big\}ds_k.$$

7. We consider first coupled magnetoacoustic waves with $\lambda=0$.

For simplicity it will be assumed that the medium is isotropic in both its elastic and magnetostrictive properties. The last condition means that $F_{ik}(\mathbf{M})$ is of the form

$$F_{ik}(\mathbf{M})=\delta_{ik}M^2F_1(M^2)+M_iM_kF_2(M^2), \tag{23}$$

where F_1 and F_2 are certain functions of M^2.

Assuming that at equilibrium $u_{ik}=0$, from the minimum energy condition it is easy to show that $F_{ik}(\mathbf{M}_0)=0$.

Linearizing Eq. (22) and assuming for simplicity that F_1 and F_2 are constant, we obtain the following expressions for $\mathbf{H}^{(e)}$ and $\mathbf{f}$:

$$\mathbf{H}^{(e)}=\mathbf{h}-\beta\boldsymbol{\mu}_\perp-\boldsymbol{\mu}\frac{H_0}{M_0}-a\mathbf{n}(\boldsymbol{\mu}\,\mathbf{n})+\alpha\Delta\boldsymbol{\mu}-\delta_1\mathbf{M}_0\operatorname{div}\mathbf{u}-\frac{1}{2}\delta_2M_0((\mathbf{n}\nabla)\cdot\mathbf{u}+\nabla(\mathbf{n}\cdot\mathbf{u})),$$
$$f_i=\rho c_t^2\Delta u_i+\rho(c_l^2-c_t^2)\frac{\partial}{\partial x_i}\operatorname{div}\mathbf{u}+\frac{1}{c}[\mathbf{j}\times\mathbf{B}_0]_i+\mathbf{M}_0\frac{\partial}{\partial x_i}\mathbf{H}^{(e)}+\delta_1\frac{\partial}{\partial x_i}(\mathbf{M}_0\cdot\boldsymbol{\mu})+\frac{\delta_2}{2}\Big(M_{0i}\operatorname{div}\boldsymbol{\mu}+M_{0k}\frac{\partial\mu_i}{\partial x_k}\Big),$$

where δ_1 and δ_2 are the magnetostriction constants

$$\delta_1=2F_1 \text{ and } \delta_2=2F_2.$$

The linearized equations of motion for the magnetization and elasticity are of the form:

$$\frac{\partial\boldsymbol{\mu}}{\partial t}+\mathbf{M}_0\operatorname{div}\dot{\mathbf{u}}=gM_0[\mathbf{n}\times\mathbf{H}^{(e)}],$$
$$\ddot{\mathbf{u}}=c_t^2\Delta\mathbf{u}+(c_l^2-c_t^2)\nabla\operatorname{div}\mathbf{u}+\frac{\delta_1}{\rho}\nabla(\mathbf{M}_0\,\boldsymbol{\mu})+\frac{\delta_2}{2\rho}\Big(\mathbf{M}_0\operatorname{div}\boldsymbol{\mu}+(\mathbf{M}_0\nabla)\cdot\boldsymbol{\mu}\Big)+\frac{1}{c\rho}[\mathbf{j}\times\mathbf{B}_0]+\frac{1}{\rho}\nabla\mathbf{M}_0\cdot\mathbf{H}^{(e)}, \tag{24}$$

where $\mathbf{B}_0=\mathbf{H}_0+4\pi\mathbf{M}_0$, c_l and c_t are the velocities of the longitudinal and transverse acoustic oscillations. Assuming that all quantities vary as $e^{-i(\omega t-\mathbf{k}\cdot\mathbf{r})}$ and that $\sigma=0$, we obtain the following dispersion relation

$$(v^2-c_t^2)^2(v^2-\tilde{c}_l^2)(v^2-\Omega\Omega_1/k^2)-\zeta(v^2-c_t^2)f_1-\zeta^2f_2=0, \tag{25}$$

where

$$v=\frac{\omega}{k},\ \zeta=\frac{M_0^2}{4\rho}\delta_2^2,\ \tilde{c}_l^2=c_l^2-\frac{M_0^2}{\rho}\Big[2(\delta_2-2\pi)\cos^2\vartheta+2\delta_1-a-\frac{H_0}{M_0}-\alpha k^2\Big],$$
$$f_1=\frac{gM_0\Omega_1}{k^2}\Big\{v^2-c_t^2+(c_t^2-c_l^2)\cos^22\vartheta+\frac{\Omega}{\Omega_1}(v^2-c_l^2)\cos^2\vartheta+\frac{16\pi^2}{\delta_2^2}\Big(1-\frac{\delta_2}{2\pi}\Big)(v^2-c_t^2)\sin^22\vartheta\Big\},$$
$$f_2=\frac{(gM_0)^2}{k^2}(c_t^2-c_l^2)\cos^2\vartheta\cos^22\vartheta.$$

From this equation it is easy to obtain the velocities for the longitudinal (v_1) and transverse (v_2, v_3) acoustic waves:

$$v_1^2=\tilde{c}_l^2\Big\{1+\frac{gM_0^3}{4\rho c_l^2}(\delta_2-4\pi)^2\frac{\Omega_1}{\omega^2-\Omega\Omega_1}\sin^22\vartheta\Big\},$$
$$v_2^2=c_t^2\Big\{1+\frac{gM_0^3}{8\rho c_t^2}\delta_2^2\frac{\Omega_1\cos^22\vartheta+\Omega\cos^2\vartheta+[(\Omega_1\cos^22\vartheta-\Omega\cos^2\vartheta)^2+4\omega^2\cos^2\vartheta\cos^22\vartheta]^{1/2}}{\omega^2-\Omega\Omega_1}\Big\}, \tag{26}$$
$$v_3^2=c_t^2\Big\{1+\frac{gM_0^3}{8\rho c_t^2}\delta_2^2\frac{\Omega_1\cos^22\vartheta+\Omega\cos^2\vartheta-[(\Omega_1\cos^22\vartheta-\Omega\cos^2\vartheta)^2+4\omega^2\cos^2\vartheta\cos^22\vartheta]^{1/2}}{\omega^2-\Omega\Omega_1}\Big\}.$$

These formulas apply if ω^2 is not close to $\Omega\Omega_1$.

If ω^2 is approximately the same as $\Omega\Omega_1$ the magnetic and acoustic branches of the oscillations "cross"; this effect is now considered for $\vartheta = \pi/2$ and $\vartheta = 0$.[6]

If $\vartheta = \pi/2$, the roots of the dispersion equation are:

$$v^2 = \begin{cases} \tilde{c}_l^2 \\ c_t^2 \\ \frac{1}{2}\left(c_t^2 + \frac{\Omega\Omega_1}{k^2}\right) + \frac{1}{2}\left[\left(c_t^2 - \frac{\Omega\Omega_1}{k^2}\right)^2 + \zeta\frac{gM_0\Omega_1}{k^2}\right]^{1/2}, \\ \frac{1}{2}\left(c_t^2 + \frac{\Omega\Omega_1}{k^2}\right) - \frac{1}{2}\left[\left(c_t^2 - \frac{\Omega\Omega_1}{k^2}\right)^2 + \zeta\frac{gM_0\Omega_1}{k^2}\right]^{1/2}. \end{cases} \tag{27}$$

The first two roots determine the phase velocities of the longitudinal and one of the transverse acoustic waves. When $kc_t < \sqrt{\Omega\Omega_1}$ the third root determines the phase velocity of the magnetic wave while the fourth determines that of the other transverse wave. When $kc_t > \sqrt{\Omega\Omega_1}$, on the other hand, the fourth root determines the phase velocity of the magnetic wave while the third determines the phase velocity of the acoustic wave.

If $\vartheta = 0$, the roots of the dispersion equation are:

$$v = \begin{cases} \tilde{c}_l \\ c_t - \frac{1}{2}\frac{\zeta}{c_t}\frac{gM_0}{kc_t + \Omega_1} \\ \frac{1}{2}\left(c_t + \frac{\Omega_1}{k} - \frac{1}{2}\frac{\zeta}{c_t}\frac{gM_0}{kc_t + \Omega_1}\right) + \frac{1}{2}\left[\left(c_t - \frac{\Omega_1}{k}\right)^2 + \zeta\frac{gM_0}{kc_t}\frac{3kc_t + \Omega_1}{kc_t + \Omega_1}\right]^{1/2} \\ \frac{1}{2}\left(c_t + \frac{\Omega_1}{k} - \frac{1}{2}\frac{\zeta}{c_t}\frac{gM_0}{kc_t + \Omega_1}\right) - \frac{1}{2}\left[\left(c_t - \frac{\Omega_1}{k}\right)^2 + \zeta\frac{gM_0}{kc_t}\frac{3kc_t + \Omega_1}{kc_t + \Omega_1}\right]^{1/2}. \end{cases} \tag{28}$$

When $kc_t < \Omega_1$, the third root determines the phase velocity of the magnetic wave while the fourth determines that of the transverse acoustic wave; if, however, $kc_t > \Omega_1$ the third root determines the phase velocity of the transverse acoustic wave while the fourth determines that of the magnetic wave.

Equations (27) and (28) apply if

$$(\lambda/gM_0)^2 \ll \delta_2^2 (M_0^2/4\rho c_t^2)\, M_0/(H_0 + \beta M_0).$$

It can be shown that the transverse waves are elliptically polarized and are of the following form:

$$\mathbf{u} = U\left\{\left(\frac{\mathbf{k}_\perp}{k_\perp^2} - \frac{\mathbf{k}_\parallel}{k_\parallel^2}\right)\cos(\omega t - \mathbf{k}\cdot\mathbf{r}) - \frac{[\mathbf{n}\times\mathbf{k}]}{k_\perp^2 \eta}\sin(\omega t - \mathbf{k}\cdot\mathbf{r})\right\};$$

$$\boldsymbol{\mu} = \frac{gM_0^2}{2c_t}\delta_2\frac{U\cos\vartheta}{\eta - \frac{\Omega_1}{\omega}\cos 2\vartheta}\left\{\frac{\mathbf{k}_\perp}{k_\perp^2}\sin(\omega t - \mathbf{k}\cdot\mathbf{r}) + \frac{[\mathbf{n}\times\mathbf{k}]}{k_\perp^2\eta}\cos 2\vartheta\cos(\omega t - \mathbf{k}\cdot\mathbf{r})\right\}, \tag{29}$$

where U is a constant,

$$\eta = \frac{\Omega_1}{\omega}\left\{1 + \frac{2\cos^2\vartheta}{\Omega_1^2}\frac{\omega^2 - \Omega\Omega_1}{\cos^2 2\vartheta + \frac{\Omega}{\Omega_1}\cos^2\vartheta \pm \left[\left(\cos^2 2\vartheta - \frac{\Omega}{\Omega_1}\cos^2\vartheta\right)^2 + 4\frac{\omega^2}{\Omega_1^2}\cos^2\vartheta\cos^2 2\vartheta\right]^{1/2}}\right\}\cos 2\vartheta \tag{30}$$

and $k_\parallel$ and $k_\perp$ are the longitudinal and transverse components of $\mathbf{k}$ (with respect to $\mathbf{M}_0$). The ratio of the semi-axes of the ellipse is

$$b/a = \cos\vartheta/\eta \tag{29'}$$

(the semi-axis a lies in the $(\mathbf{n}, \mathbf{k})$ plane and is perpendicular to $\mathbf{k}$).

When $\theta = 0$ we obtain two cylindrically polarized waves; with $\theta = \pi/2$ we have two linearally polarized waves for which the vector $\mathbf{u}$ has components along $\mathbf{M}_0$ and $\mathbf{M}_0 \times \mathbf{k}$.

8. We can analyze coupled magnetoacoustic oscillations in the high-conductivity case $\sigma \gg \omega c^2/c_t^2$ in similar fashion.

Here we present only the formulas for the phase velocities of the acoustic waves for $\vartheta = 0$ and $\vartheta = \pi/2$. If $\vartheta = 0$,

$$v_1 = c_l\left[1 - \frac{M_0^2}{2\rho c_l^2}\left(2\delta_1 + 2\delta_2 - a - 4\pi - \frac{H_0}{M_0}\right)\right],$$

$$v_2 = v_3 = c_t\left\{1 + \frac{M_0^2}{2\rho c_t^2}\left[\frac{B_0^2}{4\pi M_0^2} - \frac{1}{4}\frac{gM_0}{\Omega_1 + 4\pi g M_0}\left(\delta_2 - 2\frac{B_0}{M_0}\right)\right]\right\}. \tag{31}$$

When $\vartheta = \pi/2$

$$v_1 = c_l\left[1 - \frac{M_0^2}{2\rho c_l^2}\left(\delta_2 - \frac{H_0}{4\pi M_0}\right)^2\right],$$

$$v_2 = c_t\left[1 - \frac{M_0^2}{8\rho c_t^2}\frac{gM_0}{\Omega_1 + 4\pi g M_0}\right], \quad v_3 = c_t, \tag{31'}$$

Taking $M_0 = 0$ and $\delta_1 = \delta_2 = 0$ in these expressions we obtain the velocity of acoustic waves in a high conductivity metal located in an external magnetic field H_0. If $H_0^2/8\pi\rho c_t^2 \ll 1$ these expressions assume the following form: for $\vartheta = 0$

$$v_1 = c_l, \quad v_2 = v_3 = c_t\,(1 + H_0^2/8\pi\rho c_t^2); \tag{32}$$

for $\vartheta = \pi/2$

$$v_1 = c_l\,(1 + H_0^2/8\pi\rho c_l^2), \quad v_2 = v_3 = c_t. \tag{32'}$$

9. In the previous formulas we have neglected terms containing λ and σ. Taking these terms into account leads, firstly, to damping and, secondly, to an additional variation in phase velocity. We consider the second problem, having in mind a number of effects for which the condition $(\lambda/gM_0)^2 \gg \delta_2^2\,(M_0^2/4\rho c_t^2)\,M_0\,(H_0 + \beta M_0)^{-1}$ is satisfied. It can be shown that if this condition is satisfied the dispersion equation leads to the following expression for the phase velocity of the acoustic wave when $\vartheta = 0$

$$v_2^2 = v_3^2 = c_t^2\left[1 - \frac{\zeta}{c_t^2}gM_0\frac{\omega - \Omega_1}{(\omega - \Omega_1)^2 + (\lambda/gM_0)^2\,\Omega_1^2}\right]. \tag{33}$$

The relative change in the acoustic velocity $\Delta v/v$ for $\lambda/gM_0 \sim 10^{-1}$ and $\beta \sim 10^{-1}$ is approximately 0.1%.

10. We now consider the absorption factor for magnetoacoustic oscillations. For this purpose, in accordance with Eq. (15), we compute the quantities $\overline{\mathcal{H}}$ and $\overline{d\mathcal{H}/dt}$ with values of the field corresponding to $\lambda = 0$ and $\sigma = 0$. Here we present only the final results.

For small σ the absorption factors for longitudinal $\Gamma^{(l)}$ and transverse $\Gamma^{(t)}$ waves are

$$\Gamma^{(l)} = \lambda\omega^2\frac{M_0^2}{4\rho c_l^2}(4\pi - \delta_2)^2\frac{\omega^2 + \Omega_1^2}{[\omega^2 - \Omega\Omega_1]^2}\sin^2\vartheta$$
$$+\sigma\frac{M_0^2}{\rho c^2}\left\{\left[\frac{H_0}{M_0} + (4\pi - \delta_2)\frac{4\pi g M_0 \Omega_1}{\omega^2 - \Omega\Omega_1}\cos^2\vartheta\right]^2 + (4\pi - \delta_2)^2\left[\frac{4\pi g M_0\omega}{\omega^2 - \Omega\Omega_1}\right]^2\cos^2\vartheta\right\}\sin^2\vartheta,$$

$$\Gamma^{(t)} = \lambda\omega^2\frac{M_0^2}{4\rho c_t^2}\delta_2^2\frac{\eta^2 + \cos^2 2\vartheta}{\eta^2 + \cos^2\vartheta}\frac{\cos^4\vartheta}{[\omega\eta - \Omega_1\cos 2\vartheta]^2} + \frac{\sigma B_0^2}{\rho c^2}\frac{\cos^2\vartheta}{\eta^2 + \cos^2\vartheta}$$
$$\times\left\{\left[1 - \frac{2\pi M_0}{B_0}\delta_2\frac{gM_0\cos^2\vartheta}{\eta\omega - \Omega_1\cos 2\vartheta}\right]^2\eta^2 + \left[1 - \frac{2\pi M_0}{B_0}\delta_2\frac{gM_0\cos 2\vartheta}{\eta\omega - \Omega_1\cos 2\vartheta}\right]^2\cos^2\vartheta\right\}. \tag{34}$$

It is apparent that the acoustic absorption is highly anisotropic and that the absorption is especially high at resonances at which the magnetic and acoustic frequencies coincide.

To determine the acoustic absorption coefficient in the resonance region it is necessary to use the exact dispersion equation, which takes account of the conductivity σ and the absorption factor λ. It can be shown that in this case the absorption factor is given by (30) and (34) if in the denominators of these expressions we replace $[\omega^2 - \Omega\Omega_1]^2$ by $[\omega^2 - \Omega\Omega_1]^2 + \Gamma\Omega\Omega_1$ where Γ is the damping factor for the magnetic wave as given by Eq. (16).

The longitudinal absorption factor for acoustic waves at resonance when $\vartheta = \pi/2$ is given by

$$\Gamma^{(l)} \approx \frac{M_0^2}{\rho c_l^2}\frac{(4\pi - \delta_2)^2}{4\pi}\frac{\omega_0^2}{\lambda}, \quad \omega_0^2 = \Omega\Omega_1. \tag{35}$$

We compare the damping at resonance with the damping due to thermal conductivity. The latter is determined from the expression[7]

$$\gamma \approx (\Omega\Omega_1/C^2)\,\varkappa T\alpha_T^2\rho, \tag{36}$$

where κ is the thermal conductivity, T is the temperature in degrees, C is the heat capacity per unit volume and α_T is the thermal expansion coefficient.

The ratio $\Gamma^{(l)}/\gamma$ is

$$\Gamma^{(l)}/\gamma \approx (4\pi - \delta_2)^2\, C^2 M_0^2/4\pi\alpha_T^2 \lambda\varkappa c_l^2 T; \tag{37}$$

with

$$\alpha_T = 10^{-5},\ C = 10^6,\ \varkappa \sim 10^6,\ T \sim 10^2\,{}^\circ\mathrm{K},$$

$$\lambda/gM_0 \sim 10^{-1},\ \delta_2 \sim 1,\ c_l \sim 5\cdot 10^5,\ M_0 \sim 10^3,$$

this ratio becomes $\Gamma^{(l)}/\gamma \sim 10^2$.

Finally, we present the formulas which determine the acoustic absorption in ferromagnetic media of high conductivity $\sigma \gg c^2\omega/c_t^2$. When $\vartheta = 0$

$$\Gamma^{(t)} = \lambda\left(\delta_2 - 2\frac{B_0}{M_0}\right)^2 \frac{M_0^2}{4\rho c_t^2}\left(\frac{M_0}{B_0 + \beta M_0}\right)^2\left(\frac{\omega}{gM_0}\right)^2 \tag{38}$$

$$+ \frac{\omega^2}{\sigma}\frac{M_0^2}{\rho c_t^2}\left(\frac{c}{c_t}\right)^2\left\{\frac{B_0}{4\pi M_0} + \frac{1}{2}\frac{M_0}{B_0 + \beta M_0}\left(\delta_2 - 2\frac{B_0}{M_0}\right)\right\}^2,\ \Gamma^{(l)} = 0.$$

when $\vartheta = \pi/2$

$$\Gamma_2^{(t)} = 0,\ \Gamma_3^{(t)} = \lambda\omega^2 (M_0^2/4\rho c_t^2)\,\delta_2^2/\Omega^2, \tag{38'}$$

$$\Gamma^{(l)} = (\omega^2/16\,\pi^2\sigma)\,(c/c_l)^2\, H_0^2/\rho c_l^2.$$

We see that the acoustic absorption is non-resonant at high values of σ.

We compare this absorption factor (38) with that associated with thermal conductivity and internal viscosity. In metals the latter is determined from the expression:[8]

$$\gamma \sim \tau\omega^2 n\varepsilon_0/\rho c_t^2,\quad \omega\tau \ll c_t/v_0, \tag{39}$$

where ϵ_0 is the Fermi energy and v_0 is the corresponding limiting electron velocity, n is the number of conduction electrons per unit volume and τ is the relaxation time. The ratio of the absorption factors (38) and (39) is

$$\frac{\Gamma}{\gamma} \sim \left(\frac{gM_0}{\sigma}\right)^2\left(\frac{c^2}{c_t v_0}\right)^2\left[\frac{B_0}{4\pi M_0} + \frac{1}{2}\frac{M_0}{B_0 + \beta M_0}\left(\delta_2 - 2\frac{B_0}{M_0}\right)\right]^2. \tag{40}$$

With $\sigma \sim 10^{17}$ and $\omega < 10^7$ both factors are of the same order of magnitude.

11. In conclusion, we consider the problem of exciting magnetic waves by means of an external acoustic field.

Suppose that the half-space $z > 0$ is filled with a ferromagnetic medium at the external surface of which ($z = 0$) is applied a displacement $\mathbf{u} = \mathbf{u}_0 e^{-i\omega t}$ or a stress $\sigma_{i3} = f_i e^{-i\omega t}$ ($\mathbf{u}_0$ and $\mathbf{f}$ are assumed constant). It is required to determine $\mathbf{u}(\mathbf{r}, t)$ and $\mathbf{u}(\mathbf{r}, t)$.

Since we wish to consider low resonance frequencies, the magnetization vector $\mathbf{M}_0$ will be taken perpendicular to the boundary $z = 0$ (to satisfy this condition we imagine a magnetization equal to $\mathbf{M}_0$ in the region $z < 0$).

It follows from Eq. (28) that when $\vartheta = 0$ the interaction of the transverse sound with magnetic waves is distinctive only in the resonance effect. Hence we will assume that $\mathbf{u}_0$ and $\mathbf{f}$ are in the (x, y) plane which contains the vector $\boldsymbol{\mu}$.

In our case, the basic equations (24) can be written in the form:

$$\ddot{\mathbf{u}} - c_t^2\frac{\partial^2 \mathbf{u}}{\partial z^2} - \frac{M_0\delta_2}{2\rho}\frac{\partial \boldsymbol{\mu}}{\partial z} = 0,$$

$$\dot{\boldsymbol{\mu}} = gM_0\left[\mathbf{n}\times\left(-\frac{\omega_0}{gM_0}\boldsymbol{\mu} + \mathbf{h}^{(s)}\right)\right] + \lambda\left(\frac{\omega_0}{gM_0}\boldsymbol{\mu} - \mathbf{h}^{(s)}\right) = 0,$$

where

$$\mathbf{h}^{(s)} = -\frac{\delta_2 M_0}{2}\frac{\partial \mathbf{u}}{\partial z},\quad \omega_0 = gM_0\left(\beta + \frac{H_0}{M_0}\right)$$

It follows from the second equation that

$$\boldsymbol{\mu} = \hat{\chi}\mathbf{h}^{(s)},$$

where the tensor $\hat{\chi}$ is of the form:

$$\hat{\chi} = \frac{-gM_0\omega_0}{\omega_0^2 - \left(\omega - i\omega_0\frac{\lambda}{gM_0}\right)^2}$$

$$\times\begin{pmatrix} 1 - i\frac{\lambda}{gM_0}\frac{\omega}{\omega_0} + \left(\frac{\lambda}{gM_0}\right)^2 & i\frac{\omega}{\omega_0} \\ -i\frac{\omega}{\omega_0} & 1 - i\frac{\lambda}{gM_0}\frac{\omega}{\omega_0} + \left(\frac{\lambda}{gM_0}\right)^2 \end{pmatrix}.$$

Assuming that $\mathbf{u}$ and $\boldsymbol{\mu}$ are proportional to $e^{-i\omega t}$, we have

$$\omega^2\mathbf{u} + (c_t^2 - \zeta\hat{\chi})\,\partial^2\mathbf{u}/\partial z^2 = 0,$$

whence

$$\mathbf{u} = \mathbf{c}_1 e^{ik_1 z} + \mathbf{c}_2 e^{ik_2 z},$$

where $\mathbf{c}_1$ and $\mathbf{c}_2$ are integration constants and

$$k_1^2 = \frac{\omega^2}{c_t^2}\frac{c_t^2}{c_t^2 - \zeta\left(\chi_{xx} - \sqrt{\chi_{xy}\chi_{yx}}\right)},\quad k_2^2 = \frac{\omega^2}{c_t^2}\frac{c_t^2}{c_t^2 - \zeta\left(\chi_{xx} + \sqrt{\chi_{xy}\chi_{yx}}\right)}.$$

If $\mathbf{u}|_{z=0} = \mathbf{u}_0$ is given,

$$c_{1x} = c_{2x} = ic_{1y} = -ic_{2y} = u_0/2$$

($\mathbf{u}_0$ is taken along the x axis) and

$$\mathbf{u}(z,t)=\frac{1}{2}u_0\{e^{i(k_1z-\omega t)}+e^{i(k_2z-\omega t)}\}\,\mathbf{i}+\frac{i}{2}u_0\{e^{i(k_1z-\omega t)}-e^{i(k_2z-\omega t)}\}\,\mathbf{j},$$

$$\boldsymbol{\mu}(z,t)=-{}^1/_2\,M_0\hat{\delta}_2\chi\partial\mathbf{u}/\partial z.$$

Here $\mathbf{i}$ and $\mathbf{j}$ are unit vectors along the x and y axes. Whence:

$$\mu_x(z,t)=\frac{\delta_2\omega\omega_0 gM_0^2u_0}{2c_t\left[(\omega^2-\omega_0^2)^2+4\omega^2\omega_0^2\left(\frac{\lambda}{gM_0}\right)^2\right]}\left\{(\omega^2-\omega_0^2)\sin(\omega t-kz)-2\omega\omega_0\frac{\lambda}{gM_0}\cos(\omega t-kz)\right\},$$

$$\mu_y(z,t)=\frac{\delta_2\omega^2 gM_0^2u_0}{2c_t(\omega^2-\omega_0^2)^2+4\omega^2\omega_0^2(\lambda/gM_0)^2]}\left\{(\omega^2-\omega_0^2)\cos(\omega t-kz)+2\omega\omega_0\frac{\lambda}{gM_0}\sin(\omega t-kz)\right\}, \tag{41}$$

$$u(z,t)=u_0\cos(\omega t-kz)$$

(here we have made use of the fact that k_1 and k_2 are approximately the same; thus $k_1=k_2=k=\omega/c_t$).

The equations in (41) apply if:

$$\lambda/gM_0\gg(\zeta/c_t^2)\,gM_0/\omega_0.$$

At resonance:

$$\mu_{\mathrm{res}}/M_0\approx\delta_2(gM_0)^2u_0/\lambda c_t.$$

Assuming $\lambda/gM_0\sim10^{-1}$, $\omega_0\sim10^7$, and $\omega_0u_0/c_t\sim10^{-6}$ we find $\mu_{\mathrm{res}}/M_0\sim10^{-2}$.

If the stress is given at the boundary,

$$\mu_x(z,t)=\frac{\delta_2\omega_0 gM_0^2f}{2\rho c_t^2}\,\frac{(\omega^2-\omega_0^2)\cos(\omega t-kz)+2\omega\omega_0(\lambda/gM_0)\sin(\omega t-kz)}{(\omega^2-\omega_0^2)^2+4\omega^2\omega_0^2(\lambda/gM_0)^2},$$

$$\mu_y(z,t)=-\frac{\delta_2\omega_0 gM_0^2f}{2\rho c_t^2}\,\frac{(\omega^2-\omega_0^2)\sin(\omega t-kz)-2\omega\omega_0(\lambda/gM_0)\cos(\omega t-kz)}{(\omega^2-\omega_0^2)^2+4\omega^2\omega_0^2(\lambda/gM_0)^2},$$

$$u(z,t)=\frac{-f}{\rho\omega c_t}\sin(\omega t-kz).$$

These formulas apply when $\lambda/gM_0\gg(\zeta/c_t^2)\,gM_0/\omega_0$.

The authors wish to express their gratitude to Academician L. D. Landau for a number of valuable comments and to M. I. Kaganov for illuminating discussions.

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Translated by H. Lashinsky
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