

INFLUENCE OF DEFORMATIONS ON OSCILLATION EFFECTS IN METALS AT LOW TEMPERATURES

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The influence of elastic strains on the properties of the electron gas in a metal is considered. It is shown that if the metal contains electron groups with significantly different numbers of electrons, the de Haas — van Alphen effect is very sensitive to deformations of the metal. An explanation is offered for the oscillations of the thermodynamic quantities, which can be produced, by changing the external pressure, in a metal placed in a constant magnetic field.

MANY recent articles deal with experimental investigations of the influence of elastic deformations (particularly uniform compression) in metals on the physical phenomena that are due to the character of the energy spectrum of the conduction electrons.¹⁻⁴ In the present work we shall study certain effects in deformed metals, starting with a semi-phenomenological account of the influence of elastic deformations on the electron spectrum.

1. Following Akhiezer et al.⁵ we shall assume that the influence of elastic deformation on the law of electron dispersion can be taken into account in the form of a small addition to the electron energy in the undeformed metal:*

$$\epsilon^\alpha(\mathbf{p}) = \epsilon_0^\alpha(\mathbf{p}) + g_{ik}^\alpha(\mathbf{p}) u_{ik}, \quad (1)$$

where $\epsilon^\alpha(\mathbf{p})$ and $\epsilon_0^\alpha(\mathbf{p})$ are the energies of the electrons of group α in the deformed and undeformed metal respectively,† u_{ik} is the deformation tensor, and $g_{ik}^\alpha(\mathbf{p})$ is a tensor function of the quasi-momentum \mathbf{p} , characteristic for the given group. In writing (1) it is assumed that the inhomogeneity in the deformation field is small,

*Since the electron energy $\epsilon(\mathbf{p})$ in the metal is a periodic function of \mathbf{p} with the period of the reciprocal lattice, the deformation of a metal changes not only the form of the function $\epsilon(\mathbf{p})$ itself, but also its period. In the case of electron trajectories in quasi-momentum space, located in one or several neighboring elementary cells of the reciprocal lattice, both these changes in $\epsilon(\mathbf{p})$ can be described, in the case of small deformations, by formula (1). If, however, the electron trajectory covers a very large number of reciprocal-lattice cells, then a small change in the period of the deformation cannot be taken into account in the form of a small addition to the function $\epsilon(\mathbf{p})$; however, we shall not be interested henceforth in such trajectories.

†Hereinafter, zero in the subscript will denote the corresponding quality in the undeformed metal.

i.e., that u_{ik} changes substantially over distances that exceed considerably the average de-Broglie wavelength for the electron. Note that if u_{ik} in (1) is taken to be the field of the sound wave, then the second term in (1) becomes the classical analogue of the electron-phonon interaction Hamiltonian. Therefore the $g_{ik}^\alpha(\mathbf{p})$ are connected with the effective cross sections for the absorption and emission of a phonon by an electron.

Starting with (1), it is easy to calculate the spectral characteristics of the electron gas in a homogeneously-deformed metal: the number of states of an electron of group α with energies less than ϵ is $n^\alpha(\epsilon)$ and the total number of states is $n(\epsilon) = \sum_\alpha n^\alpha(\epsilon)$. It turns out that in an approximation linear in u_{ik} we have

$$n^\alpha(\epsilon) = n_0^\alpha(\epsilon) - u_{ik} \bar{g}_{ik}^\alpha \rho_0^\alpha(\epsilon), \quad \rho_0^\alpha(\epsilon) = dn_0^\alpha(\epsilon) / d\epsilon,$$

where \bar{g}^α is the average value of $g^\alpha(\mathbf{p})$ over the equal-energy surface $\epsilon_0^\alpha(\mathbf{p}) = \epsilon$:

$$\bar{g}^\alpha = \oint g^\alpha \frac{d\Omega}{v_0} / \oint \frac{d\Omega}{v_0},$$

(integration over the surface $\epsilon_0^\alpha(\mathbf{p}) = \epsilon$), and also

$$n(\epsilon) = n_0(\epsilon) - u_{ik} \bar{g}_{ik} \rho_0(\epsilon), \quad (2)$$

where

$$\bar{g} = \sum_\alpha \bar{g}^\alpha \rho_0^\alpha(\epsilon) / \rho_0(\epsilon), \quad \rho_0(\epsilon) = \sum_\alpha \rho_0^\alpha(\epsilon).$$

Let us consider an electron gas in a homogeneously-deformed metal of volume V at low temperatures ($kT \ll \zeta$, where ζ is the chemical potential of the electron gas at $T = 0^\circ\text{K}$), when the number of electrons per unit volume can be

considered equal to $n(\zeta)$. From (2), and from the condition that the number of electrons is constant,

$$Vn(\zeta) = V_0 n_0(\zeta_0) = \text{const}$$

we can then determine the change in the chemical potential $\delta\zeta$, due to the deformation of the metal

$$\delta\zeta = -\alpha_{ik} u_{ik}, \quad \alpha_{ik} = \frac{n_0(\zeta_0)}{\rho_0(\zeta_0)} \delta_{ik} - \bar{g}_{ik}, \quad (3)$$

where δ_{ik} is the Kronecker symbol.

Naturally, the coefficients α_{ik} are determined in terms of the functions g_{ik} for all electron groups. If the metal contains electron groups with substantially differing numbers of electrons then, generally speaking, α_{ik} is determined essentially by the groups with large numbers of electrons i.e., $\alpha_{ik} \sim \zeta^{\text{gr}}$, where ζ^{gr} is the chemical potential reckoned from the "bottom" of the energy band of the fundamental group of electrons. Assuming the usual value of ζ^{gr} for metals, the order of magnitude of α_{ik} can be estimated to be 1 eV.

Since α_{ik} is a symmetrical tensor of second rank, the number of its independent components, meaning also the number of independent experimental measurements of $\delta\zeta$ from which α_{ik} can be established, is determined by the symmetry of the crystal. For example, for cubic metals α_{ik} has one independent component ($\alpha_{ik} = \alpha \delta_{ik}$), and for uniaxial metals it has two independent components.

In case of uniform hydrostatic compression at a pressure p , Eq. (3) becomes

$$\delta\zeta = ap, \quad a = \alpha_{ik} s_{ikhll}, \quad (4a)$$

where s_{ikhll} is the tensor of the coefficients of elasticity of the metal (repeated indices are summed).

The value of the scalar coefficient a in (4) can be readily estimated from the following considerations. Were the examined "electron gas in the metal" a free gas, then at constant temperature it would obey the relation $\delta\zeta = v_0 \delta p$, where v_0 is the volume per gas particle. For an electron gas in metal, this relation can be satisfied only in order of magnitude, i.e., $a \sim v_0$. Considering that in a metal there is approximately one electron for each atom, we obtain, in order of magnitude, $a \sim 10^{-23} \text{ cm}^3$. This estimate, as expected, corresponds to the values of the coefficients α_{ik} estimated above.

In the case of uniaxial compression of a metal at a pressure p (for example, along the x_3 axis), formula (3) becomes even simpler:

$$\delta\zeta = b_{33} p, \quad b_{33} = \alpha_{ik} s_{ikh33}. \quad (4b)$$

Finally, we note that the change in the electron work function of the deformed metal follows from (3) and (4). As a result, a contact potential difference arises between two specimens of the same metal, if one specimen is elastically deformed.

2. Consider a metal having one or several groups with an anomalously small number of electrons. For such a group, $\zeta^\beta \ll \zeta^{\text{gr}}$ (ζ^β is the chemical potential, reckoned from the bottom of the β energy band). As is known, the presence of such a group of electrons leads to a very strong de Haas - van Alphen effect. Here the period of the corresponding oscillations due to a change in magnetic field H is determined by the extremal area $S_m^\beta(\zeta^\beta)$ of the intersection between the Fermi surface for the anomalously small group and the plane perpendicular to H .⁶ In the deformed metal, the area cut from the Fermi surface varies both because of change in the form of the equal-energy surfaces and because of the change in ζ . Taking account of both factors that influence the variation of S_m , we readily obtain from (1) the area of the central section of the Fermi surface after deformation.

$$S_m^\beta(\zeta^\beta) = S_m^{0\beta}(\zeta_0^\beta + \delta\zeta) \{1 + \beta_{ik} u_{ik}\}; \quad (5)$$

Here β_{ik} is a coefficient that takes into account the change in the form of the β -equal-energy surface. This coefficient depends on the orientation of the plane of intersection. If the intersection considered is fully contained in one cell or covers only several cells of the reciprocal lattice, then

$$\beta_{ik} = -2\pi m_\beta^0 \bar{g}_{ik}^\beta / S_m^{0\beta}, \quad 2\pi m_\beta^0 = dS_m^{0\beta} / d\zeta_\beta,$$

where \bar{g}^{β} is the average value of $g^\beta(\mathbf{p})$ along the trajectory which encloses the area $S_m^{0\beta}(\zeta_0^\beta)$ in the quasi-momentum space. Generally speaking, $\beta_{ik} \sim 1$.

Since, in order of magnitude, $\delta\zeta \sim u_{ik} \zeta^{\text{gr}}$, we can use in (5)

$$\delta\zeta / \zeta_0^\beta \sim u_{ik} (\zeta^{\text{gr}} / \zeta_0^\beta).$$

For anomalously small electron groups we usually have $\zeta^{\text{gr}} / \zeta_0^\beta \sim 10^2$ to 10^3 , and therefore for deformations $u_{ik} \sim 10^{-2}$ to 10^{-3} it turns out that $\delta\zeta \sim \zeta_0^\beta$, i.e., the change in the chemical potential is commensurate with ζ_0^β . In this case it follows from Eq. (5) that

(a) with varying H , the oscillation period changes, because of $\delta\zeta$, by an amount comparable with the period itself, in spite of the fact that the deformation, and hence the relative changes of the crystal-lattice constants, is small;

(b) anisotropic changes in the period of the oscillation determined by the second term in the

curly brackets of (5) give a relatively small contribution to the total change in the period.

This means that of the two indicated factors that influence the oscillations in a deformed metal, the change in the chemical potential of the electron gas is the more significant. The change in the form of the β Fermi surface affects very little the oscillations due to the β group of electrons.

If the electron dispersion in the undeformed metal is not quadratic, then the effective mass m_β^0 of the electron depends in general on the energy, and should therefore be considerably changed by the above deformations. This leads, in particular, to a considerable change in the temperature dependence of the amplitude of the oscillation that is determined by the quantity m_β^0 .

A similar type of period variation and a similar temperature dependence of the deformation were observed for single crystals of zinc by Dmitrenko, Verkin, and Lazarev,^{2,3}

Let us note that simultaneous measurements of the changes in the oscillation periods, i.e., δS_m , and of the effective mass of the electron m^0 in one form of deformation makes it possible to determine $\delta\zeta$, and consequently the coefficients a or b in (4). With several loads it is possible to determine the coefficient α_{ik} in (3). In fact, we have

$$\delta S_m^0 \approx 2\pi m_\beta \delta\zeta = 2\pi [m_\beta^0(\zeta_0) + \delta m_\beta] \delta\zeta, \quad (6)$$

where δm_β is the average change in the electron mass due to the deformation. Relation (6), together with (3) and (4), solves the above problem.

If the coefficient a (or b) is known, it is possible to establish the energy dependence of S_m^0 , over a considerable range of energies, from the dependence of the period on p . This energy dependence makes it possible to establish the law of electron dispersion in the anomalously small group for a sufficiently large region of the Fermi boundary energy.

Finally, since $S_m^{0\beta}(\epsilon)$ is a monotonic function of energy for closed equal-energy surfaces, then the following is true:

(a) In the presence of anomalously small groups of carriers of opposite polarity, the corresponding periods will change upon deformation in different directions (increase for particles of one sign and diminish for those of the opposite sign).

(b) Changing the sign of the pressure in uniaxial loading, as can be seen from (4b), leads to a change in the sign of δS_m^0 for anomalously small groups (in the case of a quadratic dispersion law, the effect should be symmetrical, accurate to terms $\beta_{ik} u_{ik}$).

A change in the sign of δS_m^0 was observed by Verkin and Dmitrenko upon reversal of the uniaxial loading of single crystals of zinc.⁷

3. Equation (5) leads also to an explanation of another possible oscillation effect, namely oscillations of thermodynamic quantities in a constant magnetic field due to changes in the external load applied to the metal.* Actually, at low temperatures, oscillations produced in any thermodynamic quantity by the β group of electrons are determined by a factor of the form⁶

$$\sin(cS_m^0(\zeta_\beta)/e\hbar H + \varphi), \quad (7)$$

where φ is a certain practically-constant phase.

A change of 2π in the argument of (7) corresponds to one period of oscillation. As can be seen from (5) and (7), this change can result, at constant H , from a change in ζ due to the deformation of the metal. In the presence of an anomalously small group of electrons, it is easy to determine from (5) and (7) what change in chemical potential corresponds to one period of oscillations:

$$\Delta\zeta = 2\pi e\hbar H / c(dS_m^0/d\zeta) = e\hbar H / m_\beta c = \mu_\beta H. \quad (8)$$

Naturally, (8) coincides with the distance between the electron energy levels in the magnetic field. Formula (8) becomes quite obvious in the case of a quadratic dispersion law, when the argument of (7) contains $2\pi\zeta_\beta/\mu_\beta H$.

In the case of uniform hydrostatic compression, the change in pressure p , corresponding to the oscillation period, is

$$\Delta p = \Delta\zeta/a = \mu_\beta H/a. \quad (9a)$$

In uniaxial compression along the x_3 axis, the period of the pressure oscillations is determined by the formula

$$\Delta p = \mu_\beta H/b. \quad (9b)$$

Formulas (8) and (9) show (a) that the period of oscillations due to changes in applied pressure p is proportional to the magnetic field H , and (b) that the anisotropy of the period and its dependence on p are determined by the anisotropy and pressure dependence of the effective mass m averaged over the period.

Measurement of the oscillation periods due to a change in pressure in weak magnetic fields, when the energy period of $\Delta\zeta = \mu_\beta H$ is small, makes it possible to determine directly the effective mass of the electrons of anomalously small groups and its dependence on p (meaning also on the energy).

The possibility of experimental observation of

*That such oscillations can be observed is suggested also by Alekseyevskiy, Brandt, and Kostina.⁴

the above oscillations is confirmed by the fact in references 2 and 3, at $p = 1500$ atmos, the change $\delta\zeta$ is comparable with ζ_β for an anomalous group of electrons in zinc, while in weak fields usually $\mu\beta H \ll \zeta_\beta$.*

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*All the calculations of Secs. 2 and 3 of this article were made assuming $\mu H \ll \zeta$. If this more stringent inequality is satisfied we can, in particular, disregard the quantum oscillations of ζ .

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⁷B. I. Verkin and I. M. Dmitrenko, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 291 (1958), Soviet Phys. JETP **8**, 200 (1959) (this issue).

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ON A POSSIBLE LIMIT ON THE APPLICABILITY OF QUANTUM ELECTRODYNAMICS

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Processes that can compete with electromagnetic processes at high energies are considered. It is shown that these can be processes associated with four-fermion interactions.

1. INTRODUCTION

It has been shown in reference 1 that the application of the present method of renormalization in quantum electrodynamics leads to a difficulty in principle — to the vanishing of the renormalized charge. Although objections have been raised² against the unconditional cogency of the proof, nevertheless it seems to be quite convincingly demonstrated that there are difficulties in prin-

ciple in the range of energies E defined by the condition $\alpha \ln(E/mc^2) \sim 1$ ($\alpha = e^2/\hbar c$). The typical length corresponding to this energy, $l \sim (\hbar/mc) e^{-3\pi/\alpha}$, lies far beyond the limit of the gravitational radius of the electron, as was first shown in reference 3. The limiting energy itself is enormously large ($E_0 \sim mc^2 e^{3\pi/\alpha}$).

It can therefore be expected that in actual fact the limits of the applicability of the present electrodynamics will show up considerably earlier,