the above oscillations is confirmed by the fact in references 2 and 3, at $p = 1500$ atmos, the change $\delta\zeta$ is comparable with ζ_β for an anomalous group of electrons in zinc, while in weak fields usually $\mu_\beta H \ll \zeta_\beta$.*

In conclusion, I use this opportunity to thank I. M. Lifshitz for counsel and discussions, and also B. I. Verkin and I. M. Dmitrenko for discussing the results of the work.

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*All the calculations of Secs. 2 and 3 of this article were made assuming $\mu H \ll \zeta$. If this more stringent inequality is satisfied we can, in particular, disregard the quantum oscillations of ζ.

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ON A POSSIBLE LIMIT ON THE APPLICABILITY OF QUANTUM ELECTRODYNAMICS

D. I. BLOKHINTSEV

Joint Institute for Nuclear Research

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Processes that can compete with electromagnetic processes at high energies are considered. It is shown that these can be processes associated with four-fermion interactions.

1. INTRODUCTION

It has been shown in reference 1 that the application of the present method of renormalization in quantum electrodynamics leads to a difficulty in principle — to the vanishing of the renormalized charge. Although objections have been raised[2] against the unconditional cogency of the proof, nevertheless it seems to be quite convincingly demonstrated that there are difficulties in principle in the range of energies E defined by the condition $\alpha \ln(E/mc^2) \sim 1$ ($\alpha = e^2/\hbar c$). The typical length corresponding to this energy, $l \sim (\hbar/mc)e^{-3\pi/\alpha}$, lies far beyond the limit of the gravitational radius of the electron, as was first shown in reference 3. The limiting energy itself is enormously large ($E_0 \sim mc^2 e^{3\pi/\alpha}$).

It can therefore be expected that in actual fact the limits of the applicability of the present electrodynamics will show up considerably earlier, for example through a possible change of the space-time structure in regions of space and time that are small, but still markedly larger than l.

There is, however, another limitation on the meaningfulness of quantum electrodynamics, more accessible to theoretical analysis.

Besides the purely electrodynamic interactions of photons, electrons, and positrons there occur processes involving mesons and nucleons. These processes can be brought about in a purely electrodynamic way, for example by the interaction of a photon with an electron.

If it should turn out that the contribution of these nonelectromagnetic processes exceeds that of the electromagnetic processes, then we should lose the possibility of dealing with pure electrodynamics without including other types of interactions in an essential way. In particular, beginning at a certain energy E_{cr} it would become meaningless to think of an expansion in powers of $e^2/\hbar c$.

We shall show that the weak four-fermion Fermi interaction can be a competing interaction of this kind. The validity of this interaction in the high-energy region has not been experimentally verified, and various theoretical doubts as to the applicability of this interaction for energies $E \gg mc^2$ can be put forward. We shall, however, start from the assumption that this interaction can be used right up to very high energies, and we shall consider the consequences following from this assumption.

A physical peculiarity of the pure fermion interactions is the fact that the matrix elements of these interactions do not fall off with increase of the energies of the fermions taking part in the process, whereas the matrix elements of processes that involve bosons (photons, π mesons and K particles) decrease with increasing energy of the bosons. The reason for this is that with increasing boson energy the boson field falls off like $k^{-1/2}$:

$$\Phi_k = \sqrt{\hbar/2k}e^{ikx}b_k + \text{conjugate};$$

(here k is the momentum of the boson and b_k is the creation operator of the boson), whereas the fermion field remains constant with increasing energy of the fermion:

$$\psi_k \sim u_k e^{ikx} a_k + \text{conjugate},$$

(u_k is a spinor amplitude, and a_k is the creation operator of the fermion).

In what follows we shall show that because of this peculiarity of the fermion interactions they become important in electromagnetic processes considerably before the logarithmic limit $E \sim mc^2 e^{3\pi/\alpha}$ is reached.

2. THE FERMION-ELECTROMAGNETIC INTERACTION

Let us consider the process of interaction of a photon (k) with an electron (e) leading to the formation of a μ meson (μ) and two neutrinos (ν, $\tilde{\nu}$):

$$k + e \rightarrow \mu + \nu + \tilde{\nu}. \quad (1)$$

Such a process will be described by the interaction Lagrangian W:

$$W = eW_e + eW_\mu + gW_{\mu e\nu}, \quad (2)$$

where $eW_e = (J_e A)$ is the interaction of the electron (J_e is the electron current) with the electromagnetic field (A is the vector potential); eW_μ has the same meaning for the μ meson. Finally, $gW_{e\mu\nu}$ is the four-fermion interaction of electron, μ meson, and neutrino; $g = \hbar c\Lambda_0^2 \approx 10^{-49}$ erg-cm^3 is the Fermi constant ($\Lambda_0 = 6 \times 10^{-17}$ cm), and

$$W_{e\mu\nu} = (\bar{\psi}_e O_1 \psi_\mu)(\bar{\psi}_\nu O_2 \psi_\nu) + \text{conjugate}$$

Here ψ_e, ψ_μ, ψ_ν are the spinor fields of electrons, μ mesons, and neutrinos, respectively, and O_1 and O_2 are certain spinor operators.

The total effective cross-section for the process (1) will be:

$$\sigma_\mu = \frac{2\pi}{\hbar c}\int |W_{af}|^2 \frac{p_\nu^2 dp_\nu d\Omega_\nu \tilde{p}_\nu'^2 dp_\nu' d\Omega_\nu'}{(2\pi\hbar)^6 dE_f}, \quad (3)$$

where W_{af} is the matrix element of the interaction energy (2) for the process (1), p_ν, $\tilde{p}_\nu$ are the momenta of the neutrino and antineutrino, and E_f is the energy of the final state. The structure of this matrix element is such that in the first nonvanishing approximation it is given by:

$$W_{af} = eg\sum_i \left\{ \frac{(a|W_e|c)(c|W_{e\mu\nu}|f)}{E_0 - E_c} + \frac{(a|W_{e\mu\nu}|c)(c|W_\mu|f)}{E_0 - E_c'} \right\}, \quad (4)$$

where E_0 is the energy of the initial state and E_c that of the intermediate state. In the center-of-mass system of the photon and electron $E_0 - E_c \sim \hbar ck$ (k is the wave vector of the photon); $(a|W_e|c) \sim k^{-1/2}$, $(c|W_\mu|f) \sim k^{-1/2}$. Therefore $|W_{af}|^2 \sim e^2 gk^{-3}$. The weight factor in Eq. (3) is proportional to k^5. Thus the total cross-section is

$$\sigma_\mu \approx \alpha\Lambda_0^4 k^2 F, \quad (5)$$

where F is a factor of the order 1 which depends weakly on k.*

In a similar way we can consider the process of collision of two electrons with simultaneous transformation into two mesons,

$$e' + e'' \rightarrow \mu' + \mu''. \qquad (1')$$

The differential cross-section (in the center-of-mass system) for this process will be

$$d\sigma_{\mu\mu} \cong \Lambda_0^8 q^4 p^2 F \, d\Omega, \qquad (6)$$

where q is the momentum transfer and p is the initial momentum of the electron, measured in reciprocal-length units.

On the other hand, the cross-sections for the purely electromagnetic processes are given by ($\alpha = e^2/\hbar c$):

$$\sigma_c = \frac{\pi\alpha^2}{2k^2}\left(\ln\frac{4k^2}{k_c^2} + \frac{1}{2}\right) \qquad (7)$$

for the Compton effect,

$$d\sigma_{ee} = \alpha^2 (p^2/q^4)\, d\Omega \qquad (8)$$

for the elastic collision of electrons,

$$\sigma_p = \frac{28}{9}\frac{\alpha^3}{k_c^2}\left(\ln\frac{4k^2}{k_c^2} - 3.5\right) \qquad (9)$$

for pair production (here $k_c = mc/h$), and

$$\sigma_\gamma = \frac{4\alpha^3}{k_c^2}\left|\ln\frac{4k^2}{k_c^2} - 3.5\right| \qquad (10)$$

for bremsstrahlung from electron collisions.

A comparison of these cross-sections with those for the mixed processes (1) and (1′) shows that

$$\sigma_\mu > \sigma_c \quad \text{for} \quad k \gg \alpha^{1/4}/\Lambda_0; \qquad (11)$$

$$\sigma_\mu > \sigma_p \quad \text{for} \quad k \gg \alpha^{1/2}(\alpha/\Lambda_0 k_c)/\Lambda_0; \qquad (12)$$

$$d\sigma_{\mu\mu} > d\sigma_{ee} \quad \text{for} \quad q \gg \alpha^{1/4}/\Lambda_0; \qquad (13)$$

$$\sigma_{\mu\mu} > \sigma_\gamma \quad \text{for} \quad q \sim p \gg (\alpha^3/k^2\Lambda_0^2)/\Lambda_0. \qquad (14)$$

As can be seen from these inequalities, if the four-fermion interaction can be regarded as applicable in the energy region $k > 1/\Lambda_0$, then the processes with the production of neutrinos and μ mesons are more intense than the purely electromagnetic processes. For this the corresponding energies of the photons or electrons in the center-of-mass system must be larger than $\hbar c/\Lambda_0 \sim 2$ Bev. This is an enormous energy, but it is nevertheless much smaller than that from the logarithmic formula.

It must be noted that the production of pairs of nucleons and mesons will play a much smaller role, since their production cross-section will be smaller by a factor $(m/M)^2$ than that for the production of electron-positron pairs.

Processes with production of neutrinos and boson mesons will also become important only at higher energy because of the above-mentioned difference in the behaviors of the matrix elements of bosons and fermions.

Thus the fermion interaction may be the one which limits the range of applicability of electrodynamics to dimensions larger than Λ_0. For smaller lengths, and consequently for energies larger than $\hbar c/\Lambda_0$, there is no sense at all in studying electrodynamics without directing attention to processes with μ mesons and neutrinos and to the Fermi constant g as well as $e^2/\hbar c$.

*These qualitative conclusions are confirmed by more detailed calculations carried out by Doctor M. Meier (Romania), to whom the writer expresses his thanks.

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