AN ESTIMATE OF THE ENERGY OF SHOWER-PRODUCING PARTICLES WITH ALLOWANCE FOR THEIR ENERGY SPECTRUM

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THE importance of accounting for the energy spectrum of shower-producing particles in estimating their energy from the angular distribution of the secondary particles has been stressed in a number of discussions.* One can account for the spectrum without recourse to any theory of multiple-meson production.

Let θ^* and θ represent the angle of π meson emission in the c.m. and laboratory systems respectively. We have then

$$\gamma_c \tan\theta_i = \sin\theta^*/(\cos\theta^* + \beta_c/\beta^*),$$

where β_c/β^* is the ratio of the velocity of the c.m.s. to the velocity of the meson in that system, and $\gamma_c = (1-\beta_c)^{-1/2}$. Calculating hence $\gamma + 1 = 2\gamma_c^2$, taking the logarithm and summing over all shower particles, we obtain

$$\ln(\gamma+1) = \frac{1}{n_s}\sum_{i=1}^{n_s} \ln\frac{2}{\tan^2\theta_i} + \frac{1}{n_s}\sum_{i=1}^{n_s} \ln\left[\frac{\sin\theta_i^*}{\cos\theta_i^* + \beta_c/\beta_i^*}\right]^2$$

Let us introduce the notation

$$x_i = \ln\frac{2}{\tan^2\theta_i}, \qquad \mu = \ln(\gamma+1),$$

$$U_i = -\ln\left[\frac{\sin\theta_i^*}{\cos\theta_i^* + \beta_c/\beta_i^*}\right]^2.$$

Let us assume further that, corresponding to the assumptions of Castagnoli et al.,[1] the U_i are random quantities. We then have

$$U_i = x_i - \mu = \ln(2/\tan^2\theta_i) - \ln(\gamma+1).$$

Let us first assume, for the sake of simplicity, that $\beta_c/\beta = 1$. Let us assume, besides, that the mesons are emitted symmetrically in the c.m.s. This means that the angle θ^* occurs as often as does $\pi - \theta^*$. We have then $\gamma + 1 = 2$ and the average of U_i equals zero, i.e., the random U_i are distributed uniformly about a zero mean value.

We have investigated, furthermore, the distribution of the experimentally measured values $V_i = x_i - \bar{x}$ for a number of showers. The analysis shows that in a number of cases the distribution law of random V_i coincides to a satisfactory degree with the normal distribution law

$$\varphi(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(x-\bar{x})^2}{2\sigma^2}\right\},$$

where σ^2 is determined for each shower from the measurements of the angles of the shower particles with respect to the direction of the primary particle. Basically, a normal distribution of the V_i is not a necessary condition. If we assume that $\bar{x} \approx \mu$, then $V_i \equiv U_i$, which fully corresponds to the physical sense of U_i.

The quantity $\bar{x}$ is itself random and subject to statistical fluctuations; in other words, it varies as the n measurements, on which it is based, are repeated. According to the theory of probability, $\bar{x}$ has a normal distribution for all n, with a mean value μ and a standard deviation $\sigma' = \sigma/\sqrt{n}$. We shall call the quantity

$$\varphi(\bar{x}) = \frac{1}{\sqrt{2\pi}\sigma'}\exp\left\{-\frac{[\bar{x}-\mu(\gamma)]^2}{2\sigma'^2}\right\}$$

the probability $P(\bar{x}/\gamma)$ that an event $\bar{x}$ takes place after the event $\mu(\gamma)$. In other words, it represents the probability of observing $\bar{x}$ for a given value γ of the energy of a shower-producing particle.

On the other hand, the spectrum of shower-producing particles is given by the expression $P(5) = A\gamma^{-2.7}$. We calculate

$$P(\gamma/\bar{x}) = P(\gamma)P(\bar{x}/\gamma)/\int P(\gamma)P(\bar{x}/\gamma)\,d\gamma,$$

and obtain from the condition $\partial P(\gamma/\bar{x})\,\partial\gamma = 0$

$$2.7\sigma'^2(\gamma+1) = \gamma[\bar{x} - \ln(\gamma+1)]$$

$$\text{or } \ln(\gamma+1) \approx \bar{x} - 2.7\sigma'^2.$$

This means, for example, that the energy of a shower-producing particle, found equal to 5500 Bev without accounting for the spectrum, must be reduced to 4500 Bev if $\sigma^2 \approx 1$.

We have assumed, in the above, that $\beta_c/\beta = 1$. It can be easily seen that this assumption is unnecessary. It has been shown by several authors[2,3] that the introduction of the energy spectrum leads to a change of $\ln(\gamma+1) = \ln B^2 + \bar{x}$. In consequence, in all previous formulae, one has to substitute $\bar{x}$ for $\bar{x} + \ln B^2$. The energy of the primary particle is then found from the relation

$$\ln(\gamma+1) = \bar{x} + \ln B^2 - 2.7\,\sigma^2/n_s;$$

the value of σ^2 is determined experimentally from the usual formula $\sigma^2 = \frac{1}{n_s}\sum(x-\bar{x})^2$. It is then immaterial whether the distribution of x is normal or not. Such an estimate accounts for the energy spectrum $E_0^{2.7}$ of shower-producing particles, and for the energy spectrum of secondary shower particles (through the factor B). The factor B depends also on the angular distribution of shower particles. (cf. references 2 and 3).

More detailed data on actual energies of primary particles for individual showers and fluctuation curves will be given in a work devoted to the study of showers detected in emulsions at high altitudes.

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*The necessity of this has been demonstrated by N. L. Grigorov. A number of important observations has been made by G. P. Zhdanov.

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[1] Castagnoli, Cortini, Franzinetti, Manfredini, and Moreno, Nuovo cimento **10**, 1939 (1953).

[2] W. Heisenberg, Kosmische Strahlung, Berlin-Göttingen-Heidelberg (1953).pp. 563-564.

[3] L. v. Lindern, Z. Naturforsch IIa, 5, 340 (1956).

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CONTRIBUTION TO THE THEORY OF THE POMERANCHUK EFFECT IN He^3

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POMERANCHUK[1] predicted that the melting curve of He^3 would have a minimum on the p-T diagram, and that below this minimum the heat of melting would be negative. Since this effect has been observed experimentally,[2] it is interesting to examine this problem by using the thermodynamic functions calculated on the basis of the Fermi-liquid model, as proposed by Landau.[3] Our purpose is to reconstruct the left branch of the melting curve from the experimentally-known portion of the curve above the minimum.

The equation relating the two melting temperatures, for equal pressure, is of the form

$$\Phi^{I}(p, T_1) - \Phi^{I}(p, T_2(p)) = \Phi^{II}(p, T_1) - \Phi^{II}(p, T_2(p)), \tag{1}$$

where $\Phi^{I}(p, T_1)$ and $\Phi^{II}(p, T_1)$ are the thermodynamic potentials below the minimum point for the liquid and solid phases respectively, and $\Phi^{I}(p, T_2(p))$ and $\Phi^{II}(p, T_2(p))$ are the corresponding quantities above the minimum point.

It is known that above 0.5°K the entropy of liquid He^3 is essentially of spin origin. On the other hand, the spin entropy should increase with increasing pressure, owing to the increase in the exchange interaction that contributes to the parallel orientation of the spins[3,4] and competes with the Fermi tendency towards the anti-parallel spin arrangement. From the equality $(\partial S/\partial p)_T = -(\partial V/\partial T)_p$ we see that the coefficient of expansion is negative in that region of temperatures, in which $(\partial S/\partial p)_T > 0$. Consequently, the density of liquid He^3 should have a maximum, as indeed was observed experimentally[5] (the temperature of the maximum is $T_0 \approx 0.4$°K). In view of the fact that the density of liquid He^3 has a maximum near the minimum point of the pT diagram, it is easy to show that the effect of the change in volume can be neglected. Assuming that the coefficient of expansion of solid He^3 is of the same order of magnitude as that of He^4, the change in volume can also be neglected in the solid phase. Equation (1) can then be replaced by

$$F^{I}(T_1, V) - F^{I}(T_2,(p), V) = F^{II}(T_1, V) - F^{II}(T_2(p), V), \tag{2}$$

where F(T, V) is the free energy.

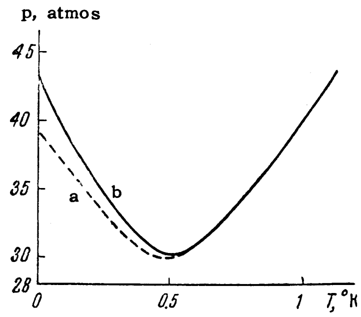

FIG. 1

Using the results of Khalatnikov and Abrikosov,[4] it is possible to calculate the free energy of liquid He^3 for two possible forms of the spectrum

$$\varepsilon(p) = p^2/2m, \tag{3a}$$
$$\varepsilon(p) = (p - p_0)^2/2m. \tag{3b}$$

The calculations yield, respectively

$$F^{I} = RT\{-2I_{3/2}/3I_{1/2} + \ln A\}, \tag{4a}$$
$$F^{I} = RT\{-2I_{1/2}/I_{-1/2} + \ln A\}, \tag{4b}$$

where