THE π-MESONIC ATOM AND CORRECTIONS TO THE DISPERSION RELATIONS

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RECENTLY there appeared a series of papers[1] devoted to various corrections to the dispersion relations. All those effects due to the mass difference between the neutral and the charged mesons and to the electromagnetic interaction proved to be too small to explain the Puppi-Stanghellini puzzle.[2] This consists in the fact that the dispersion relations for the process $\pi^- + p \rightarrow \pi^- + p$ agree with experiment with the choice of $f^2 = 0.04$ for the coupling constant of the nucleon-meson interaction up to the resonance, whereas $f^2 = 0.08$ after the resonance. Such a marked energy dependence of f^2 contradicts the results of other methods for the determination of f^2, which give the value $f^2 = 0.08$ for the above energy region. The aim of this paper is to estimate the magnitude of the corrections due to the π mesonic atom.

The appearance of a state with only a nucleon and a photon in the expansion of the antihermitian part of the scattering amplitude in terms of a complete system of functions does not fully account for the electromagnetic interaction. In the case of π^--p scattering we have to consider the π mesonic atom. For this purpose the forward scattering amplitude $f_-(\omega)$ for the process $\pi^- + p \rightarrow \pi^- + p$ has to be investigated more carefully. It can be represented as the sum of three terms: (1) the Rutherford amplitude, (2) a purely nuclear term, and (3) an interference term of the preceding two. The last term is small for energies of some 10 Mev, since it is proportional to α/η (α is the fine-structure constant, $\eta = p/m_\pi c$). But in the dispersion relations it is important to know the scattering amplitude for small $\omega - m_\pi > 0$, in which case the interference term is not small. We note that the dispersion relations are strictly proven[3] only for the nuclear part of the scattering amplitude. Therefore we have to regard the interference term as a correction to the usual dispersion relations for the processes $\pi^\pm + p \rightarrow \pi^\pm + p$.

The interference term contains poles corresponding to the bound states of the system π^-, p. The correction to the dispersion relations due to these states is equal to

$$\Delta\left(\frac{D^b_\mp}{r_0}\right) = \pm\frac{\eta\eta^b}{\omega - 1}\left[\frac{8\pi^2\alpha}{9}(4a_1^2 + a_3^2 + 4a_1a_3)\right.$$
$$\left.\times\sum\frac{(-1)^n}{n!\,(n+1)!} + O(\alpha)\right],$$

where . $D_\pm = \mathrm{Re}\, f_\pm(\omega)$, $r_0 = h/m_\pi c$, and the index b implies taking the value in the center-of-mass system. The sum arises from the inclusion of all bound states. These states are considered stable, the finite level width effecting only the terms $0(\alpha)$. The quantities a_1, a_2 are equal to α_1/η, α_3/η and are taken from the paper of Orear.[4] Numerical calculations show that the correction coming from the π mesonic atom is a small effect and accounts for only 4% of the deviation of the coupling constant f^2 in the region of 120 Mev.

In conclusion I express my gratitude to D. V. Shirkov for helpful discussions and interest in this work.

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EFFECT OF UNIAXIAL ELASTIC DEFORMATIONS ON THE MAGNETIC PROPERTIES OF ZINC CRYSTALS AT LOW TEMPERATURES

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UNIFORM compression of zinc crystal at an approximate pressure of 700 kg/cm^2 increases the period of oscillations of its magnetic susceptibility by 40 to 50%, and reduces correspondingly the number of mobile charge carriers in the anomalously small group by more than 40%.[1] Thanks to the anisotropy of the compressibility of the zinc crystal, uniform compression leads to a reduction in the ratio c/a of the crystal axis and thus makes its lattice structure closer to that corresponding to hexagonal dense sphere packing.

We considered it important to investigate the influence of uniform elastic deformations of the lattice on the de Haas — van Alphen effect in zinc crystals, in order to establish above all the connection between the change in the number of charges in the anomalously small group and the sign of variation of c/a.

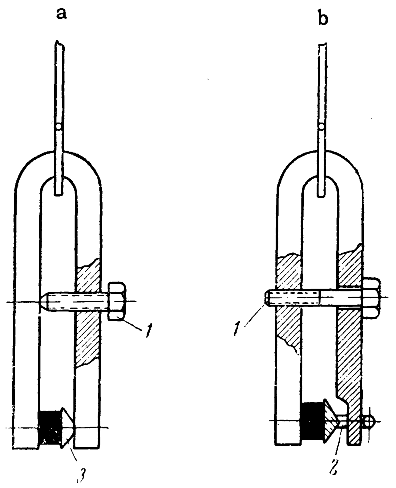

FIG. 1. Clamping springs for the investigation of the anisotropy of magnetic properties of uniaxially elastically-deformed crystals.

Figure 1 shows an instrument that produces elastic uniaxial compression or tension in zinc crystals at low temperatures. Special springs made of very pure beryllium bronze were calibrated at low temperature. Having determined the elastic constant of such a spring it became possible to establish by means of screw 1, the fixed value of spring tension or compression required to apply a specified load to the crystal. Crystals of the required size were pricked out of a large cylindrical zinc crystal, freely grown in a vessel of pyrophyllite on a plate in which a temperature gradient was produced. One spallation surface of one specimen investigated was glued to the smooth arm of the clamp spring, and the second surface was glued either to a small thimble 2 (for uniaxial tension of the crystal — Fig. 1b) or to a knife edge 3 with a polished base (for uniaxial compression of the crystal — Fig. 1a). The clamping spring was attached together with the crystal to the suspension system of the usual instrument used to investigate the anisotropy of magnetic properties.[2]

TABLE I

Uniaxial compression			Uniaxial tension		
$\theta°$	$T\cdot10^4$, Oe^{-1}		θ	$T\cdot10^4$, Oe^{-1}	
	$p=0$	$p\approx100$kg/cm^2		$p=0$	$p\approx20$kg/cm^2
20	0.492	0.520	20	0.506	0.494
30	0.455	0.478	30	0.465	0.454

We first investigated the anisotropy of the magnetic properties of the unloaded zinc crystal. The prestressed spring was then unloaded by rotating screw 1 and the zinc crystal was subjected respectively to either uniaxial compression or uniaxial tension along the hexagonal axis c. We then investigated, at the same crystal orientation in the field, the anisotropy of the magnetic properties of the uniaxially deformed zinc.

During the investigation the orientation of the zinc crystals in the field was such that the axis of suspension of the spring and of the specimen was perpendicular to the hexagonal and binary axes of the crystal. The load does not exceed 100 kg/cm^2 in uniaxial compression or 20 kg/cm^2 in uniaxial tension.

The results of the measurement (the periods T of the oscillations of the susceptibility due to the small group of charges) are given in Table I (θ is the angle between the field vector **H** and

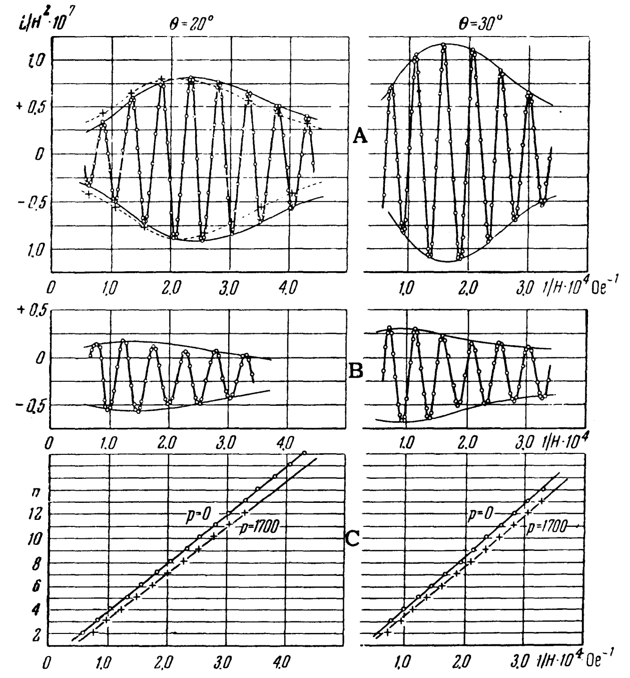

FIG. 2. Curves of L_y/H^2 (which is proportional to $\Delta\chi = \chi_3 - \chi_2$) vs. 1/H for free and uniaxially-compressed zinc crystals; T = 4.2°K; A — free crystal (p = 0); o — prior to loading, + — after removal of load; B — uniaxially-compressed crystal (p ~ 100 kg/cm^2); C — n(1/H) for free and uniaxially-compressed crystal at θ = 20 and 30 degrees.

the principal axis of the crystal).

In the graphs of Fig. 2, the influence of uniaxial elastic compression of the zinc crystals is illustrated by the shapes of the curves $\Delta\chi(1/H)$. The same figure shows the straight line $n(1/H)$, which determine the period of oscillation of susceptibility of a free and compressed crystal at $\theta = 20$ and $30°$.

The uniform compression of zinc crystals in the region $\theta \leq 30°$ increases the period of susceptibility oscillations by 4 to 5%. With increasing θ, the oscillation period increases less, at $\theta = 70°$ it stays unchanged, and at $\theta = 80°$ the period of oscillation even diminishes somewhat ($\Delta T/T \sim 1\%$).

Uniaxial tension of zinc crystals in the region $\theta \leq 30°$ decreases the period of the susceptibility oscillations by 2 or 3%. The amplitude of the susceptibility oscillations diminishes several-fold in uniaxial elastic deformation of the crystal. After removing the load the period and the amplitude of the oscillations return to their initial values.

The investigations performed have thus shown that reducing c/a for elastic deformation of the crystal leads to an increase in the periods of the susceptibility oscillations, leading in turn to an increase in the number of charges in the anomalous group. On the contrary, an increase in c/a causes a reduction in the oscillation periods and a corresponding increase in the number of charges in this group.

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INFLUENCE OF MULTIPLE SCATTERING ON THE DEVELOPMENT OF HIGH-ENERGY ELECTRON-PHOTON CASCADES IN LEAD

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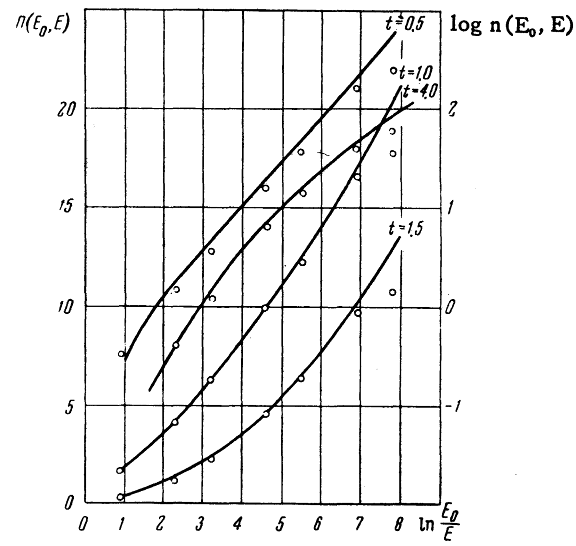
THE influence of the multiple scattering of electrons on atomic nuclei on the processes of bremsstrahlung and pair production was investigated in recent years by several authors.[1-4] It was found that the cross-sections for the above processes for particle energies $> 10^{11}$ ev are considerably different from the Bethe-Heitler cross sections.[5] For an experimental test of the theoretical predictions, it is necessary to recalculate the development of cascade showers using the new values of the cross-sections for the basic processes in the early stages of development.

We carried out calculations of the longitudinal development, for two cascade units, of 154 showers initiated by 10^{12} ev electrons, and of 40 showers initiated by an electron or a photon in lead, for four cascade units. The calculations were carried out by the Monte-Carlo method using the "Strela" electronic computer. The cross sections for bremsstrahlung and pair production were taken from reference 4, accounting for the fact that the refraction index of the medium is different from unity. Ionization losses were neglected.

The circles in the figure represent the mean energy spectrum of electrons at the depth of 0.5,



Mean energy spectra of electrons at various depths. Solid curves are taken from reference 5; circles represent calculations of the authors. The scale indicated refers to the curves for t = 0.5, 1.0, and 1.5. In order to obtain numerical values the ordinate of the first curve should be multiplied by 10^{-1}, of the second – by 3×10^{-1}, and of the third – by 1. The curve for t = 4 is represented in a logarithmic scale. The latter curve is calculated according to reference 5, since that given in reference 4 is subject to a ~ 30 % error.